Circuit Theory of
Electron Tubes
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EE 607

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Chapter I

Diodes, Triodes, and Equivalent Circuits

Introduction

The study of electron tubes has become an important and an essential part of any curriculum in electrical engineering. The uses of electronic devices are so numerous and so varied in complicated communication and industrial control systems that it becomes necessary to consider the electron tube in rather general terms as a circuit element of certain particular properties. These circuit properties of electron tubes may be analyzed in their dependence upon the design of the component elements of the tube, or they may be determined for practical application by a consideration of the tube in its envelope as a "black box" from which extend certain terminals at which energy, force, voltage, etc. may be impressed and the resulting response measured. The latter point of view has the distinct advantage of permitting the intelligent use of the electron tube as a circuit component without a prior knowledge of its interior complexities.

Engineering consists of a combination of the processes of analysis and synthesis. Analysis usually precedes synthesis, but it is of extreme importance that analysis is carried only so far as may be required for intelligent and efficient synthesis. For example, one may analyze a building designed to be a home in terms of its functional parts -- kitchen, living room, bedrooms, bath, laundry, furnace room, and so on. The analysis may proceed much farther, however, to include the concrete block, brick, cement, lumber, nails, pipes, and other components. The degree to which the analysis is carried depends upon the purpose to be served. Most electrical engineers who use electron tubes are concerned with the utilization of these tubes as component parts of systems. For such use, it is unnecessary to carry analysis to the level of complexity which becomes essential where the synthetic product is the tube itself. The tube engineer and the physicist must analyze the electron tube in terms of its dependence upon the materials of which it is built, the physical nature of these materials, and the effect of electric and magnetic field forces upon the motion of electrons within the device. Such analysis is complicated and difficult and may be postponed until the student has become familiar with the circuit behavior of electron tubes as determined at the available terminals of the tube.

1-1. Electron Tube Definitions.

Before proceeding with the circuit analysis of electron tubes, it is important to define certain terms in common use. It is assumed that the word "electron" has a definite meaning to the student from previous study or reading in physics, but it is not proposed to define the electron in this text. The observed properties of the electron and of ions will be presented in a later chapter when it appears desirable to carry the analysis to the ultimate components of electron tubes.

The standard definitions proposed by the Institute of Radio Engineers for several important terms are given in the following list:

Electron Tube. An electron tube is a device in which conduction of electricity takes place between two or more electrodes through an enclosed vacuum, gas, or vapor.

Vacuum Tube. A vacuum tube is an electron tube evacuated to such a degree that its electrical characteristics are essentially unaffected by the presence of residual gas or vapor.

Gas Tube. A gas tube is an electron tube in which the electrical characteristics are substantially affected by the presence of an enclosed gas or vapor.

Element (Of an Electron Tube). An element of an electron tube is any integral part of the tube which contributes to its operation.

Electrode (Of an Electron Tube). An electrode of an electron tube is a conducting element that performs one or more of the functions of emitting, collecting, or controlling by an electric field the movements of electrons or ions.

Thermionic Tube. A thermionic tube is an electron tube in which one of the electrodes is heated for the purpose of causing electron or ion emission from that electrode.

Cathode. A cathode of an electron tube is an electrode through which a primary stream of electrons enters the interelectrode space.

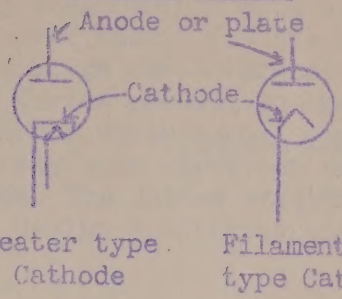
Anode. An anode of an electron tube is an electrode through which a principal stream of electrons leaves the interelectrode space.

Grid. The grid of an electron tube is an electrode having one or more openings for the passage of electrons or ions. (ASA* definition, 1941.) It is the function of the grid to control the flow of electrons between electrodes by controlling the electric field in the interelectrode space.

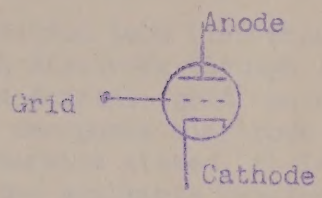
1-2. Vacuum Tube Symbols.

Vacuum tubes are designated in circuit diagrams by certain standard symbols. A few of these, essential for present use, are shown in the following tabulation:

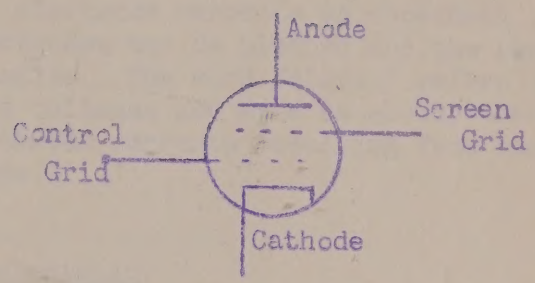
* American Standards Association.

Vacuum Tube	Circuit Symbol
Two-electrode tube or diode	

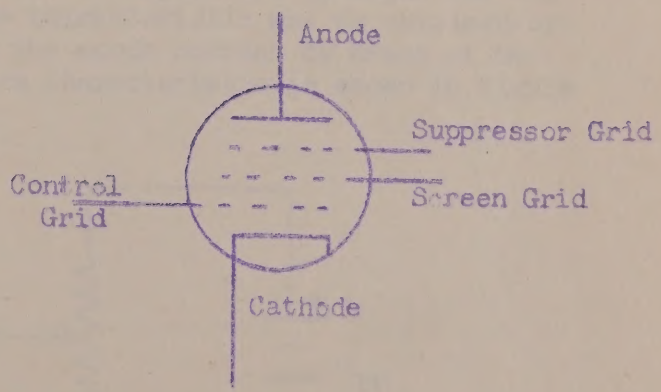
Three-electrode tube or triode



Four-electrode tube or tetrode



Five-electrode tube or pentode



The first part of this text will be devoted to the study of the high vacuum tube and its circuit behavior. Later chapters will be concerned with the gas tube and associated circuits.

A considerable knowledge of the circuit behavior of vacuum tubes may be developed from data obtained by measurements made at the available terminals of the tube if it is assumed that: (1) the electrons set free at a hot cathode are free to move through the interelectrode space under the influence of electric field forces present in that space; (2) the electron current to or from an electrode under normal operating conditions of the vacuum tube is dependent only upon electrode potentials and not upon the nature of the heated cathode. The latter statement is true for tubes operated in accordance with usual design specifications.

1-3. Vacuum Tube Static Characteristics.

"The electrode current of an electron tube (IRE standard definition) is the current passing to or from an electrode through the interelectrode space. NOTE: The terms cathode current, grid current, anode current, plate current, etc., are used to designate electrode currents for these specific electrodes. Unless otherwise stated, it is understood that an electrode current is measured at the available terminal."

The measured values of electrode currents as dependent upon voltages applied to the tube electrodes may be plotted and the resulting curves are termed the tube static characteristics. The word "static" refers to the fact that the changes in electrode voltages are made so slowly that there are negligible effects upon electrode currents resulting from the capacitance existing between electrodes.

1-4. The Diode Current-Voltage Characteristic.

With a thermionic diode connected as shown in Figure 1-1, and with the cathode or filament heated to the rated temperature by adjusting the filament current, the volt-ampere tube characteristic may be obtained by varying the anode voltage and reading the anode current by means of the milliammeter, A. A typical volt-ampere characteristic is shown in Figure

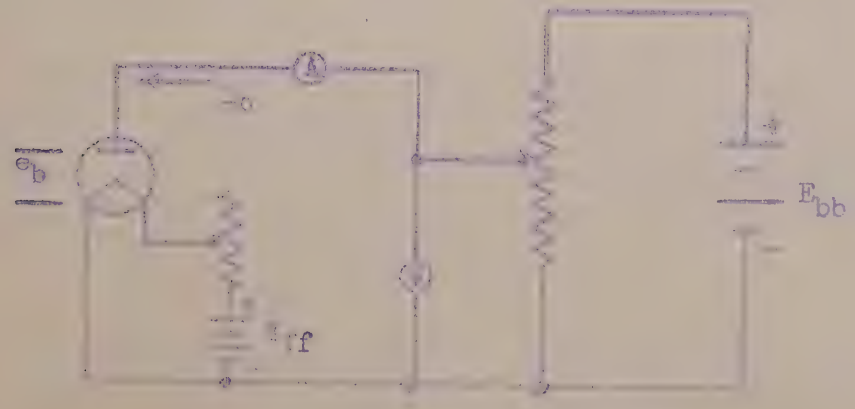


Figure 1-1.

Circuit for obtaining the diode characteristic.

1-2, in which the anode or plate voltage, e_b, is plotted as abscissa, and the plate current i_b as ordinate. This curve is referred to as a plate characteristic.

It is evident from Figure 1-2 that the diode is a non-linear circuit element, since the derivative di_b/de_b is not constant. The characteristic of a linear circuit element such as a resistor is a straight line through the origin. The slope of the line, di_b/de_b, is

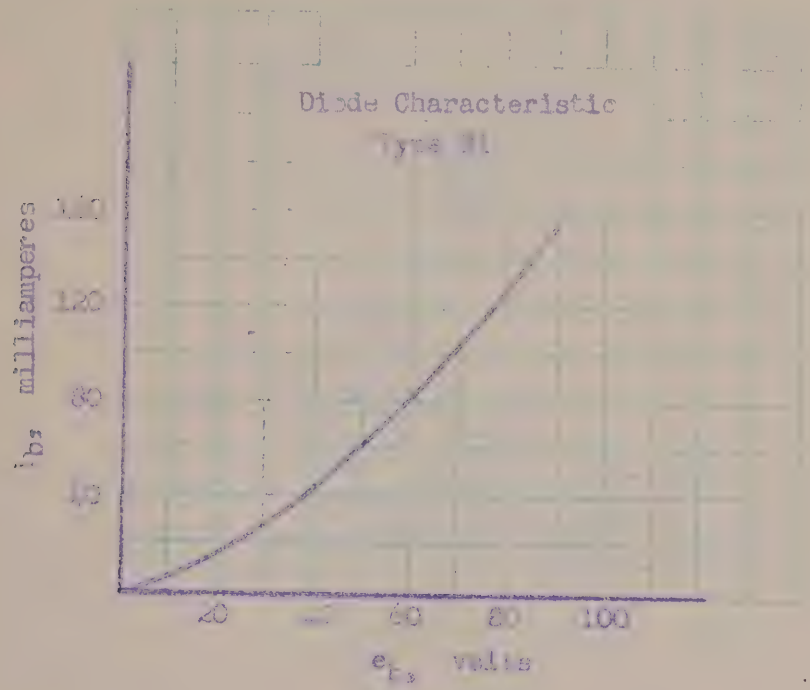


Figure 1-2.

constant for a linear circuit element and equal to the variational conductance of the element. For the diode, the quantity di_b/de_b is a function of e_b, and is called the plate conductance, g_p. Its reciprocal $de_b/di_b = r_p$ is defined as the plate resistance.

The plate resistance, r_p, as just defined is sometimes referred to as the a-c or variational plate resistance. It is used as a circuit constant when the anode voltage varies by such small amounts above and below a steady average value that the portion of the characteristic involved is approximately a straight line. For a fixed direct voltage, the d-c plate resistance would be used, and is defined as

$$R_b = \frac{E_b}{I_b}$$

where E_b is any plate voltage and I_b is the corresponding value of plate current. From the diode characteristics of Figure 1-2, the a-c plate resistance of the type 81 at 80 volts is given exactly by the slope of the curve at 80 volts, and is found to be approximately 410 ohms. At the same

plate voltage the d-- plate resistance is $R_p = \frac{\Delta}{130 \times 10^{-3}} = (illegible)$

If a diode is connected in an alternating-current circuit as shown in Figure 1-3, the anode will remain positive with respect to the cathode only during the positive half-cycle of the supply voltage. When the anode becomes negative, the positive the cathode becomes positive. Since electrons are negative, no electron is permitted to leave the cathode. (Heater connections are not shown in

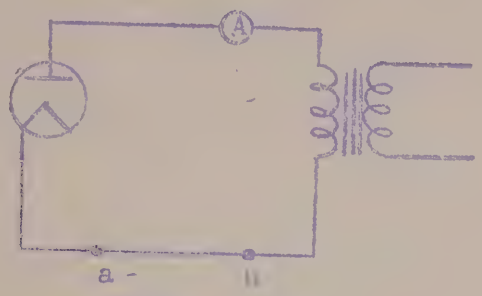


Figure 1-3.

Diode rectifier circuit.

the diagram.) Thus, current flows when the anode is positive and does not flow when the anode is negative. The diode is then acting as a rectifier, and a battery inserted between a and b with its positive terminal at a will receive a pulsating emerging current which flows only in the direction a to b. If the applied voltage wave-form is shown and is plotted to the same voltage scale used for e_p, the current wave-form may be obtained by use of the plate characteristic, as shown in Figure 1-4. Point 1 on the plate voltage wave corresponds to point 1' on the plate current wave. The time axes are plotted to the same scale. Projecting vertically upward from point 1 to the tube characteristic, and thence horizontally to the current axis, the tube current corresponding to the voltage at point 1 is determined. This current is then properly located on the time axis by laying off the angular distance θ as shown in Figure 1-4.

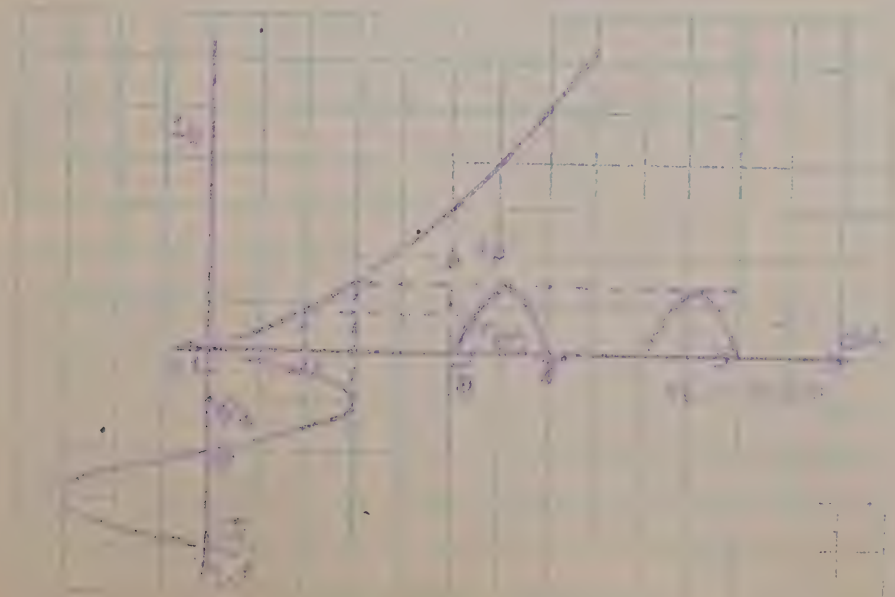


Figure 1-4.

Graphical Determination of plate current wave form.

If a resistance load is inserted in series with the diode of Figure 1-3, it is necessary to determine the new d-c characteristic before the plate current wave form can be obtained by a technique similar to that employed in Figure 1-4. The new d-c characteristic is determined by the voltage drops in both tube and resistance load, and may be defined as the composite characteristic. Since an exact mathematical expression is not available for the non-linear tube characteristic, the composite tube-and-circuit characteristic is determined graphically, as shown in Figure 1-5, in which the tube voltage e_b is added to the voltage drop across the resistor for a given i_b in obtaining a point on the composite characteristic. Thus, the applied voltage is $e = i_b R + e_b$ for points on the composite characteristic. The tube characteristic must first be obtained experimentally. For a given applied alternating plate voltage, the resulting plate-current wave form may now be determined point-by-point from the composite characteristic as previously shown in Figure 1-4.

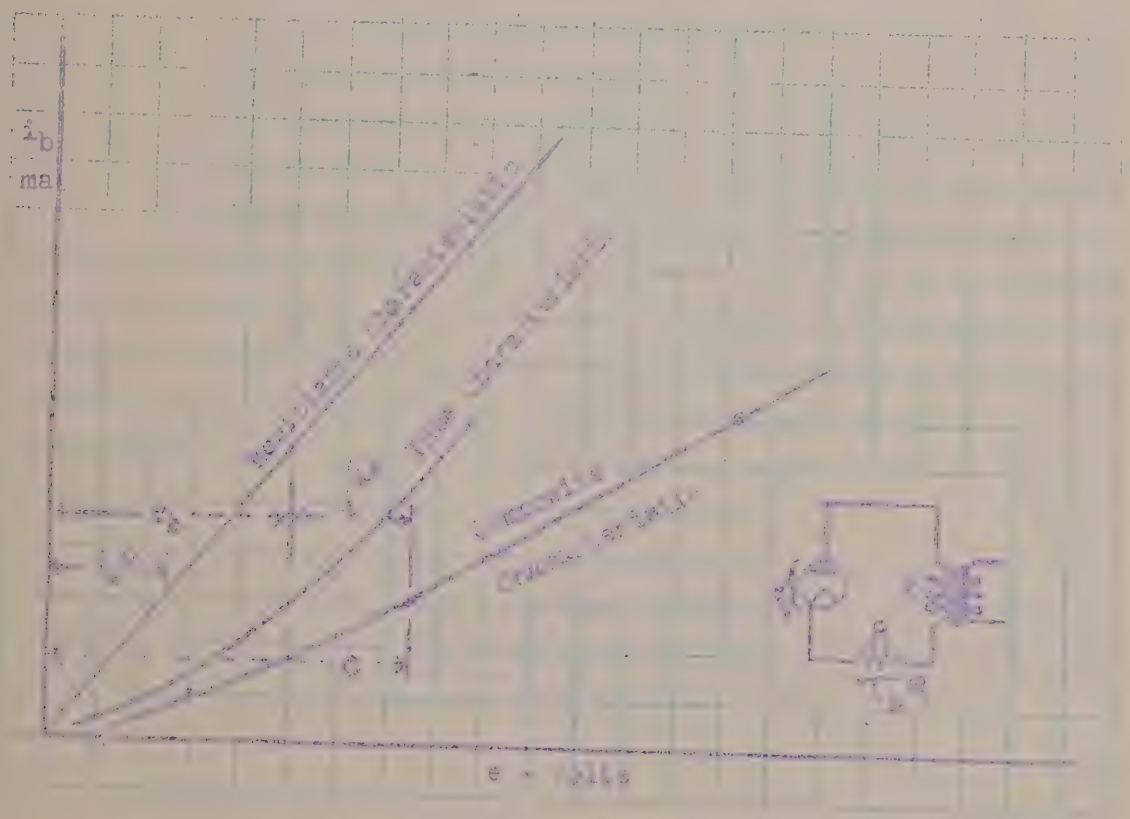


Figure 1-5

In Figure 1-6 is shown a circuit similar to that of Figure 1-5 except that a battery has been included. The effect of the battery, as shown, has been to shift the origin of the alternating plate voltage to point Q on the composite characteristic. The instantaneous value of the a-c component of plate voltage referred to the origin at Q is represented by the standard symbol e_p, and the corresponding instantaneous value of the a-c component of plate current referred to the same origin is represented by i_p. The resulting plate current now consists of two components,

one alternating, one direct. The value of the direct-current component if e_p is made zero is I_Q; I_{av} is the average or d-c component resulting from the application of e_p. Both I_{av} and I_Q differ from zero only because the tube-and-circuit characteristic is not linear. If the maximum

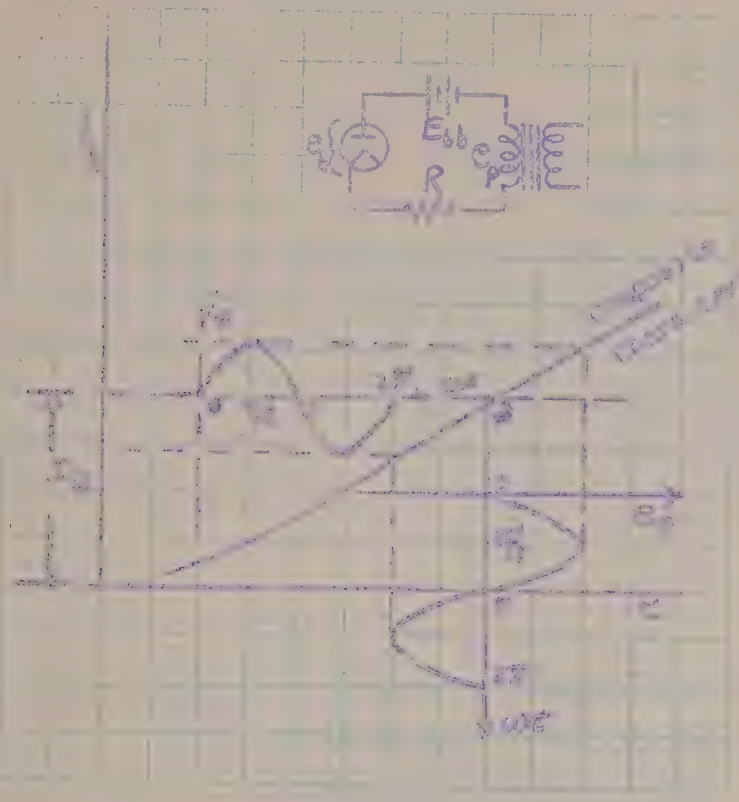


Figure 1-6.

value of the alternating plate voltage is small, the characteristic approximates a straight line, and $I_Q = I_{av}$. As far as the superposed alternating voltage is concerned, the approximately linear characteristic is described by an a-c resistance given by the reciprocal of its slope. If R were zero, this a-c resistance would be r_p.

1-5. Triode Static Characteristics.

The diode current-voltage characteristic of Section 1-4 may be described mathematically and a derivation of an equation for the curve $i_b = f(e_b)$ will be given in a later chapter. It should be evident from the experimental data, however, that the diode behaves as a variable resistance or valve depending upon the magnitude of the applied anode voltage provided that the frequencies of any ac components of plate voltage are low enough that the positive ion current is negligible compared with the

$_{pk}$ e_p

is the capacitance between anode and cathode of the tube and $\omega = 2\pi f$ is the angular frequency of any a-c component of plate voltage); (2) the cathode is capable of supplying at least as many electrons to the plate as are called for by the voltages applied. Under these conditions it is found, as will be shown later, that the relation

$$i_b = k\, e_b^{3/2} \qquad (1\text{-}1)$$

where k is a constant, is a good approximation for the diode static characteristic.

The use of a third electrode between the cathode and the anode of the high-vacuum electron tube was introduced by DeForest in 1907. In modern three-electrode tubes or triodes, the third electrode consists of a grid of fine wires which permit free passage of the electrons through the spaces between the wires. The electric field at the cathode surface is controlled by controlling the electric potential of the grid relative to the cathode, with the result that the anode current is dependent upon the grid voltage, and can be varied by changing either the plate or grid voltage. The anode current i_b may therefore be a function of two voltages, or symbolically,

$$i_b = f\,(e_b, e_c)\,, \qquad (1\text{-}2)$$

where e_b is the anode voltage, and e_c the grid voltage, each referred to the cathode as reference. With three variables involved, a plot of the function i_b would be a surface of three dimensions. Because a surface is rather inconvenient to obtain, it is perhaps preferable to hold one variable constant while plotting the variation of another variable as a function of the third. In this way three families of curves characteristic of the triode are obtained:

(a) the plate characteristics, $i_b = F_1\,(e_b)$, e_c constant;

(b) the mutual or transfer characteristics, $i_b = F_2\,(e_c)$, e_b constant;

(c) the anode voltage-grid voltage characteristics, $e_b = F_3\,(e_c)$, with i_b constant.

Typical plate, mutual, and anode-grid families for the type 6J5 tube are shown in the sketches, Figures 1-7, 1-8, and 1-9.

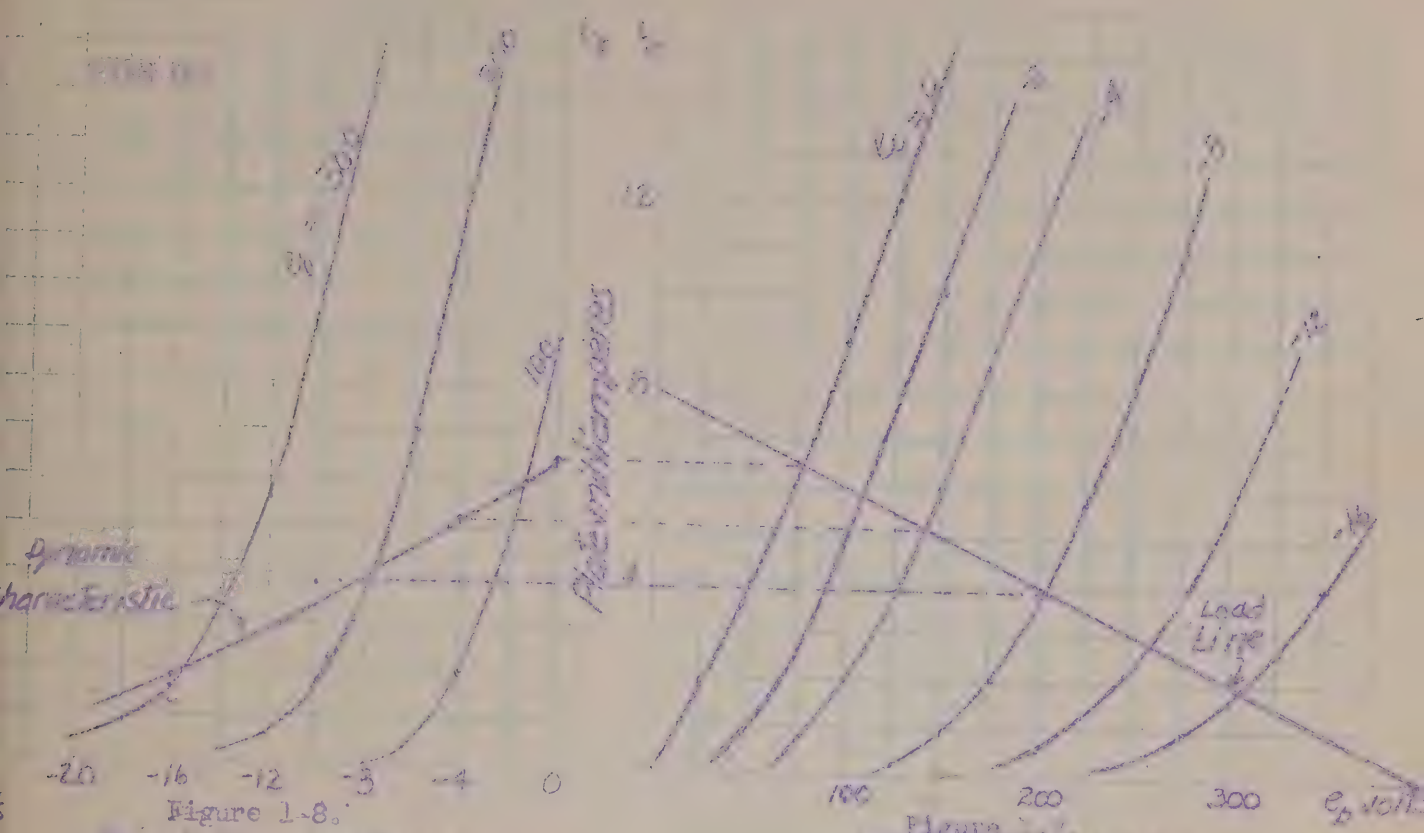


Figure 1-8.

Figure 1-9.
Plate Characteristics

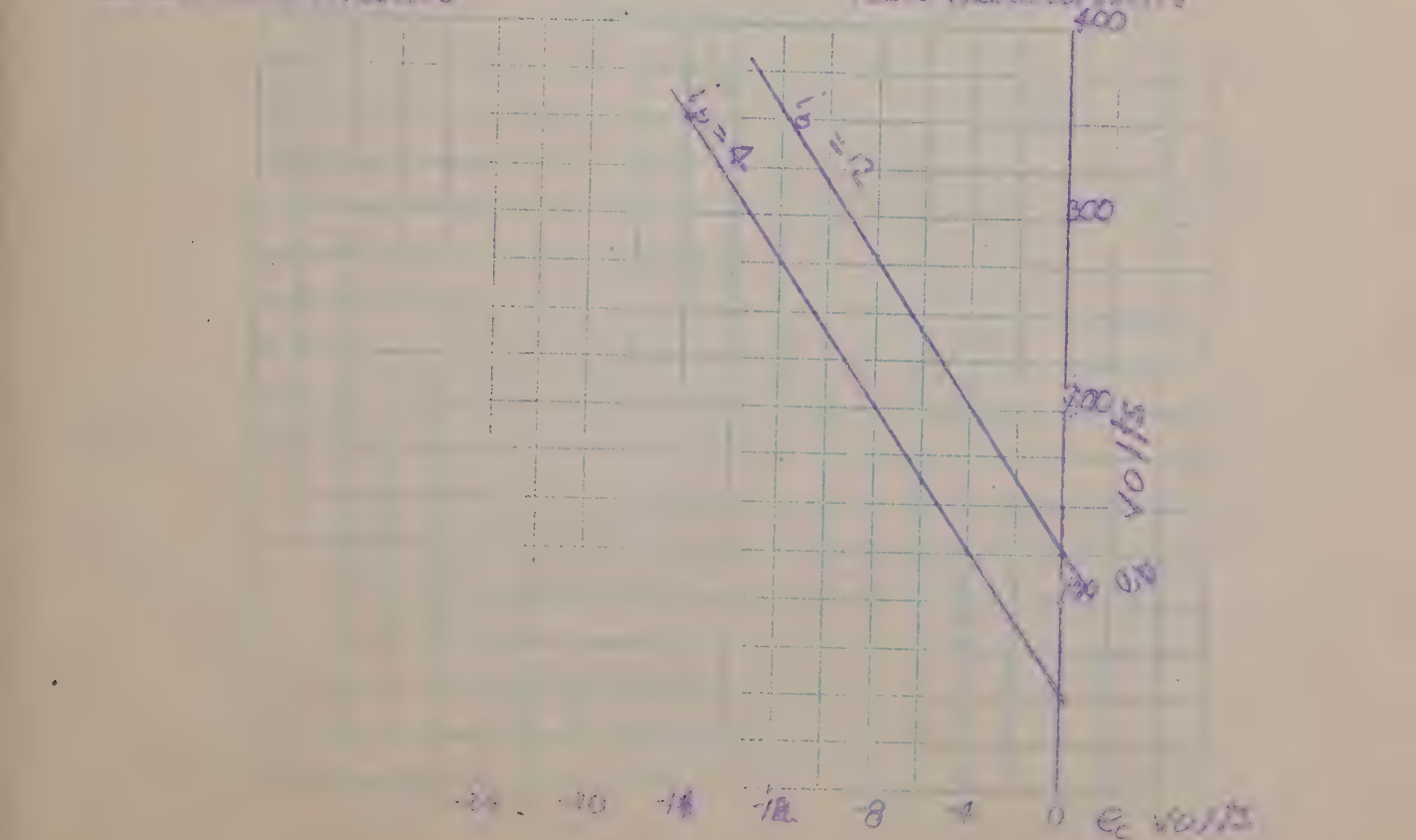


Figure 1-4

Anode Voltage - Grid Voltage Characteristics

1-6. The Triode Coefficients.

The slopes of the families of triode characteristics are significant. In the case of the plate family, Figure 1-7, the plate conductance, g_p, is defined by the relation

$$g_p = \frac{\partial i_b}{\partial e_b} \qquad (1\text{-}3)$$

and the plate resistance

$$r_p = \frac{1}{g_p} = \frac{1}{\left(\frac{\partial i_b}{\partial e_b}\right)} \qquad (1\text{-}4)$$

Examination of the curves shows that, as for the diode, these quantities are not constant. From the mutual family, Figure 1-8, the slope

$$\frac{\partial i_b}{\partial e_c} = g_m \qquad (1\text{-}5)$$

is defined as the grid-plate transconductance. In regard to the grid-anode family of Figure 1-9, it is evident that the slope is a pure number, and the ratio $\partial e_b / \partial e_c$, for one of the curves at constant i_b, is a measure of the relative effectiveness of the grid and anode in controlling the plate current, i_b. Thus, for a change Δe_c in increasingly negative grid voltage, a positive change Δe_b in anode voltage is required if the current is to remain constant. Since the grid wires are much closer to the cathode than is the anode, $\Delta e_b \gg \Delta e_c$, and the ratio is greater than 1. The quantity

$$-\frac{\partial e_b}{\partial e_c} = \mu \qquad (1\text{-}6)$$

is defined as the tube amplification factor, and is fairly constant for negative grid voltages

The similarity in shape of the triode plate characteristics with the diode characteristic should be noted. In each case the current obeys a three-halves power law. Both theoretical and experimental justification exists for the following equation applying to triode plate current:

$$i_b = B\,(e_b + \mu e_c)^{n} \qquad (1\text{-}7)$$

The quantities D and n are constants which are easily determined experimentally. For triodes with indirectly heated, equipotential cathodes, n is approximately 1.5. The quantity $(e_b + \mu e_c)$ is called the equivalent diode voltage; diode current-voltage equations to be derived later are found to apply well to triodes if the diode plate voltage is replaced by $(e_b + \mu e_c)$. It should be noted that $i_b = 0$ for $e_c = -\frac{e_b}{\mu}$, which is a convenient relation for determining the required negative grid voltage for plate current cut-off, or conversely, for determining μ (approximately) if the cut-off voltage is determined experimentally.

1-7. Relation Between the Triode Coefficients.

Although the relation between i_b and the electrode voltages e_b and e_c is usually empirical, one may utilize the functional expression to obtain the relation between μ, r_p, and g_m. The total differential of the plate current obtained from the relation

$$i_b = f(e_b, e_c)$$

is

$$di_b = \frac{\partial i_b}{\partial e_b}\, de_b + \frac{\partial i_b}{\partial e_c}\, de_c , \qquad (1\text{-}8)$$

If now i_b is held constant, as in Figure 1-9, $di_b = 0$, and,

$$\left.\frac{de_b}{de_c}\right]_{(i_b \text{ constant})} = \frac{\partial e_b}{\partial e_c} = -\left[\frac{\left(\frac{\partial i_b}{\partial e_c}\right)}{\left(\frac{\partial i_b}{\partial e_b}\right)}\right], \qquad (1\text{-}9)$$

or

$$\boxed{\mu = g_m r_p} \qquad (1\text{-}10)$$

The most obvious method of obtaining values for the triode coefficients is to plot from the experimental data the families of characteristics and measure their slopes. The data for plotting such tube characteristics may be obtained by connecting tube and meters as shown in Figure 1-10. High resistance voltmeters should be used.

Figure 1-10

Numerous other methods for determining triode coefficients have been suggested and are described in detail in the literature and in books on electron tubes. Useful bridge methods of determining μ, r_p, and g_m will be described in Section 1-14.

1-8. The Load Line. The Dynamic Characteristic.

If the tube whose static plate characteristics were given in Figure 1-7 is connected in a circuit as indicated in Figure 1-11, the drop of voltage in the load resistor, R_L, may be shown on the sheet with the plate characteristics in a manner analogous to that in which the composite characteristic was obtained for the diode. In the plate circuit of Figure 1-11, applying Kirchhoff's voltage law,

$$E_{bb} = i_b R_L + e_b$$

or

$$e_b = E_{bb} - i_b R_L \qquad (1\text{-}11)$$

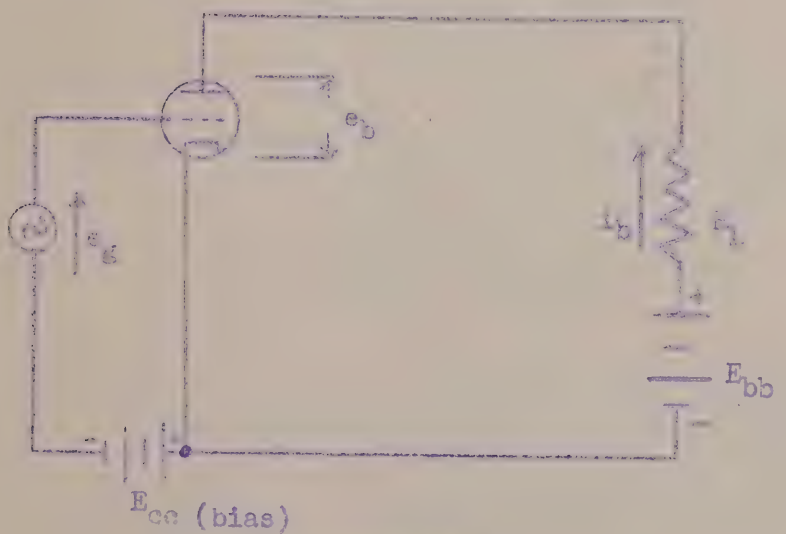


Figure 1-11.

Now if the relation $e = i_b R_L$ is plotted on i_b - e rectangular coordinate paper the result is a straight line, of slope $\frac{1}{R_L}$, passing through the origin of coordinates. This is the method used for the diode, Figure 1-5, in combining the characteristic curves of circuit and tube to obtain the composite characteristic. Again the objective is to obtain a

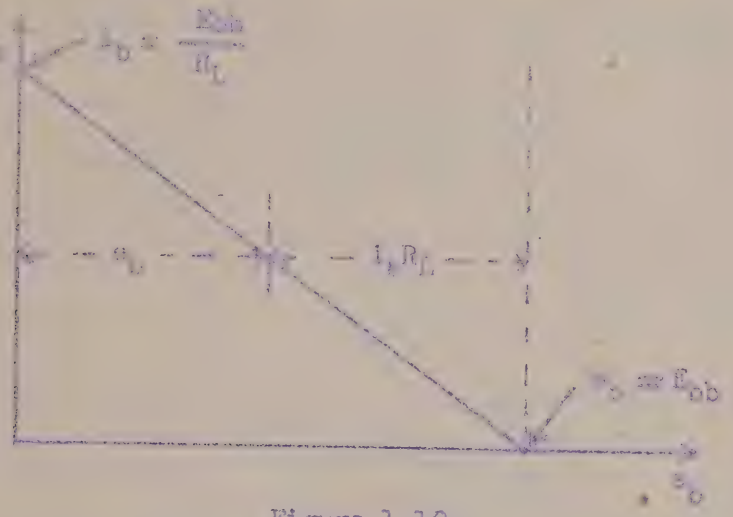


Figure 1-12.
Sketch of the Load Line $e_b = E_{bb} - i_b R_L$.

The load line, Equation (1-11), may be readily sketched by finding the intercepts on the coordinate axes. Such a typical load line has been sketched on Figure 1-7, for E_{bb} = 400 volts, R_L = 50,000 ohms. It should be carefully noted that the load line is completely independent of the tube and its characteristics. It is the load resistance characteristic analogous to the resistance characteristic used in Figure 1-5 with the diode.

Intersections of the load line with the plate characteristics of Figure 1-7 represent points for which the plate current is such that the voltage across the tube, plate-to-cathode, plus the voltage drop across the resistor add up to the plate battery voltage. If all such points are transferred to the mutual characteristic graph of Figure 1-8, the resulting locus is truly a tube-and-circuit characteristic, and is called the *dynamic characteristic*.* It is the intersection of the load line with the tube characteristic, i_b, and e_c for the particular characteristic will be known and may be transferred to the mutual characteristic graph. Load line and dynamic characteristic will be used in tube-and-circuit analysis to follow.

* The term "dynamic" as used here is somewhat misleading, although it is in common use. Actually, the so-called dynamic characteristic includes those points (on the mutual diagram) which are composite to tube and circuit resistance in the same sense as used already for the diode.

1-9. IRE Standard Symbols.

The Institute of Radio Engineers in 1938 adopted a standard system of letter symbols for electron tube currents, voltages, and parameters. The diagram of Figure 1-12 and the tabulation following this paragraph illustrate the use of the standard symbols. The complete definitions of current and voltage symbols are also given in detail on page 16 of [illegible]

Quantity	Letter	Subscript
Voltage	E - d-c, rms a-c, average, quiescent, or maximum value (with extra subscript m) e - instantaneous component or total value	b - plate; total, or [illegible] bb - plate battery p - plate: instantaneous a-c component or rms value
Current	I - d-c, rms a-c, average, quiescent, or maximum value (with extra subscript m) i - instantaneous component or total value	c - grid: total, d-c, or quiescent cc - grid battery g - grid: insvantaneous a-c component or rms value

The circuit diagram of Figure 1-13 shows a triode connected as an amplifier. A source of alternating voltage is connected in series with the grid bias battery in the grid circuit, and there is a load resistance R_L in the plate circuit in series with the plate battery. The latter is frequently referred to as the "B" supply. Static characteristics of the tube and the load resistance characteristic or load line are shown on the $i_b - e_b$ or plate diagram, and the dynamic characteristic has been drawn from points transferred to the mutual from the plate diagram. Time axes for the plotting of instantaneous values of grid voltage, plate current, and plate voltage are shown, and the two latter quantities have been obtained by graphical methods identical with those used for the diode rectifier in Figure 1-4.

Definitions of Standard Symbols. Shown in the diagram, Figure 1-13, are the load line and dynamic characteristics for a triode connected as a simple amplifier with resistance load, R_L. The input, signal, or excitation voltage is the alternating voltage, e_g, in series with the grid bias battery, of voltage E_{cc}. The plate battery voltage is E_{bb}. The significance of the various symbols is shown, and the relations between total values and their components are given. The definitions follow:

E_{bb} = plate supply voltage.

E_{cc} = grid supply voltage.

e_c = instantaneous total grid voltage.

e_b (volts)
Plate characteristics

e_b = instantaneous total plate voltage.

i_c = instantaneous total grid current.

i_b = instantaneous total plate current.

E_c = average or quiescent value of grid voltage

E_b = average or quiescent value of plate voltage

I_c = average or quiescent value of grid current

I_b = average or quiescent value of plate current

e_g = instantaneous value of the a-c component of grid voltage.

e_p = instantaneous value of the a-c component of plate voltage.

i_g = instantaneous value of the a-c component of grid current

i_p = instantaneous value of the a-c component of plate current.

E_g = rms value of the a-c component of grid voltage.

E_p = rms value of the a-c component of plate voltage

I_g = rms value of the a-c component of grid current.

I_p = rms value of the a-c component of plate current.

r_p = a-c plate resistance.

g_m = grid-plate transconductance.

μ = amplification factor.

1-10. Slope of the Load Line. Resistance Load.

The equation of the load line is $e_b = E_{bb} - i_b R_L$; the slope $\left[\frac{di_b}{de_b}\right]$ of the load line is $-\frac{1}{R_L}$. At any point P, Figure 1-13, within the range of operation along the load line, the slope of the line can be obtained as follows:

$$\text{slope} = \frac{\Delta i_b}{\Delta e_b} = \frac{i_p}{-e_p} = -\frac{1}{R_L}. \quad (1\text{-}12)$$

In equation (1-12), Δi_b and Δe_b are the increments or changes in plate current and plate voltage from their zero signal values, and are therefore identical with the instantaneous a-c values, i_p and e_p. Then,

$$e_p = - i_p R_L \qquad (1-13)$$

or in terms of the rms values, for sinusoids,

$$E_p = - I_p R_L \qquad (1-14)$$

Evidently e_p and i_p are 180° out of phase; e_p is in phase with e_g and therefore e_p and e_g also differ in phase by 180°. An examination of Figure 1-4 will show that a positive increase of grid voltage is accompanied by a proportional increase in plate current in the positive direction, if the dynamic characteristic is linear. By position direction of increase of plate current is meant an increase of actual, instantaneous plate current flowing from anode to cathode as shown on the sketch of plate current, Figure 1-1. Referring to the circuit diagram and to the load line, it is evident that an increase (in the positive direction) of i_p, or of I_p, will result in an increased voltage drop in load resistor R_L and in a corresponding decrease in plate voltage. For a pure resistance load, the result is the phase reversal of Equation (1-12) or Equation (1-13). An important interpretation of these equations is simply this: they show that the alternating component of plate voltage appears across the load resistor.

The following relations illustrate the behavior of an amplifier operating in the linear region of its dynamic characteristic:

For a sinusoidal input voltage,

$$e_g = \sqrt{2}\, E_g \sin \omega t. \qquad (1-15)$$

The total grid voltage is then

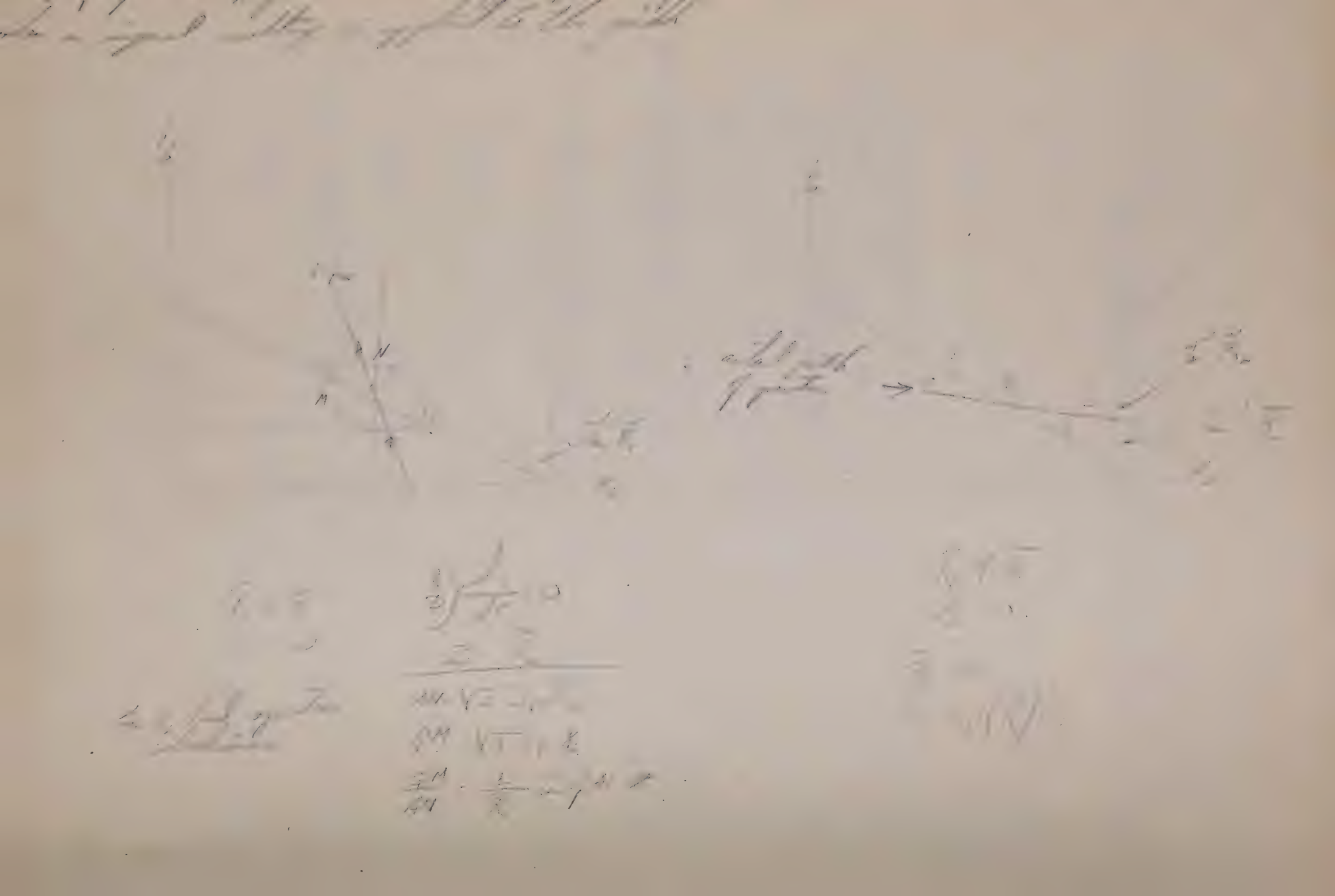
$$e_c = E_c + \sqrt{2}\, E_g \sin \omega t. \qquad (1-16)$$

The instantaneous alternating plate current is (for linear dynamic characteristic)

$$i_p = \sqrt{2}\, I_p \sin \omega t. \qquad (1-17)$$

The instantaneous total current is

$$i_b = I_b + \sqrt{2}\, I_p \sin \omega t. \qquad (1-18)$$



The instantaneous alternating component of plate voltage appearing across R_L is then

$$e_p = - i_p R_L = -\sqrt{2}\ I_p R_L \sin \omega t. \quad (1\text{-}19)$$

The instantaneous total plate voltage is

$$e_b = E_b + e_p = E_b - \sqrt{2}\ I_p R_L \sin \omega t. \quad (1\text{-}20)$$

1-11. Effect of Load Impedance. Reactive Load.

In the following analysis, sinusoidal a-c components of voltage and current are assumed. If the load impedance (Figure 1-14) is $Z_L = R_L + jX_L$, the voltage across Z_L will depend upon both the rate of change and the magnitude of the plate current, and the operating locus on both plate and mutual diagrams becomes more complicated. From Kirchhoff's Law,

$$E_{bb} = i_b R_L + L \frac{di_b}{dt} + e_b \quad (1\text{-}21)$$

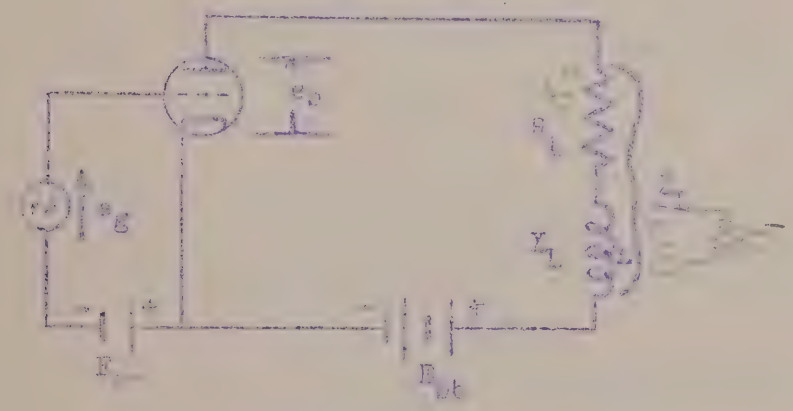


Figure 1-14.

Since $i_b = I_b + i_p$, and $\frac{dI_b}{dt} = 0$,

Equation (1-21) solved for e_b becomes

$$e_b = E_{bb} - I_b R_L - i_p R_L - L \frac{di_p}{dt} \quad (1\text{-}22)$$

Also, $$E_b = E_{bb} - I_b R_L \quad (1\text{-}23)$$

so that $$e_b = E_b - i_p R_L - L \frac{di_p}{dt} \quad (1\text{-}24)$$

$$= E_b + e_p \quad \text{(by definition)}.$$

$$e_p = -\left(\sqrt{2}\ I_p R_L \sin \omega t + \sqrt{2}\ \omega L\ I_p \cos \omega t\right)$$

$$= -\sqrt{2}\ I_p \sqrt{R_L^2 + (\omega L)^2}\ \sin(\omega t + \theta)$$

$$= -\sqrt{2}\ I_p\ |Z_L|\ \sin(\omega t + \theta) \qquad (1\text{-}26)$$

where $\theta = \tan^{-1} \frac{\omega L}{R_L}$, as shown in the sketch which defines θ.

Now the load lines is given by the two equations: (See 1-24, 1-25, and 1-21)

$$\left.\begin{aligned} e_b &= E_b - \sqrt{2}\ I_p\ |Z_L|\ \sin(\omega t + \theta) \\ i_b &= I_b + \sqrt{2}\ I_p \sin \omega t \end{aligned}\right\} \qquad (1\text{-}27)$$

Equations (1-27) are the parametric equations of an ellipse. The operating locus on both plate and mutual diagrams is an ellipse provided that R_L, r_p, and g_m are constants over the operating range. Such an elliptical operating locus is undesirable in amplifiers.

An example of the determination of the load locus for an inductive load impedance is given in an example problem solution at the end of Section 1-14.

1-12 The Alternating-Current Equivalent Anode Circuit of a Triode Amplifier.

The plate current in a vacuum-tube circuit usually consists of one or more a-c sinusoidal components of different frequencies superposed upon a direct-current component. The purpose of the direct components of current and voltage is to establish operating points and to supply power for operation. The useful output in the vacuum-tube circuit is usually alternating current or voltage, or a-c (average) power. For these reasons, it is convenient and important to reduce the actual vacuum tube circuits wherever possible to equivalent alternating current circuits. This is accomplished by applying the idea of the superposition principle. The equivalent, alternating current circuit applies only to the sinusoidal a-c components of current and voltage. The results of analysis of the equivalent a-c circuit are superposed upon the d-c components to obtain the overall or total values of current and voltage.

If a vacuum triode is operated with negative grid bias and positive anode voltage of such values that the operating point is located on the straight portion of the dynamic characteristic, and if the alternating input grid voltage is of such value that the grid does not swing positive or beyond the linear range of the dynamic characteristic, then the plate current of the tube will have a linear dependence upon the grid voltage and an alternating current equivalent circuit can be obtained and used. The conditions shown graphically in Figure 1-13 comply with these requirements.

There are several methods of obtaining a mathematical expression for the sinusoidal alternating component of plate current. One such method assumes that the plate characteristics are a family of parallel straight lines, and utilizes an equivalent diode voltage and applies the superposition principle to separate alternating current components from direct current components. Perhaps the simplest method of all utilizes the relation $i_b = f(e_b, e_c)$ which may be expanded about the operating point in a Taylor's Series. Only two terms of the series are needed if operation is confined to the linear portion of the dynamic characteristic. Then,

$$\Delta i_b = \frac{\partial i_b}{\partial e_b} \Delta e_b + \frac{\partial i_b}{\partial e_c} \Delta e_c, \qquad (1\text{-}28)$$

in which $\frac{\partial i_b}{\partial e_b}$, and $\frac{\partial i_b}{\partial e_c}$ are constants, and where Δi_b, Δe_b, and Δe_c may be as large as may be desired so long as the requirement of linearity, as specified in the preceding paragraph, is maintained.

When a signal voltage is applied at the input of the amplifier of Figure 1-13, the values of e_c, i_b, and e_b change from their quiescent values. The change from quiescence of any of these quantities is its

a-c value, where the origins of the coordinates e_g, i_p, taken at the quiescent points on the mutual and plate diagrams.

$$\Delta i_b = i_p \,, \qquad \Delta e_b = e_p = e_{KP}, \qquad \Delta e_g = e_g = e_{KG} \,,$$

where the symbols e_{KP} and e_{KG} are used to explicitly specify the assumed direction of the instantaneous values of voltage rise.

$$\frac{\partial i_b}{\partial e_b} = \frac{1}{r_p}, \qquad \frac{\partial i_b}{\partial e_g} = g_m = \frac{\mu}{r_p}$$

Equation (1-28) then becomes

$$i_p = \frac{e_{KP}}{r_p} + \frac{\mu}{r_p} e_{KG} \qquad (1\text{-}29)$$

For a load resistance R_L, the alternating component of plate voltage, e_p, becomes (see Equation 1-13)

$$e_{KP} = - i_p R_L \,. \qquad (1\text{-}13)$$

Equivalent relations for r-m-s values may be written using the effective values of current and voltage instead of the instantaneous values Equation (1-29) written with effective values is,

$$\frac{E_{KP}}{I_p} + \frac{\mu}{r_p} E_{KG} = \frac{- I_p R_L}{I_p} + \frac{\mu}{r_p} E_{KG}$$

Solving for I_p, the effective value of the alternating component of plate current is

$$I_p = \frac{\mu E_{KG}}{r_p + R_L} \,. \qquad (1\text{-}31)$$

The form of Equation (1-31) suggests that, so far as the alternating currents are concerned, the circuit of Figure 1-11 may be represented by the equivalent circuit of Figure 1-10 in which a generator of effective voltage μE_{KG} and internal impedance r_p is connected to a load resistance R_L. The batteries are not shown since their impedances are negligible compared to r_p and R_L. The grid circuit generator is connected in the diagram only since, with negative grid bias, the grid current is negligible and the grid circuit input impedance infinite.

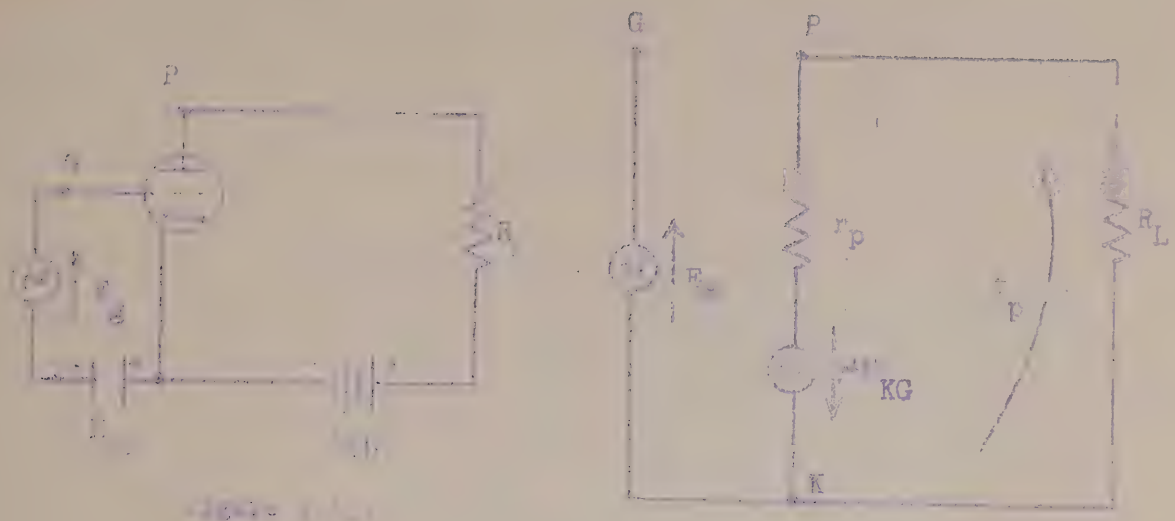


Figure 1-16

By the use of the alternating current equivalent circuit of Figure 1-16, the [illegible] circuit theory can be made available for solving these [illegible] problems. Problem operation is restricted to the region in which the [illegible] characteristics are linear. Equation (1-31) can then be written as A similar equation using the effective values of current and voltage, but may be expressed in terms of instantaneous values as

$$i_p = \frac{\mu\, e_{KG}}{r_p + R_L} \qquad (1\text{-}32)$$

The circuit of Figure 1-15 is that of a voltage amplifier. The output voltage of the amplifier is the a-c component of voltage across the load resistor R_L; the input voltage is the voltage E_g. The voltage gain of the amplifier may be obtained from the equivalent plate circuit. In the circuit of Figure 1-15 with the load resistance R_L replaced (for greater generality) by a load impedance, Z_L, the voltage drop across Z_L is $I_p Z_L$, the output voltage. The voltage gain may be defined as

$$\text{voltage gain} = A = \frac{\text{voltage drop across the load impedance}}{\text{voltage drop across the input}}$$

Then,

$$A = \frac{I_p Z_L}{E_g}$$, and I_p may be obtained from the equivalent a-c circuit, Figure 1-17, where $E_{KG} = E_g$.

Thus,

$$I_p = \frac{\mu E_{KG}}{r_p + Z_L} = \frac{\mu E_g}{(r_p + R_L) + j X_L} \qquad (1\text{-}33)$$

* It is here assumed, as will be shown later, that the equivalent a-c anode circuit may be used for any linear load impedance, and is not restricted to circuits using pure resistance loads.

where $Z_L = R_L + j X_L$ and it is to be understood that these relations have meaning only for sinusoidal voltages and currents. The voltage gain then becomes

$$A = - \frac{\mu}{r_p + Z_L} \qquad (1\text{-}34)$$

Equation (1-34) is a convenient one for gain calculations. It should be noticed that the gain as defined is a vector quantity. It is customary to take the voltage drops in the gain equation as from cathode to plate and from cathode to grid. This accounts for the negative sign before the grid voltage, E_g; since E_g is the rms value of the a-c component of voltage

1-13. Use of the a-c Equivalent Plate Circuit.

It is very important to observe that the voltage of the generator in the equivalent plate circuit of Figure 1-16 is μ times the voltage

circuit of the tube. The voltage E_g, in Figure 1-16, is also the voltage rise, due to excitation, between cathode and grid inside the tube. Only a-c components are shown on the equivalent circuit, since it represents variations of current and voltage around the operating point which has been used as origin of coordinates on the plate and mutual diagrams. All batteries are deleted. The tube is replaced by the plate resistance r_p and a constant voltage generator of generated voltage μE_{KG}. Now the symbol E_{KG} is used to call attention to the fact that the actual voltage rise between cathode and grid electrodes inside the tube is the cause of a change in plate current. In the circuit of Figure 1-15, $E_{KG} = E_g$, the voltage of the generator connected to the input terminals of the amplifier. In general, however, E_{KG} is not the same as E_g. Frequently, the voltage between cathode and grid may involve a voltage drop across, for example, a resistor used to supply bias to the grid. In any case, an inspection of the actual circuit or of the equivalent a-c circuit will disclose the presence of any a-c voltage other than that of the input excitation between

must always be used as the voltage E_{KG} in the expression μE_{KG} for the generated voltage of the equivalent plate circuit generator.

The recommended procedure in drawing the a-c equivalent plate circuit is the following:

(1) Designate the terminals of the tube as P, K, and G as in Figure 1-15.

(2) Draw the plate circuit between P and K outside the tube exactly as in the actual circuit, omitting batteries or d-c sources

(3) Replace the tube between P and K by a constant voltage generator of internal impedance r_p and generated voltage μE_{Kg}.

(4) Place arrows on the diagram to show the assumed positive sense of current flow and voltage rise of the generator. In this text the positive sense of current is taken toward the plate, and the positive sense of voltage rise in the equivalent plate circuit generator is accordingly from P to K

(5) Show the external source of voltage excitation, if any, as a generator of voltage rise E_g connected exactly as in the actual circuit except the G terminal is left unconnected to the plate circuit, as in Figure 1-16 (except for conditions to be described later). Any connection between G and the plate circuit may be omitted since it has been assumed that the grid is negatively biased and that conduction and displacement grid currents are negligible

Examples of the use of the equivalent plate circuit are given in Section 1-14. The symbol V with appropriate double subscripts is used for voltage drop, E for voltage rise.

1-14. Applications of the a-c Equivalent Plate Circuit.

A. Measurement of μ. The circuit shown in Figure 1-17 has been used in the measurement of μ. The impedance Z_e is the impedance looking into the terminals of the transformer which couples the headphones into the anode circuit. It should be observed that in drawing the equivalent a-c plate circuit all batteries are deleted. Points marked P, K, and G refer to plate, cathode and grid respectively. The tube is replaced by the equivalent plate circuit generator with its internal resistance, r_p. The assumed positive sense of current flow is shown. The drop in voltage between cathode and grid is $V_{KG} = - I R_1$; the voltage rise cathode-to-grid is $E_{Kg} = - V_{KG} = I R_1$. In the plate circuit,

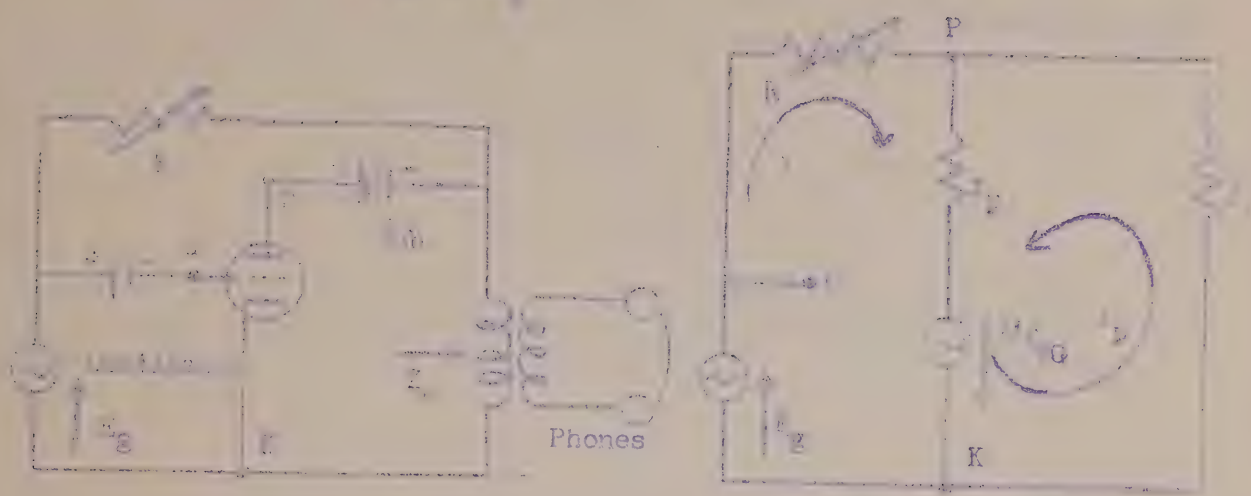
$$\mu E_{KG} = \mu I R_1 = (I_p + I) R_2 + I_p (Z_e + r_p) \qquad (1\text{-}35)$$

From (1-35),

$$\mu = \frac{(\mu R_1 - R_2) I}{R_2 + r_p + Z_e} \qquad (1\text{-}36)$$

[illegible]

[illegible] and I_p are [illegible] [illegible] [illegible]



(a) Actual Circuit (b) Equivalent Circuit

Figure 1-18

Circuit for the measurement of g_m.

6. Amplifier with Inductive Load. In the circuit of Figure 1-14, let $Z_L = 20{,}000 + j40{,}000 = 44{,}700 \angle 63.4^\circ$ ohms, $E_{bb} = 400$ volts, $E_{cc} = -8$ volts. As follows, to obtain the load lines on both plate characteristic diagrams which are helpful in the equivalent alternating current plate circuit which has been drawn in Figure 1-19(a). For

$$e_g = 2 \sin \omega t, \quad \sqrt{2}\, E_g = 2 \text{ volts},$$

$$\sqrt{2}\, I_p = \frac{\mu \sqrt{2}\, E_g}{r_p + Z_L}$$

If we take as a typical value $r_p = 30{,}000$ ohms,

$$\sqrt{2}\, I_p = \frac{40}{30{,}000 + j40{,}000} = 0.8 \times 10^{-3} \angle -53.2 \text{ amp.}$$

$$= \quad \text{volts.}$$

...ses are shown on the vector diagram of Figure 1-19(b), ... they were computed with E_g as reference vector.

In agreement with the theory of Section 1-11, where $\sqrt{2}\ I_p$ was used as the reference vector, the parametric equations of the elliptical operating locus are:

$$i_b = I_b + \sqrt{2}\ I_p \sin \omega t$$

$$e_b = E_b - \sqrt{2}\ I_p\ |Z_L| \sin (\omega t + 63.4^\circ)$$

...age $E_{c_b} = -100$, and the slope of the load line, $-1/20{,}000$. This load line has been drawn on the graph of Figure 1-20. From the graph the operating point is

$$= 1.6 \text{ ma}$$

(b) Vector Diagram

Figure 1-19.

...w total values of plate current and of plate and grid ...e as follows:

$$i_b = 1.6 + 0.8 \sin \omega t \quad \text{(milliamperes)}$$

$$e_b = 24^{\text{v}} - 35.8 \sin (\omega t + 63.4^\circ) \quad \text{(volts)}$$

$$= -8 + 2 \sin (\omega t + 53.2^\circ) \quad \text{(volts)}$$

The elliptical load lines shown in Figure 1-20 has been obtained from the equations for i_b, e_b, and e_g by assigning values to ωt. The accompanying table gives a tabulation of these calculations which have been plotted in Figure 1-20.

Tabulation of values for plotting, Figure 1-20.

ωt	e_b	i_b	e_g	e_c
90°	231	8.4	1.2	-6.8
27.[illegible]°	263	6.8	-1.2	-9.2
0	215	7.6	1.[illegible]	-6.1
180°	279	7.6	-1.6	-9.1
-63.4	247	5.88	-0.356	-8.356
(180-63.4)	247	9.32	+0.356	-7.644
206.6	283	7.[illegible]	-1.97	-9.97
26.6	211	7.98	1.97	-6.03

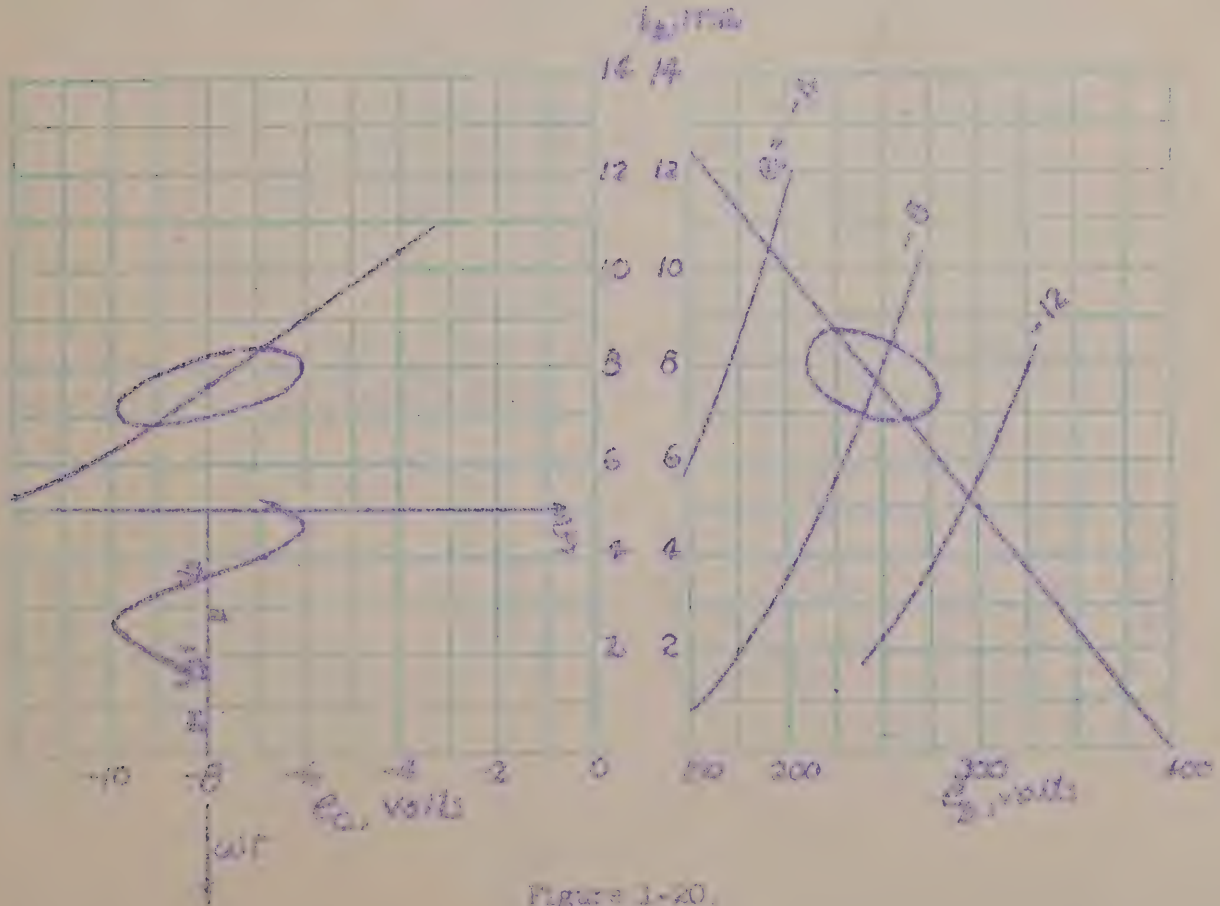


Figure 1-20.

1-14 The Constant Current Generator Equivalent Circuit

The equivalent tube circuits of Sections 1-12, 1-13, and 1-14 have used a constant voltage generator in the equivalent a-c plate circuit because, in general, the student is better acquainted with the constant voltage generator than with the constant current generator. An equivalent a-c circuit using the constant current generator may be obtained from the same fundamental relation,

$$\Delta i_b = \frac{\partial i_b}{\partial e_b} \Delta e_b + \frac{\partial i_b}{\partial e_c} \Delta e_c \,, \qquad (1\text{-}28)$$

from which the circuit of Section 1-12 was derived. It will be found that the constant current generator equivalent circuit used with Kirchhoff's current or node equations usually simplifies the analysis of tube circuits.

In (1-28), for linear operation, as explained in Section 1-12,

$$\Delta i_b = i_p, \quad \Delta e_b = e_{KP}, \quad \Delta e_c = e_{KG}, \quad \frac{\partial i_b}{\partial e_b} = g_p = \frac{1}{r_p}, \quad \frac{\partial i_b}{\partial e_c} = g_m$$

The resulting expression may then be written as

$$i_p = g_p e_{KP} + g_m e_{KG}$$

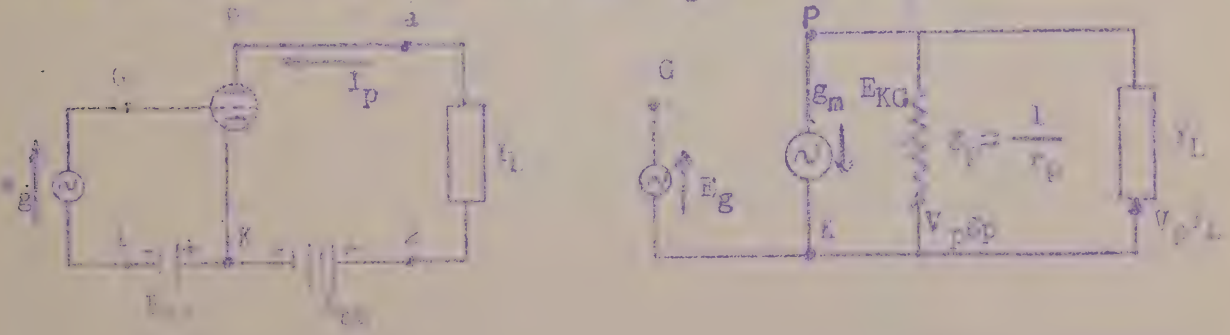
or

$$g_m e_{KG} = i_p - g_p e_{KP} \,, \qquad (1\text{-}42)$$

and this relation may be applied to the triode circuit of Figure 1-21 in which the load admittance is

$$Y_L = \frac{1}{Z_L}$$

and where e_{KG}, i_p, and e_{KP} have been replaced by the corresponding effective or rms values. In Figure 1-21(b), $E_{KG} = E_g$, and $E_{KP} = E_p$.



(a) Actual Circuit. (b) Equivalent a-c Circuit.

Figure 1-21

Then the voltage rise, cathode to plate is $E_p = - I_p Z_L$ and $I_p = - E_p Y_L$, and (1-42) becomes

$$g_m E_g = (-E_p)(g_p + Y_L). \qquad (1-43)$$

The form of (1-43) suggests that the plate conductance g_p and the load admittance, Y_L, are connected in parallel and are fed by a constant current generator of magnitude $g_m E_g$. It may be desirable to use the symbol V_p for voltage drop in the direction from the cathode to the plate side of the load impedance, so that

$$V_p = - E_p . \qquad (1-44)$$

Then,

$$g_m E_g = V_p (g_p + Y_L). \qquad (1-45)$$

The constant current generator form of the vacuum tube equivalent circuit is shown in Figure 1-21(b), in which the constant current generator of generated current $g_m E_g$ and shunt conductance g_p has been inserted between terminals P and K. The quantity E_{KG} is again the voltage rise between cathode and grid terminals inside the tube, and, for the circuit shown is equal to E_g.

The voltage gain of the amplifier circuit of Figure 1-21(a) may be obtained from the equivalent circuit 1-21(b) by applying the relation,

$$g_m E_g = V_R (g_p + Y_L) ,$$

whence

$$A = \frac{E_o}{-E_g} = - \frac{g_m}{g_p + Y_L} \qquad (1-46)$$

It may be easily shown that (1-46) is identical with (1-34) if g_m is replaced by $\frac{\mu}{r_p}$, g_p by $\frac{1}{r_p}$, and Y_L by $\frac{1}{Z_L}$. In all respects, results obtained by the use of Figure 1-21(b) will be identical with those obtained from the constant voltage generator equivalent circuit if admittances are replaced by the corresponding impedances and g_m by μ/r_p. It is important at this point to emphasize two facts of fundamental importance: (a) an equivalent circuit is never a unique representation of an actual network, i.e., there are infinitely many circuits which may be considered equivalent in the sense of having identical input and output voltages and currents; (b) in the equivalent circuits thus far used, the grid current has been assumed zero, and this assumption is not generally true, even for approximately linear operation, especially as the frequency of the input voltage, in the case of the amplifier, is increased.

1-16. The Four-Terminal Linear Network.

A linear network of electric circuit elements is a network for which the relation between applied voltage and response current is linear, i.e., may be graphed on rectangular coordinate paper as a straight line. The response of such a network may be in the form either of current or of voltage, and may result from the application somewhere in the network of voltage or of current. The network is said to be bilateral if energy flows equally well in either direction between any two pairs of terminals. Vacuum tubes are not, in general, bilateral circuit elements, but they behave as linear circuit elements if operated in the linear region of their characteristics. Although networks in general may have many pairs of terminals, there are numerous practical applications of networks which have two pairs of terminals, an input pair and an output pair. Examples are the simple T and π sections used in elementary network theory. Such networks are referred to as four-terminal networks or four-poles.

In the elementary analysis of four-terminal networks, recourse is had in the fundamental circuit laws of Kirchhoff. The use of the voltage law has, in the past, been preferred but in recent years it has been found possible to simplify the analysis of circuit problems by the use of the current law. That junction in a network at which the summation of the currents is to be found is called a node. The equation expressing the law that the vector sum of the currents flowing toward a node is equal to the vector sum of the currents flowing away from the node is referred to as a node equation. It has been found that important simplification of vacuum tube network theory result where linear four-terminal network theory and node equations are applied.

Linear four-terminal network theory will be applied to vacuum tube circuits operating in the linear region of their characteristics. A description of a general four-terminal network is needed before applying the theory to the vacuum tube. The "box" of Figure 1-22 contains two input and two output terminals. The assumed positive sense of current

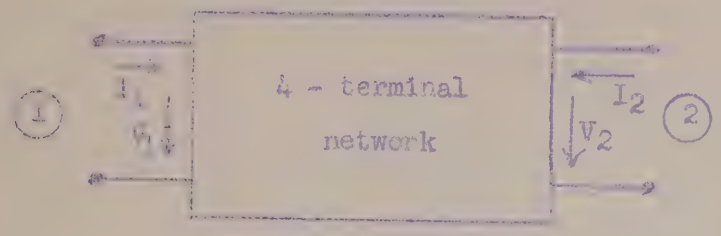


Figure 1-22.

and of voltage drop at the input and at the output terminals are shown by the arrows. If now the network inside the box behaves as a linear network, it can be shown[1,2] that the complete performance of the network can be predicted in terms of four parameters which can be measured at the available external terminals. The importance of the preceding statement will be appreciated when it is noted that a knowledge of the circuit details and physical complexities of the contents of the box of Figure 1-22 is entirely unnecessary in predicting the behavior of the four-terminal network, provided the network is linear.

The behavior of such a linear network as that in Figure 1-22 may be described by any pair of several pairs of equations expressing relations between the quantities V_1, I_1, V_2, and I_2. These equations may be said to define the properties of the 4-terminal linear network. If the network is linear, then a proportionality exists between current I_1 and voltage V_1, between current I_2 and voltage V_2, between voltage V_2 and voltage V_1, and between current I_2 and current I_1. The student has previously encountered examples of such relationships as are proposed for use here in connection with 4-terminal networks such as two winding transformers or T or π arrangements of linear impedances. For example, a two winding transformer, Figure 1-23, composed of linear elements, has constants $\vec{\mathcal{Z}}_{11} = R_1 + j\omega L_1$, $\vec{\mathcal{Z}}_{22} = R_2 + j\omega L_2$, $\vec{\mathcal{Z}}_{12} = j\omega M = \vec{\mathcal{Z}}_{21}$, where R_1 and L_1, R_2 and L_2 are respectively the resistance and inductance of coils 1 and 2 coupled with mutual inductance M. The 4-terminal network is described by the equations

1 F. Strecker & R. Feldkeller, E.N.T 6, pp 93 - 112, 1929.

2 [illegible], paper presented orally before the Winter Convention of I.R.E., IRE, January, 1945.

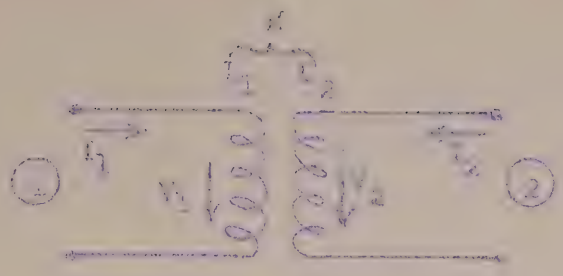


Figure 1-23
Four-terminal network.

$$\left.\begin{aligned} V_1 &= I_1 \mathcal{Z}_{11} + I_2 \mathcal{Z}_{12} \\ V_2 &= I_1 \mathcal{Z}_{21} + I_2 \mathcal{Z}_{22} \end{aligned}\right\} \qquad (1\text{-}47)$$

where, with terminals (2) open, and a voltage source connected to terminals (1), $I_2 = 0$, and $\mathcal{Z}_{11} = \frac{V_1}{I_1}$ is defined as the open-circuit driving point impedance at the input terminals, and $\mathcal{Z}_{21} = \frac{V_2}{I_1}$ = the open-circuit transfer impedance from (2) to (1). Similarly, with a voltage source connected at (2) and terminals (1) open, $I_1 = 0$, $\mathcal{Z}_{12} = \frac{V_1}{I_2}$, the open-circuit transfer impedance from (1) to (2), and $\mathcal{Z}_{22} = \frac{V_2}{I_2}$ the open-circuit driving point impedance at the output terminals. For a bilateral circuit such as that of Figure 1-23, $\mathcal{Z}_{12} = \mathcal{Z}_{21}$

Another familiar example of a linear 4-pole of general interest is an arrangement of linear impedances or admittances as a π-section, as in Figure 1-24. Since, according to a fundamental theorem of linear network, any circuit composed of linear, bilateral elements may be replaced

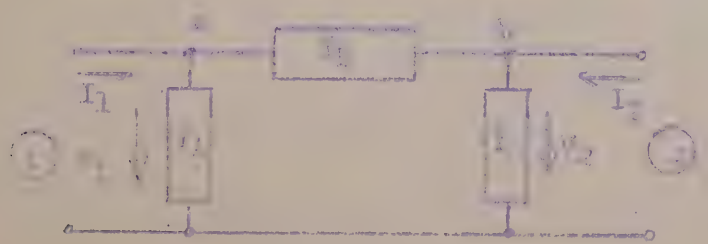


Figure 1-24.
Four-terminal network.

at a single frequency by an equivalent T or π section, the circuit of Figure 1-24 may be used with great generality for any 4-terminal linear, network, Figure 1-22. The behavior of the network may be described as follows, as may be verified by applying Kirchhoff's laws to the circuit of Figure 1-24. The following node equations are written for nodes a and b:

$$I_1 = V_1 Y_A + (V_1 - V_2) Y_B = V_1 (Y_A + Y_B) + V_2 (-Y_B)$$

$$I_2 = (V_2 - V_1) Y_B + V_2 Y_C = V_1 (-Y_B) + V_2 (Y_B + Y_C)$$

Then, by defining $y_{11} = Y_A + Y_B$

$y_{12} = -Y_B = y_{21}$

$y_{22} = Y_B + Y_C$

$y_{11} = \frac{I_1}{V_1}$

$y_{21} = \frac{I_2}{V_1}$

$y_{12} = \frac{I_1}{V_2}$

$y_{22} = \frac{I_2}{V_2}$

it is possible to obtain equations analogous to those of (1-47). Then,

$$\left.\begin{aligned} I_1 &= V_1 y_{11} + V_2 y_{12} \\ I_2 &= V_1 y_{21} + V_2 y_{22} \end{aligned}\right\} \quad (1\text{-}48)$$

are equations which define a linear 4-pole and describe its performance. The admittances y, although related to the components of the equivalent π-section as already shown, may be obtained by direct measurement as indicated for the $\mathcal{Z}$'s of Figure 1-23. The network behavior is easily predicted if either a set of impedances $\mathcal{Z}$ or the set of admittances $\mathcal{Y}$ are known. Script letters are used for the $\mathcal{Z}$'s and $\mathcal{Y}$'s to distinguish these quantities from circuit elements connected to terminals external to the box.

1-17 Definitions of the Four-Terminal Network Admittances

Either set of equations (1-47) or (1-48) may be chosen to describe the behavior of the four-terminal network, or four-pole, Figure 1-22. Equations (1-48) are chosen here because, for vacuum tubes, many circuit problems are greatly simplified when node equations and admittances are used.

The following experimental procedure may be used in the measurement of the y's, and instead as a basis for their definition:

(a) Short circuit the output terminals and measure the currents I_1 and I_2, and the voltage V_1 with a driving generator connected to input terminals (1).

(b) Short circuit the input terminals and measure the currents I_1 and I_2, and the voltage V_2 with a driving generator connected to output terminals (2).

The following definitions are suggested by the measurements (a) and (b)

$$y_{11} = \frac{I_1}{V_1} = \text{short-circuit driving point admittance;}$$

$$y_{21} = \frac{I_2}{V_1} = \text{short-circuit transfer admittance;}$$

$$y_{12} = \frac{I_1}{V_2} = \text{short-circuit feedback admittance;}$$

$$y_{22} = \frac{I_2}{V_2} = \text{short-circuit output admittance.}$$

If the measured values of y_{21} and y_{12} are different, then the four-pole is not a bilateral network. This is not a violation of the reciprocity theorem which applies only to bilateral networks.

1-18. Applications of the Four-Pole Theory to Vacuum Tube Equivalent Circuits

The circuit analysis of a vacuum tube as a linear four-terminal network follows easily from Figure 1-25, the definitions of Section 1-17, and equations (1-48). Let it be assumed that the tube and batteries, as included between input terminals G-1 and output terminals a-2, are enclosed in a box with only the input and output terminals brought out, as shown in Figure 1-25. The external terminating linear admittances are shown as Y_1 and Y_2 in Figure 1-25, and the circuit is driven by a constant-current generator of generated current I_g. It is required to determine the voltage gain and the driving point admittances at both input and output terminals, with the respective output and input terminal admittances connected. In Figure 1-25,

$$I_2 = -V_2 Y_2 = V_1 y_{21} + V_2 y_{22}. \qquad (1\text{-}49)$$

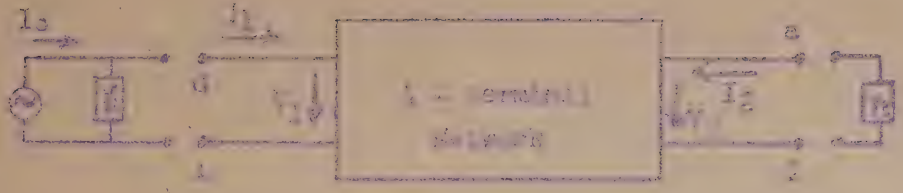


Figure 1-25.
Four-terminal network representing a vacuum tube.

The voltage gain (forward) is obtained from (1-47) and is

$$\frac{V_2}{V_1} = - \frac{y_{21}}{y_{22} + Y_2} \quad . \qquad (1\text{-}50)$$

Also,

$$I_1 = V_1 y_{11} + V_2 y_{12} \qquad (1\text{-}51)$$

Then the input admittance (if the terminated four-pole) is obtained from (1-51) and (1-50) and is

$$Y_{11} = \frac{I_1}{V_1} = y_{11} + \frac{V_2}{V_1} y_{12}$$

$$= y_{11} - \frac{y_{21} y_{12}}{y_{22} + Y_2} \quad . \qquad (1\text{-}52)$$

If the driving generator is removed from the input and connected to the output terminals, then

$$I_2 = - V_2 Y_1 = V_1 y_{11} + V_2 y_{12} \qquad (1\text{-}53)$$

The voltage gain (backward) is obtained from (1-53) and is

$$\frac{V_1}{V_2} = - \frac{y_{12}}{y_{11} + Y_1} \quad . \qquad (1\text{-}54)$$

The output admittance of the terminated four-pole is obtained from (1-49) and (1-54) and is

$$Y_{22} = \frac{I_2}{V_2} = y_{22} + \frac{V_1}{V_2}\, y_{21} = y_{22} - \frac{y_{12}\, y_{21}}{y_{11} + Y_1} \qquad (1\text{-}55)$$

The input and output admittances of a vacuum tube connected as an amplifier may be computed, along with the forward and backward voltage gains, according to (1-50), (1-52), (1-54), and (1-55) provided that the tube short-circuit admittances, y_{ij}, and the external terminations, Y_1 and Y_2, are known. Several laboratories are at present engaged in measurements of the y_{ij} admittances using equipment and methods developed originally at Bell Telephone Laboratories by J. A. Morton.[3] The work has not, however, progressed far enough as yet that sufficient data are available for all types of vacuum tubes. It is therefore necessary to identify the y_{ij} admittances with tube and tube-circuit admittances which are already available through manufacturers' ratings of their tube types. Such identification is desirable also in order to permit a comparison of the four-pole theory with older methods of vacuum tube circuit analysis. In the following section, the four-pole theory will be applied in developing the vacuum tube equivalent circuits in such a way that the relation between the y_{ij} admittances and the tube parameters will be shown for a particular frequency range.

1-19. Relation of the Four-pole Admittances y_{ij} to other Tube Parameters

Grid current was neglected in the analysis which led to the zero a-c equivalent circuits of Sections 1-12 and 1-15. Even with negative grid, however, small grid currents may be measured with a microammeter in the grid circuit. At frequencies above the audio range, the capacitances existing between various electrodes in the triode introduce reactances which are comparable with other circuit impedances. In particular, the grid-to-cathode capacitance and the grid-to-plate capacitance result in components of current in the grid circuit which flow even in the absence of any emitted electrons from the cathode. It is therefore necessary to include additional circuit elements in any equivalent a-c circuit of the triode as the operating frequency is increased. In particular, the input circuit of the triode is no longer open at the grid terminal.

The 4-pole theory is particularly useful in extending the equivalent circuit to higher frequency ranges. The analysis to be presented in the following pages will take into account both conduction and capacitive (or displacement) components of grid current. First, the 4-pole theory will be applied at such low frequencies that capacitances between electrodes may

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[3] The methods of 4-pole analysis used here were explained in an oral presentation by J. A. Morton to the Columbus Section of the IRE on April 19, 1946.

be neglected. Then the theory will be applied to the array of inter-
[illegible] with the tube [illegible] and [illegible] electron
emission. The results of the low and high frequency analyses will then
be combined to obtain a more general equivalent circuit which will de-
scribe the behavior of most triodes quite well for frequencies up to
about one megacycle.

It is found experimentally that conduction grid current depends upon both grid and anode voltages. Therefore, the relations

$$i_c = f_1 (e_g - e_c)$$

and

$$i_b = f_2 (e_b, e_c)$$

may be used exactly as in Section [illegible] and expanded about operating points in a Taylor Series. Again, the first degree terms only of the expansion are necessary as operation is restricted to the linear region of the tube characteristics. Then the relations

$$\Delta i_c = \frac{\partial i_c}{\partial e_c} \Delta e_c + \frac{\partial i_c}{\partial e_b} \Delta e_b$$

and

$$\Delta i_b = \frac{\partial i_b}{\partial e_c} \Delta e_c + \frac{\partial i_b}{\partial e_b} \Delta e_b$$

become, for a-c instantaneous components,

$$\begin{aligned} i_g &= g_g e_{KG} + g_{gp} e_{KP} \\ i_p &= g_{pg} e_{KG} + g_p e_{KP} \end{aligned} \qquad (1\text{-}56)$$

In (1-56) $g_g = -\frac{1}{r_g} = \frac{\partial i_c}{\partial e_c}$ is the grid conductance, measurable from the slope of the $i_c - e_c$ grid characteristic;

$g_{gp} = \frac{\partial i_c}{\partial e_b}$ is the plate-to-grid transconductance, measurable from the slope of the $i_c - e_b$ characteristic;

$g_{pg} = \frac{\partial i_b}{\partial e_c}$ is the grid-to-plate transconductance, measurable from the slope of the $i_b - e_c$ characteristic;

$r_p = \frac{\partial e_b}{\partial i_b} = \frac{1}{g_p}$ is the plate resistance, or reciprocal slope of the $i_b - e_b$ characteristic.

If effective, rather than instantaneous values are used in (1-56), and the new equations

$$\left.\begin{aligned} I_g &= g_g E_{KG} + g_{gp} E_{KP} \quad (a) \\ I_p &= g_{pg} E_{KG} + g_p E_{KP} \quad (b) \end{aligned}\right\} \qquad (1\text{-}57)$$

are compared with (1-48), then with $I_g = I_1$, $E_{KG} = V_1$, $E_{KP} = V_2$, $I_p = I_2$, the admittances y_{ij} are seen to be related to the tube parameters as follows:

$$\left.\begin{aligned} y_{11} &= g_g = \frac{1}{r_g} \\ y_{12} &= g_{gp} \\ y_{21} &= g_{pg} = g_m \\ y_{22} &= g_p = \frac{1}{r_p} \end{aligned}\right\} \qquad (1\text{-}58)$$

The relations of (1-58) apply very well for frequencies of the order of 1000 cps or lower. In this frequency range, an equivalent a-c circuit may be based on (1-57), as shown in Figure 1-26. For zero conduction grid current, g_g is zero (r_g infinite) and $g_{gp} = 0$, and the circuit of Figure 1-26 reduces to that previously used in Section 1-15, Figure 1-21.

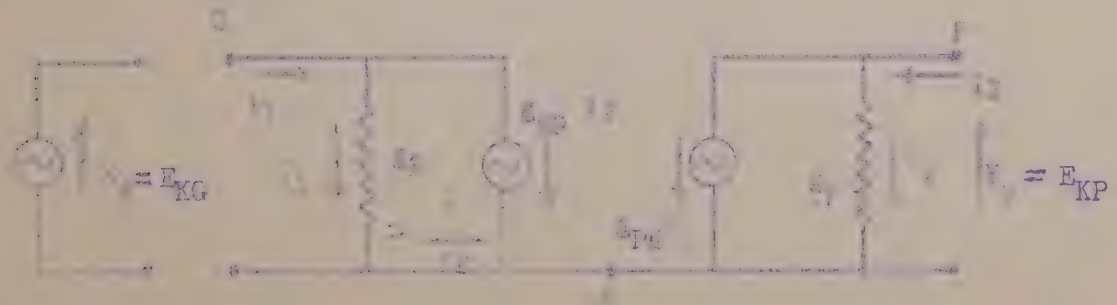


Figure 1-26.
Equivalent a-c circuit for a triode at low frequencies.

At frequencies of the order of 1000 cps to 1 or 2 megacycles per second, capacitive susceptances within the tube are not to be ignored. The interelectrode capacitances of the triode constitute a passive four-terminal network between input and output terminals as shown in Figure 1-27. The word passive, as used in connection with the network of Figure 1-27, refers to the fact that there are no generators or sources of energy within the network.

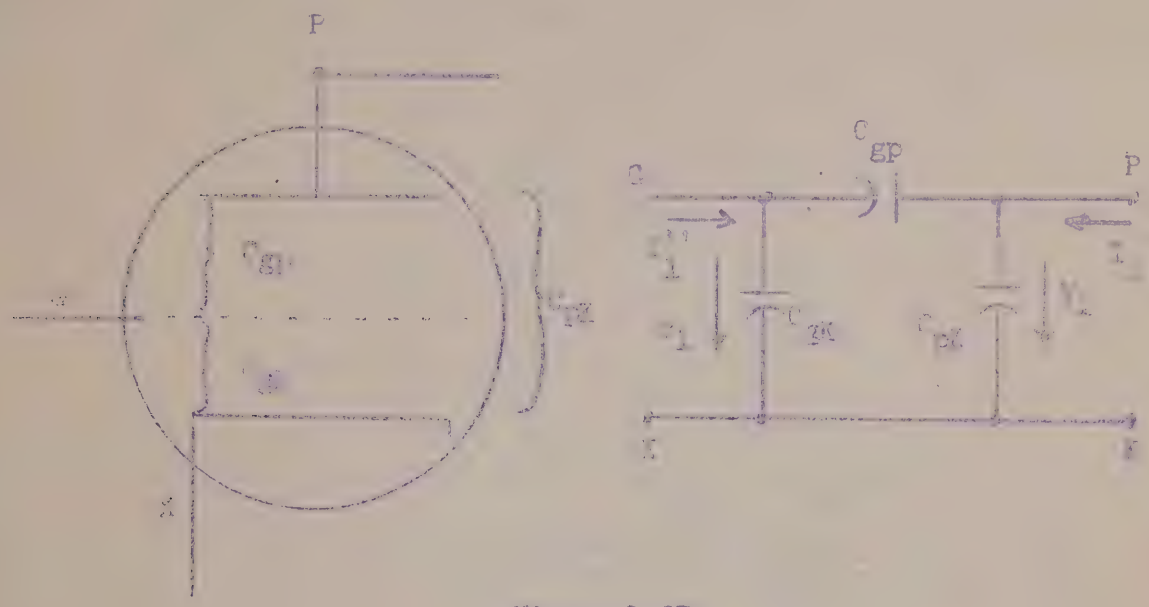


Figure 1-27.

Triode interelectrode capacitances and equivalent passive 4-terminal network.

The contributions of the elements of the passive circuit of Figure 1-27 to the admittances y_{ij} are now obtainable by superposing the currents of the circuit of Figure 1-27 with the currents of the circuit of Figure 1-26. Let the currents and admittances of Figure 1-26 be distinguished from those of Figure 1-27 by the use of single primes for Figure 1-26 and double primes for Figure 1-27. Then, for Figure 1-26,

$$I_1' = V_1 y_{11}' + V_2 y_{12}'$$

$$I_2 = V_1 y_{21} + V_2 y_{22} \quad , \quad - -$$

[illegible] in Figure 1-27

$$I_1 = V_1 y_{11}'' + V_2 y_{12}''$$

$$I_2 = V_1 y_{21}'' + V_2 y_{22}''$$

When the input currents of the two circuits are superposed, as is physically true with the two circuits connected in parallel,

$$I_1 = I_1' + I_1'' = V_1 (y_{11}' + y_{11}'') + V_2 (y_{12}' + y_{12}'')$$

and

$$I_2 = I_2' + I_2'' = V_1 (y_{21}' + y_{21}'') + V_2 (y_{22}' + y_{22}'') .$$

Finally, the y_{ij} for the complete circuit are given by

$$\left.\begin{aligned} y_{11} &= y_{11}' + y_{11}'' \\ y_{12} &= y_{12}' + y_{12}'' \\ y_{21} &= y_{21}' + y_{21}'' \\ y_{22} &= y_{22}' + y_{22}'' . \end{aligned}\right\} \qquad (1\text{-}59)$$

The values of y_{ij}' are given by (1-50). The y_{ij}'' admittances of Figure 1-27 are obtained by applying the fundamental definitions. With a short circuit placed first at the output and next at the input terminals, and with a driving generator connected at the unshorted terminals, the results are as follows:

$$\left.\begin{aligned} &\text{(a)} \quad V_2 = 0, \\ &y_{11}'' = \frac{I_1''}{V_1} = j\omega (C_{gK} + C_{gp}) \\ &y_{21}'' = \frac{I_2''}{V_1} = \frac{-V_1 (j\omega C_{gp})}{V_1} = -j\omega C_{gp} \\ &\text{(b)} \quad V_1 = 0, \\ &y_{12}'' = \frac{I_1''}{V_2} = \frac{-V_2 (j\omega C_{gp})}{V_2} = -j\omega C_{gp} \\ &y_{22}'' = \frac{I_2''}{V_2} = j\omega (C_{pK} + C_{gp}) . \end{aligned}\right\} \qquad (1\text{-}60)$$

The results of (1-59) and (1-60) are combined in accordance with (1-59) and the correct values of y_{ij} in the frequency range 1000 to 2,000,000 cps are as follows:

$$\left.\begin{aligned} y_{11} &= g_g + j\omega(C_{gK} + C_{gp}) \\ y_{12} &= g_{gp} - j\omega C_{gp} \\ y_{21} &= g_{pg} - j\omega C_{gp} \\ y_{22} &= g_p + j\omega(C_{pK} + C_{gp}) \end{aligned}\right\} \qquad (1\text{-}61)$$

1-30. Gain and Input Admittance of a Triode Amplifier.

The relations of (1-50), (1-52), and (1-61) provide the information needed to formulate the expressions for forward voltage gain and input admittance of a triode amplifier. A case of particular interest is that for which the grid bias is sufficient to reduce grid conduction current to negligible values. For this case $g_g = 0$, and $g_{gp} = 0$. The voltage gain (forward) is, from (1-50),

$$A = -\frac{y_{21}}{y_{22} + Y_2} = -\frac{g_m - j\omega C_{gp}}{g_p + Y_2 + j\omega(C_{pK} + C_{gp})} \qquad (1\text{-}62)$$

The input admittance, from (1-52) is

$$Y_{11} = y_{11} + A y_{12} = j\omega(C_{gK} + C_{gp}) + A(-j\omega C_{gp})$$

$$= j\omega\left[C_{gK} + (1 - A) C_{gp}\right] . \qquad (1\text{-}63)$$

For low frequencies, (1-62) reduces to (1-46).

A calculation of gain and input admittance for a specific case will illustrate the order of magnitude of A and of Y_{11}. Calculations will be given for the circuit of Figure 1-21, with the tube assumed to be a Type 6C5 for which $g_m = 2000$ micromhos, $r_p = 10{,}000$ ohms, $C_{gp} = 2$, $C_{gK} = 3$, and $C_{pK} = 11$ micromicrofarads. The load impedance $Z_L = 20{,}000$ ohms. Then, $g_p + Y_L = 150 \cdot 10^{-6}$, and Equation (1-62) becomes

$$A = -\frac{(2000 - j\,2\omega \cdot 10^{-6})\,10^{-6}}{(150 + j\,13\omega \cdot 10^{-6})\,10^{-6}} .$$

If the frequency is 796 cps, $\omega = 5000$ radians per second, and $j\,2\omega\ 10^{-6}$ $= j\ .01$ mho; $j\,130\omega\ 10^{-6} = j\ .065$ mho. At this low frequency the simpler relation (1-46) applies, and

$$A = -\frac{2000}{150} = -13.3$$

Also,

$$Y_{11} = j\,5000\left[3 \cdot 10^{-12} + \{1 - (-13.3)\}\ \ 2 \times 10^{-12}\right]$$

$$= j\,5000\,(31.6) \cdot 10^{-12}$$

$$= j\,0.158 \cdot 10^{-6} \text{ mho.}$$

$$Z_{11} = \frac{1}{Y_{11}} = -j\,6.34 \text{ megohms.}$$

The equivalent triode amplifier input circuit, for 796 cps, is then simply a capacitance of 31.6 $\mu\mu$f. If the frequency is increased to 1193 kilocycles per second, $\omega = 10 \cdot 10^6$, and

$$A = -\frac{2000 - j\,20}{150 + j\,130} = -7.6 + j\,6.67$$

$$= -10.1\ \angle -41.4^\circ$$

$$Y_{11} = j\,10 \cdot 10^6\left[3 + (8.6 - j\,6.67)\,2\right] \cdot 10^{-12}$$

$$= (133.4 + j\,202) \cdot 10^{-6} \text{ mho.}$$

This admittance may be represented by a pure resistance R_{11} in parallel with a pure capacitance C_{11} where

$$R_{11} = \frac{10^6}{133.4} = 7500 \text{ ohms}$$

$$X_{11} = \frac{10^6}{202} = 4950 \text{ ohms}$$

$$C_{11} = 20.2\ \mu\mu\text{f.}$$

To an external voltage source at 1593 kc, this 6C5 amplifier circuit then presents an impedance which is the equivalent of Figure 1-28.

Figure 1-28.
Equivalent input circuit of an amplifier circuit at 1593 kc.

The dependence of the input admittance of the triode amplifier upon C_{gp} is brought out by the foregoing computations. The factor (1-A), Equation (1-63), emphasizes the contribution of the grid-plate capacitance in increasing the input admittance. It is desirable that Y_{11} for an amplifier be as small as possible. This requirement has led to the inclusion of additional grids in vacuum tubes between the plate and the control grid in order to reduce the control-grid to plate capacitance [illegible] Chapter II.

1-21 A Generalized Equivalent Circuit for Linear Operation.

As stated in the preceding section, it becomes necessary to include [illegible] closely approximate the actual tube behavior in the range of frequency from 1 kilocycle to about 2 megacycles per second. The equivalent circuit applicable in this [illegible] consists of the circuits of Figures 1-26 and 1-27 superposed

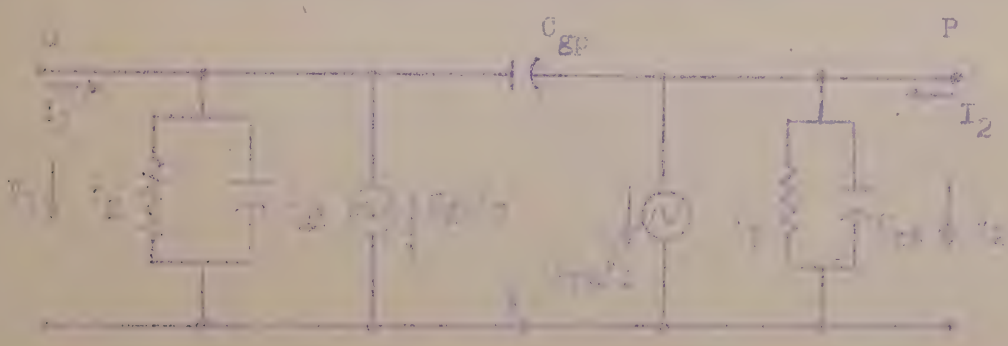


Figure 1-29
Triode Equivalent Circuit for frequencies up to 1 or 2 megacycles.

The electrodes of a high vacuum electron tube actually in or adjacent to the electron stream must be connected to the terminals available external to the tube envelope. These connecting leads have distributed inductance and capacitance of such values as to be negligible in the frequency range of applicability of the circuit of Figure 1-29, but begin to introduce increases in the driving point, transfer and feedback admittances for frequencies of the order of 10 to 100 megacycles. An attempt to represent these additional circuit complexities is made in Figure 1-30. It is evident that the circuit has now become very complicated for analysis, but of greater importance is the fact that the new circuit elements cannot be determined individually from measurements made at the external terminals.

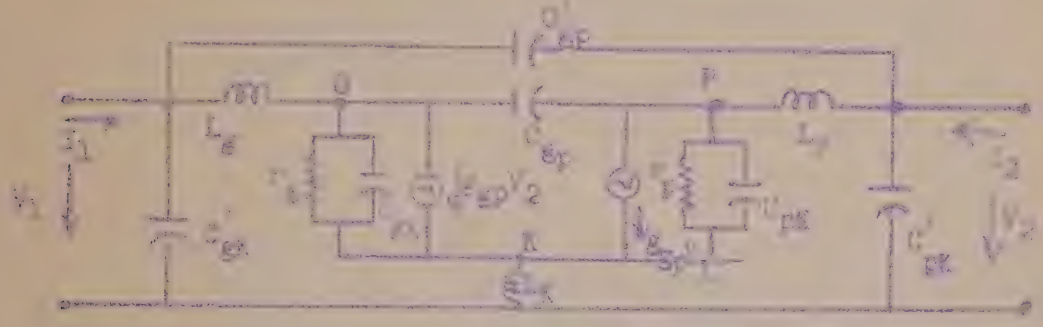


Figure 1-30
Equivalent a-c circuit as complicated by lead-wire inductances and capacitances for frequencies of the order of 10 to 100 mc.

At still higher frequencies of the order of 1000 to 10,000 megacycles, the circuit admittances y_{jj} become modified by the effects of the finite transit time of the electrons across the interelectrode space, and coupling circuit effects may still be of equal or greater importance in spite of improvements which are of necessity made in the input and output coupling circuits in the higher frequency range above 100 mc.

It is at this point that the 4-pole approach to the tube equivalent circuit offers its most important contribution. The additional circuit complexities are linear, and may be lumped together into the admittances y_{11}, y_{12}, y_{21}, and y_{22} which may be determined by direct measurement in the frequency range of interest. These admittances as measured at the external terminals contain all coupling circuit and transit-time effects. It is then possible to prepare a generalized equivalent circuit using only the admittances y_{ij}, as shown in Figure 1-31.

Figure 1-31.

Generalized equivalent a-c circuit.

At the present time several industrial electron tube research laboratories are engaged in a program of measurement of the 4-pole admittances of electron tubes used in the frequency range 100 to 10,000 megacycles. The problem of circuit design and application becomes a straightforward analysis of the circuit of Figure 1-31 when data on the values of Y_{ij} as dependent upon frequency are made available.

Chapter I. PROBLEMS

1-1. The tube characteristic of a certain diode is assumed to be linear. It is found that the anode current is 50 ma for an anode voltage of 400 volts.

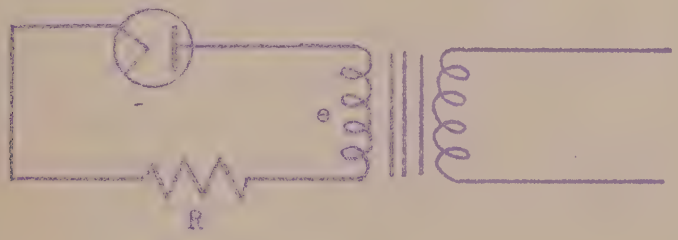
(a) Determine the equation of and sketch the anode current - anode voltage characteristic.

(b) When the tube is connected in series with the secondary of a transformer in which the secondary voltage is given by $e = 300 \sin 377t$, determine the equation of and sketch the anode current as a function of time.

1-2. Solve problem 1-1 if the tube characteristic is assumed to be a parabola, concave up, and if again, $i_b = 50$ ma for $e = 400$ volts. Compute the plate resistance for $i_b = 10$ and for $i_b = 40$ ma.

1-3. Experimental data on the tube characteristic of a Type 81 diode are given in the table. If this diode is connected as shown, determine graphically and sketch the plate current, given $R = 1000$ ohms, $e = 80 \sin \omega t$.

volts	e_b	20	40	60	80	100
ma	i_b	15	47	83	129	177



1-4. The plate current of a certain diode is given by Equation (1-1) and $i_b = 200$ ma. when the plate voltage is 100 volts. Compute the plate resistance at 25 volts and at 100 volts.

1-5. For a certain triode, the plate current is given by

$$i_b = 134 \cdot 10^{-6} (e_c + 0.125\, e_b)^{1.67}$$

For $e_c = -20$ volts, $e_b = 350$ volts, find:

(a) i_b, r_p, g_m, μ;

(b) the required plate battery voltage if the operating point is $E_b = 350$, $E_c = -20$ volts, and $R_L = 10{,}000$ ohms.

1-6. Refer to the amplifier circuit of Figure 1-11. If the tube is a 6C5, its characteristics are given by Figures 1-7, 1-8, and 1-9, or by the tube manual.

(a) Taking $E_{bb} = 400$ volts, $E_{cc} = -8$ volts, and $R_L = 40{,}000$ ohms, draw the load line, and the dynamic characteristic and locate the operating point.

(b) If $e_g = 2 \sin \omega t$, $\omega = 5000$ radians per second, determine graphically the effective value of the alternating component of the plate current assuming linear operation.

(c) Sketch e_g on the mutual characteristics and locate graphically points for sketching the instantaneous values of plate current and plate voltage. Check the phase relation between the alternating components of grid voltage, plate current, and plate voltage.

1-7. For the 6C5 in the circuit of problem 1-6,

(a) obtain data from the tube characteristics for plotting anode-grid voltage characteristics similar to Figure 1-9 for $I_b = 6$ ma.;

(b) determine the tube amplification factor;

(c) find r_p and g_m at the operating point and check the relation $\mu = r_p g_m$;

(d) find the amplifier voltage gain by graphical methods and compare the value obtained with that computed from $\frac{\mu R_L}{r_p + R_L}$.

1-8. An amplifier circuit is that of Figure 1-21(a) except that a resistor R_K is inserted in the cathode lead to provide bias voltage equal to the voltage drop $I_b R_K$ due to the d-c component of plate current flowing through R_K. The bias battery E_{cc} is removed. Derive a formula for the gain of the amplifier.

(a) using the constant voltage equivalent circuit;

(b) using the constant current equivalent circuit.

1-9. A 6C5 triode is connected as an amplifier with resistance load, $R_L = 40{,}000$ ohms. With conditions identical with those given in problem 1-6, draw the a-c equivalent plate circuit, find the alternating component of plate current (effective value) and compare with the result obtained by graphical means in problem 1-6. Draw the vector diagram and show the phase relation between E_g and the voltage across R_L.

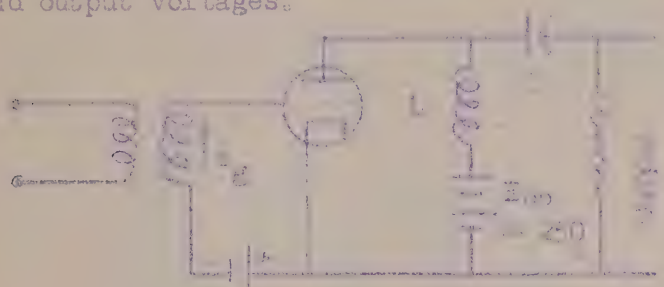
1-10. The triode in the amplifier circuit shown has $r_p = 7700$ ohms, $\mu = 20$, $C = 0.15\,\mu$fd., $R_L = 10{,}000$ ohms, $L = 5$ henrys.

(a) Draw the equivalent a-c plate circuit.

(b) Determine the effective alternating currents in choke and load resistor for $e_g = 4 \sin 5000\,t$.

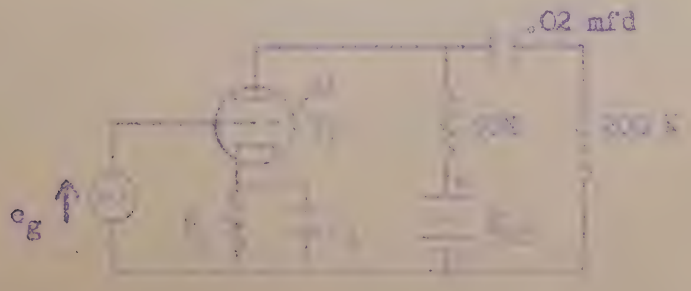
(c) Find the gain of the amplifier.

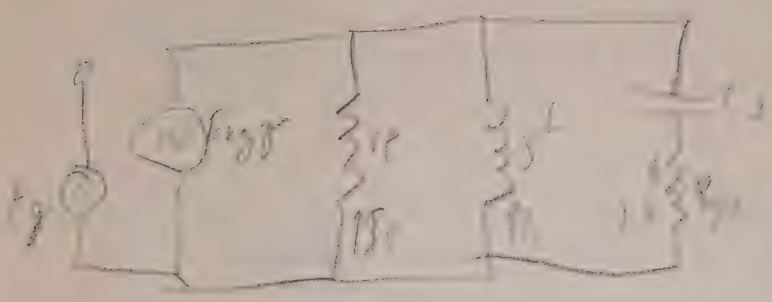
(d) Draw a complete vector diagram using the voltage across R_L as reference, and find the phase relation between the input and output voltages.

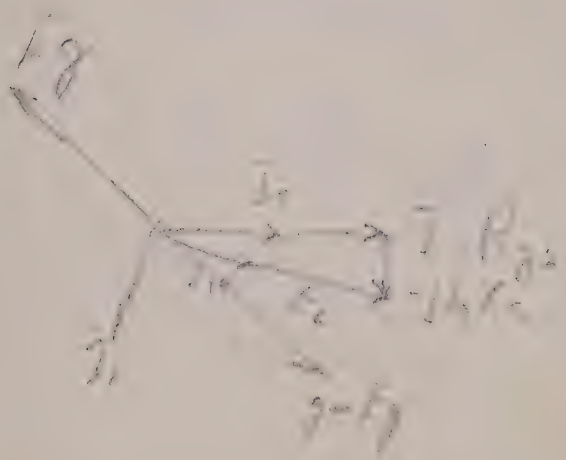


$E_{cc} = -6$ volts

1-11. In the circuit shown, $\mu = 20$, $r_p = 10{,}000$, $C_1 = 10\,\mu$fd. The drop $I_b R_1$ in the bias resistor R_1 is to be used instead of a battery for grid bias. If the quiescent point is to be $I_b = 5$ ma., $E_c = -6$ volts, determine R_1. Find the voltage gain of the amplifier and the phase angle between input and output voltages if $e_g = 4 \sin 10{,}000\,t$. Draw a complete vector diagram. $K = 1000$ ohms.







1-12. The accompanying circuit can be used in determining g_m.

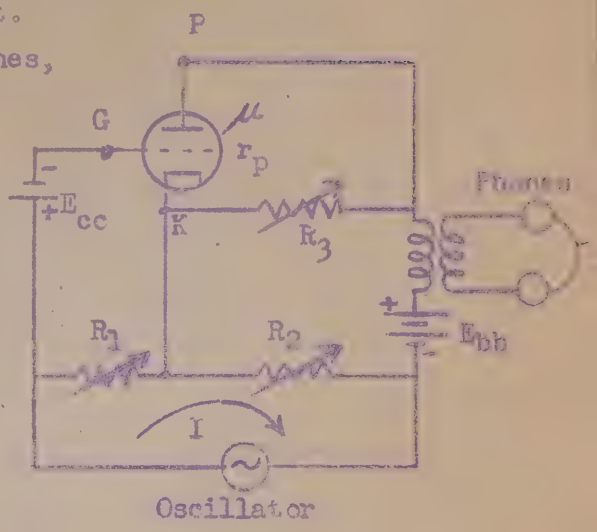
(a) Draw the equivalent a-c anode circuit.

(b) Show that for silence in the headphones,

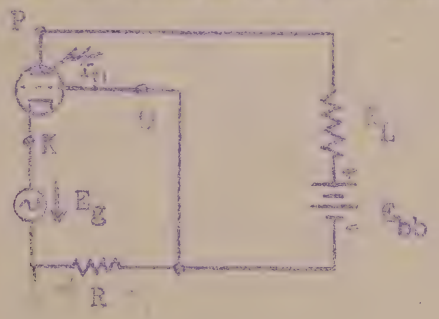
$$r_p = R_3\left(\mu\frac{R_1}{R_2} - 1\right).$$

(c) If $\mu\frac{R_1}{R_2} >> 1$, show that

$$g_m \cong \frac{R_2}{R_1 R_3}$$



1-13. Determine the gain of the circuit shown by deriving from the equivalent a-c plate circuit an expression for the gain in terms of μ, R_L, r_p, and R.



1-14. A type 2A3 triode is connected as an amplifier with cathode bias resistor (as in Problem 1-11), but with no by-pass condenser. The load resistance is 2450 ohms, $r_p = 800$ ohms, $\mu = 4.2$. The tube is to operate at $E_c = -45$ volts, $I_b = 60$ ma., $E_b = 250$ volts.

(a) Determine the necessary grid bias resistor and plate battery voltage.

(b) Compute the gain of the amplifier.

1-15. Sketch the complete equivalent circuit of a triode for which $i_c \neq 0$, and include all interelectrode capacitances.

Inside Feedback Loop

1. Input impedance varies with frequency

2 Feedback reduces distortion

KNOW

Chapter II

Tetrodes, Pentodes and Equivalent Circuits

2-1. Ideal Plate Characteristics.

Triode plate characteristics described in Chapter I show the dependence of plate current upon both plate and grid voltages. The plate voltage is ordinarily supplied from a direct current power source of fixed voltage rating, and the control of plate current is desirably a function of the grid voltage only. Ideal characteristics for many applications would consist of nearly horizontal, parallel straight lines covering the entire plate diagram. A horizontal anode current-anode voltage characteristic indicates that the plate current is independent of anode voltage. Control of plate current in such a case would depend upon grid voltage only.

Plate characteristics approximating the ideal may be obtained in vacuum tubes by using additional grids between cathode and plate. The presence of an additional grid between the grid as used in the usual triode arrangement and the anode of the tube greatly reduces the magnitude of the grid-plate capacitance. The result of such a reduction of C_{gp} is a very desirable decrease in the tube input admittance, as shown by Equation (1-63), page 42 of Chapter I.

2-2. The Tetrode or Screen-Grid Tube.

An arrangement of two grids in a vacuum tube is shown in Figure 2-1 The grid nearer to the cathode is now called the control grid, and

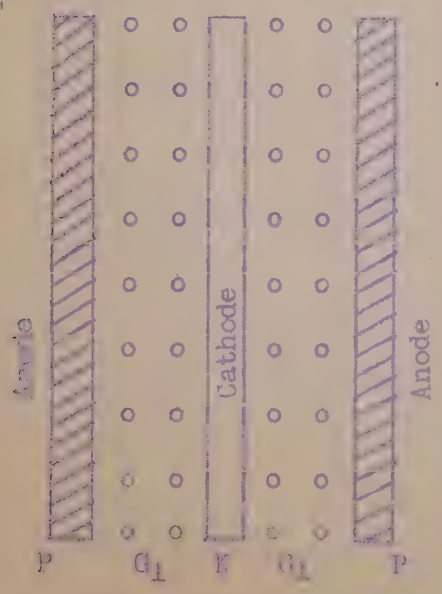


Figure 2-1.

Sectional view of electrode arrangement in a cylindrical tetrode.

G_1 and G_2 are control and screen-grid spirals.

the second grid because of its function of screening the plate from cathode and control grid is called the screen grid. It is usually necessary to number grids consecutively in multielement tubes beginning with the grid nearest the cathode as number one. The four-electrode high vacuum tube is called the tetrode.

2-3. Tetrode Static Characteristics.

The shielding effect of the two grids between plate and cathode of the tetrode is complete enough that it becomes necessary to operate the screen grid at high positive voltages in order to obtain appreciable plate current. A connection diagram for a tetrode operated as an amplifier is shown in Figure 2-2. In connecting a tetrode to its d-c power source, the screen voltage should never be applied until the plate supply is connected, thus avoiding the danger of the entire tube current flowing to the screen and wrecking the tube.

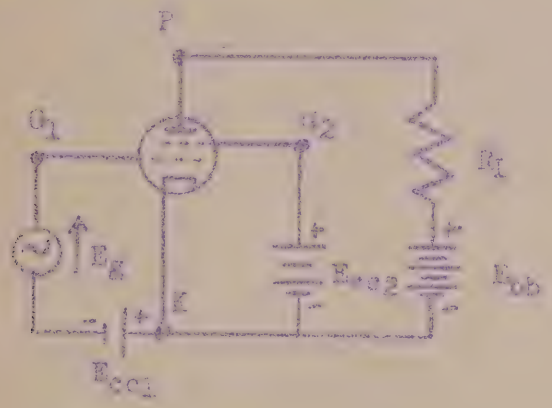


Figure 2-2.

Connection diagram for a tetrode.

Plate and mutual characteristics for a typical screen-grid tube are shown in Figure 2-3. As compared with a triode, the following differences should be observed:

(1) The slope, $\frac{\partial i_b}{\partial e_b}$ is smaller, and therefore the plate resistance is much larger than that of a triode, in the operating region to the right of $e_b = 125$ volts.

(2) For $e_b > 125$ volts, the plate current depends relatively little, compared with the triode, upon the plate voltage.

(3) The slope, g_m, of the mutual characteristics is of the same order of magnitude as for a triode.

ενδυ

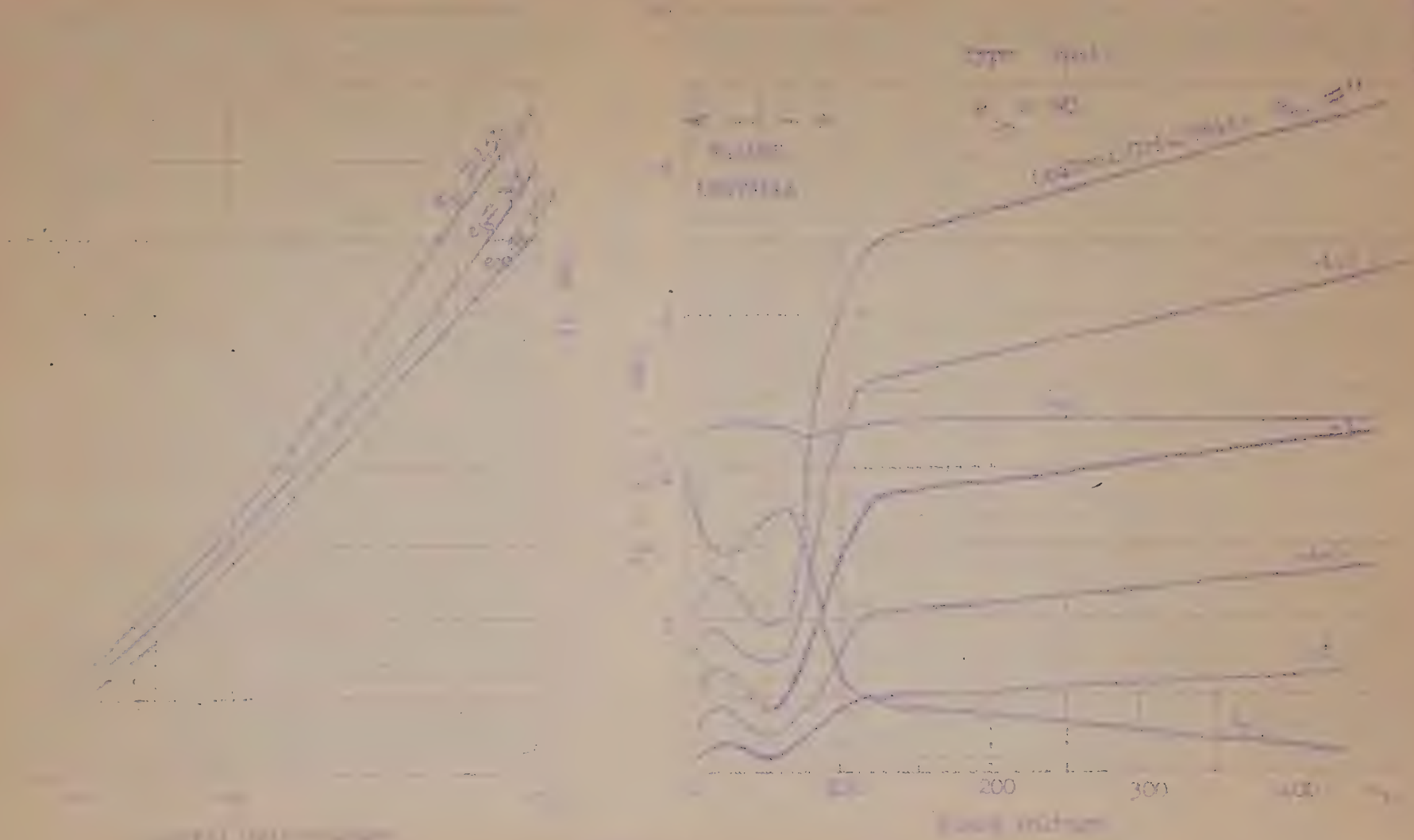


Figure 3-3.

(4) For a triode, $i_b = f(e_b, e_c)$. For the screen-grid tube, the corresponding relation is

$$i_b = F(e_b, e_{c_1}, e_{c_2}). \qquad (2\text{-}1)$$

However, e_{c_2} is held constant, and i_b is relatively independent of e_b, so that it may be said that the addition of the fourth electrode has reduced rather than increased, the number of independent variables. Approximately, then

$$i_b = f(e_{c_1}). \qquad (2\text{-}2)$$

The accuracy of the approximation is measured by the closeness of the mutual characteristics, as shown in Figure 2-3.

It should be observed that the plate characteristic for e_b between 20 and 50 volts has a negative slope, indicating a negative plate resistance. This portion of the tetrode characteristic has had practical application and use as the operating range of a dynatron oscillator. For operation as an amplifier, however, the useful plate voltage range is that above $e_b = 100$ volts. Both instability and distortion would be introduced if operation extends into the region of the plate diagram below 100 volts. The problem of distortion will be considered later in the chapter.

2-4. Secondary Emission in the Tetrode.

The plate current maximum and minimum and the negative plate resistance which are typical of the tetrode plate characteristics at low plate voltages are caused by the occurrence of secondary electron emission from the plate. Secondary electrons are emitted from an electrode only if the incident primary electrons have a certain minimum kinetic energy at impact. For the tetrode of Figure 2-3, this minimum kinetic energy is attained at approximately 20 volts. With the screen voltage at 90 volts, and the voltage of the plate lower than that of the screen, secondary electrons set free at the anode are accelerated toward the screen. As the plate voltage rises and the secondary emission yield increases, the screen current increases and the plate current decreases, as shown by Figure 2-3, until the current minimum is reached. At this plate voltage, secondary emission has its maximum effect upon plate current. After the plate voltage reaches equality with the screen grid voltage, secondary electrons leave the plate only because of initial emission velocities. For plate voltages greater than the screen voltage secondary emission from the plate still occurs, but since the plate potential is higher than the potential of the screen, secondary electrons return to the plate and no effects of secondary emission are observable. Secondary electrons may also be released at the screen. At high plate voltage, such secondary electrons

would serve to increase the current to the plate and decrease the current to the screen. Occasionally the secondary emission from a screen grid may be enough that the screen current actually reverses as the plate voltage increases, showing that a single primary electron may release more than one secondary electron upon collision with the screen. In many of the older tetrode tubes the plate current also reversed at low plate voltage because of a secondary electron yield ratio (number of secondary electrons released by one primary electron) greater than one. In the modern tubes the anodes are covered with certain surface materials such as graphite which decrease the secondary emission, and the effects of secondary emission upon anode current are considerably reduced.

2-5. Tetrode Equivalent Alternating Current Circuit.

In the operating range of the tetrode amplifier, the plate characteristics are quite linear. From the functional relation Equation (2-1), the change in plate current resulting from small changes in electrode voltages is

$$\Delta i_b = \frac{\partial i_b}{\partial e_b} \Delta e_b + \frac{\partial i_b}{\partial e_{c_1}} \Delta e_{c_1} + \frac{\partial i_b}{\partial e_{c_2}} \Delta e_{c_2} \qquad (2\text{-}3)$$

Exactly as for the triode, the change in current or in electrode voltage from quiescence is replaced by the instantaneous a-c component of current or voltage. Accordingly, $\Delta i_b = i_p$, $\Delta e_b = e_{KP}$, $\Delta e_{c_1} = e_{KG}$, and since the screen voltage is held constant $\Delta e_{c_2} = 0$. By definition,

$$\frac{\partial i_b}{\partial e_b} = \left[\frac{d i_b}{d e_b}\right] (e_{c_1}, e_{c_2} \text{ constant}) = \frac{1}{r_p} = g_p , \qquad (2\text{-}4)$$

and

$$\frac{\partial i_b}{\partial e_{c_1}} = \left[\frac{d i_b}{d e_{c_1}}\right] (e_b, e_{c_2} \text{ constant}) = g_m \qquad (2\text{-}5)$$

As for the triode, if i_b is held constant, $\Delta i_b = 0$, and Equation (2-3) provides the relation

$$\mu = - \frac{\partial e_b}{\partial e_{c_1}} = r_p g_m . \qquad (2\text{-}6)$$

Equation (2-3) then becomes

$$i_p = \frac{1}{r_p} e_{KP} + g_m e_{KG}$$

where Z_L is the equivalent impedance between K and P in the circuit outside the tube.

$$I_p = -\frac{E_p}{r_p} + g_m E_{KG} \qquad (2\text{-}7)$$

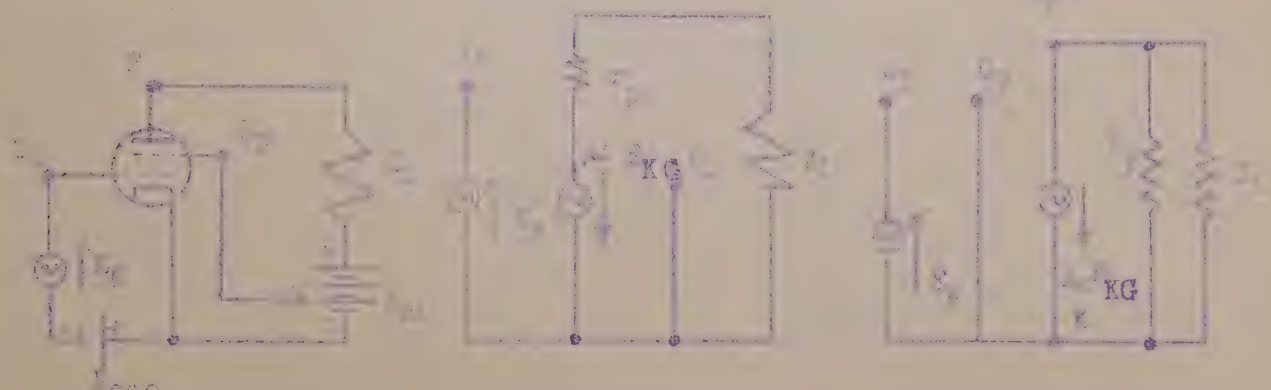
Equation (2-7) suggests either the constant voltage or constant current equivalent a-c plate circuit exactly as for the triode. Thus,

$$E_p = \mu E_{KG} - I_p Z_L$$

suggests an equivalent a-c circuit consisting of a generator of constant voltage, μE_{KG} and internal resistance r_p connected across the load impedance Z_L. If, however, Equation (2-7) is written as

$$g_m E_{KG} = I_p + \frac{I_p Z_L}{r_p} \qquad (2\text{-}8)$$

it suggests an equivalent a-c circuit consisting of a constant-current generator of current $g_m E_{KG}$ supplying current I_p and $\frac{I_p Z_L}{r_p}$ to the impedances Z_L and r_p connected in parallel across the terminals of the generator. The actual and equivalent a-c circuits of the tetrode amplifier are shown in Figure 2-4. The equivalent circuits of Figure 2-4 are identical with those used for the triode. Either the constant-voltage or the constant-current generator may be used in the equivalent a-c plate circuits of both triode and tetrode amplifiers, and the same considerations apply to both. Since the plate resistance r_p of the tetrode is much higher than that of the triode, the constant-current equivalent generator is usually employed for tetrodes. If $r_p >> |Z_L|$, the plate resistance might be omitted in Figure 2-4(c), giving an additional simplification.



(a) Actual circuit (b) Constant voltage (c) Constant current

Figure 2-4. Actual and equivalent circuits of a tetrode at audio frequencies

If all the interelectrode capacitances are included, the equivalent a-c circuit is that of Figure 2-5. Since the screen-cathode capacitance is short circuited for a-c by the screen connection to cathode through the screen supply voltage, C_{g2k} may be omitted. Capacitances C_{pk} and C_{g2p} provide parallel paths from the plate to the cathode and so may be combined. Also, C_{g1k} is in parallel with C_{g1g2} through the screen connection to cathode. By combining the parallel capacitances and neglecting C_{g1p} (which the presence of the screen makes negligible small), the circuit may be simplified as shown in Figure 2-6 in which

$$C_{11} = C_{g1k} + C_{g1g2}$$

and

$$C_{22} = C_{pk} + C_{g2p}$$

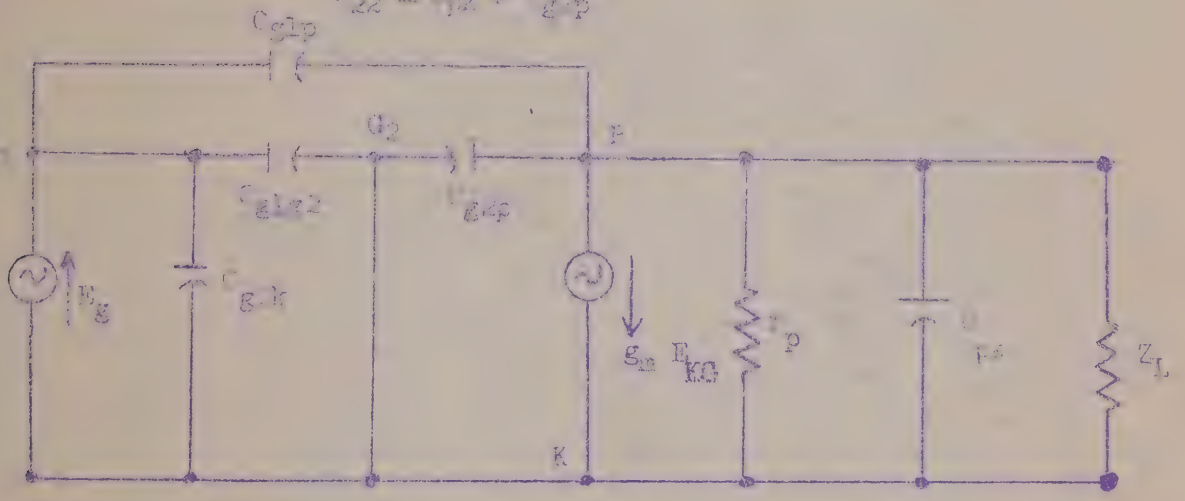


Figure 2-5.

Complete equivalent a-c circuit of a tetrode amplifier.

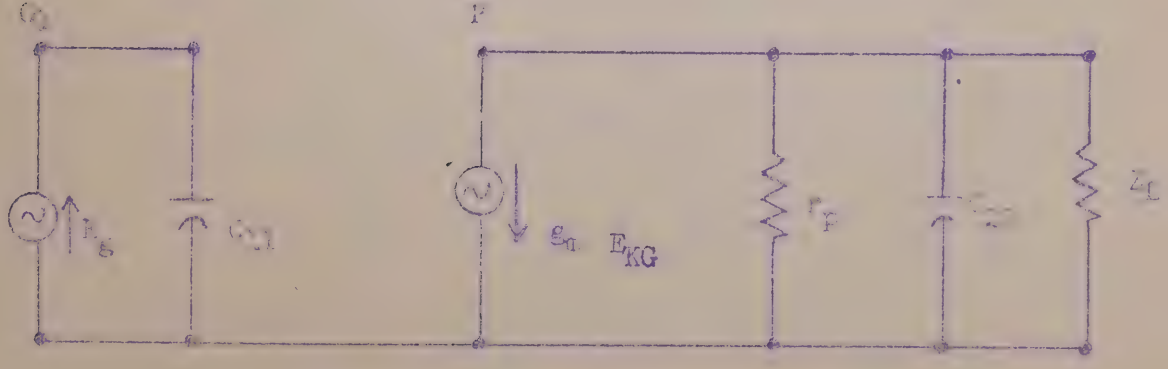


Figure 2-6

Simplified equivalent tetrode a-c circuit.

The input admittance is then

$$Y_{11} = j\omega C_{11} .$$

The output capacitance C_o is not appreciably different from C_{g2p}, because of the shielding effect of the screen between plate and cathode. The capacitances are now small enough that connecting lead wire capacitances may become equally important so that it becomes necessary to provide shielding for screen grid tubes to realize best results at frequencies much above the audio range.

2-6. Suppression of Secondary Emission. The Pentode.

Secondary emission electrons from the anode are attracted to the screen only for plate voltages which are lower than the fixed voltage applied to the screen grid. The secondary electrons released at the anode would be forced to return to that electrode if its potential were higher than any potential in the immediate vicinity. Such a space distribution of potential between a positive screen grid and a less positive anode can be obtained if a potential minimum can be provided between screen and plate. There are two methods in present use which provide the required potential minimum. One of these utilizes a grid of wires aligned with screen and control grid wires, located between screen and anode, and held at zero potential by direct connection to the cathode. The resulting five-electrode tube is called a pentode. The purpose of the third grid is to suppress the secondary emission current to the screen, and it is called the suppressor grid. The connection diagram of Figure 2-7 is drawn for a tube in which the suppressor-to-cathode connection is made externally, that is, outside the tube envelope, such as for the type 59, and the 6J7. With the suppressor connection brought out, the tube can be operated as a triode by connecting suppressor grid No. 3 to the plate and screen grid No. 2 to the control grid. In tubes such as the 6F6 the suppressor grid is connected to the cathode internally. This tube may still be operated as a triode by connecting the screen to the plate.

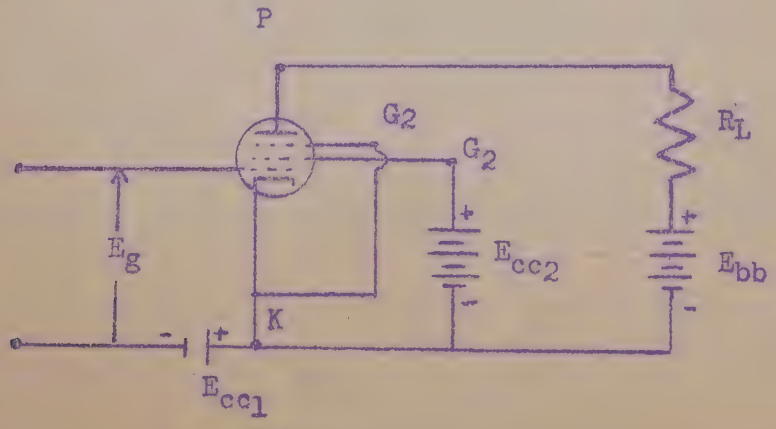


Figure 2- 7.
Connection diagram for a pentode amplifier.

Suppression of the secondary emission screen current eliminates the dip in the plate characteristic at the tetrode. Typical pentode plate characteristics are shown in Figure 2-8 for the 6F6. The dependence of the plate current upon plate voltage is less evident than for the tetrode. At a control grid bias of -30 or -35 volts, the plate current is virtually independent of plate voltage and the plate resistance is extremely high. The amplification factor, also, is much higher than for the triode or tetrode, but the control grid-plate transconductance is approximately the same. The plate resistance of the pentode is much more constant over the operating range of plate voltage than is the case for triodes. Depending upon the purpose for which the pentode is designed, r_p for different pentodes varies from about 10^5 to 10^6 ohms, and μ from 100 to 1500. Although the range of linearity of the pentode plate family extends over a greater area of the i_b-e_b graph sheet, the dynamic characteristic has somewhat the shape of the graph of a cubic equation and this results in a pronounced third harmonic in the pentode output, to be discussed in Section 2-10. It is chiefly the presence of the third harmonic which limits the undistorted power output of the pentode. It has been shown that the curved portions of the i_b-e_b characteristics for low plate voltages are responsible for the third harmonic in the pentode output. Since this portion of the characteristic is the result of the suppressor grid action being non-uniform across the path of the electrons, a type of suppressor was sought which did not require a mechanical grid. This research resulted in the beam power[1], or critical distance[2], tube.

2-7. Beam Power Tube.

Although the beam power tube is a tetrode, effects of secondary emission have been eliminated. Suppressor action in the beam power tube for example the 6L6, is obtained by the use of the effect of electron space charge in lowering the potential between screen and plate, and providing the required potential minimum at low plate voltages. Such a potential minimum between screen and plate will result from the presence of electron space charge if a sufficient current flows into this region. By careful alignment of control and screen grid wires and the use of beam forming plates at cathode potential, a sufficient electron space charge density is secured between screen and anode to produce a potential minimum in that region. In other words, the electrons in transit from screen to plate provide enough negative charge in the interelectrode space between these electrodes that a potential minimum is produced. The uniformity of the space charge density in the plane of the potential minimum provides a much more uniform suppressor effect than a mechanical grid with its inevitable potential variation from grid wire to grid wire.

Typical plate characteristics of a beam power tetrode are shown in Figure 2-9 Certain values of plate voltage and current in the region

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1. Schede, O. H., Proc. IRE - 26, 137-181, 1938.
2. Harries, H. H. O., Wireless Engineer 13, 190-199, 1936.

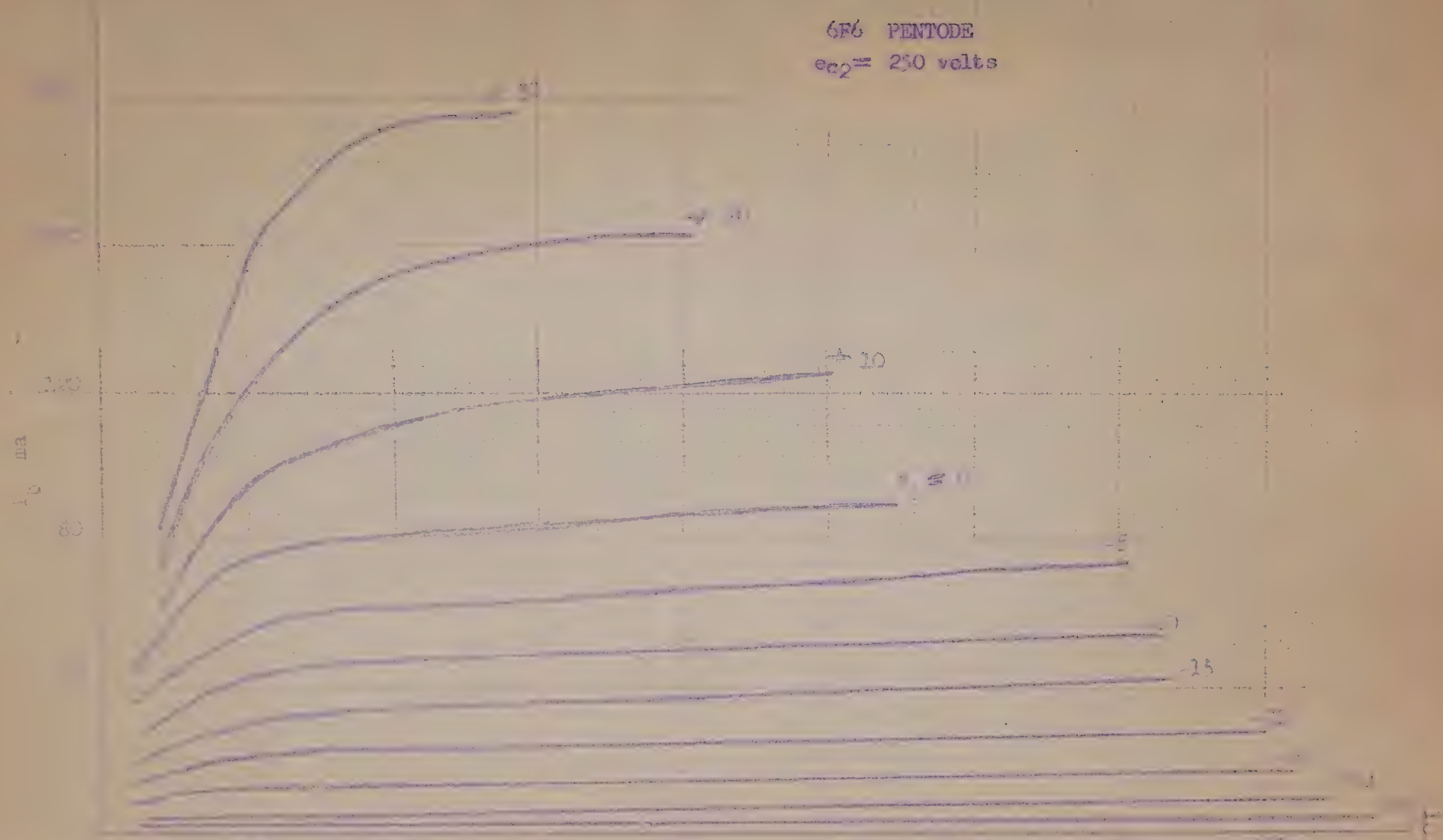

Figure 2-8.

of low plate voltage are critical in the operation of beam power tubes. By this is meant that sudden changes or instabilities occur in the plate current-plate voltage relation so that the current in this region is not a single-valued function of the plate voltage. Linear and stable amplifier operation is therefore confined to the region of the flat portions of the characteristics, but as Figure 2-9 shows, this occupies the greater part of the plate diagram. The electrode arrangement in a beam power tube is shown in the RCA Receiving Tube Manual.*

The term "critical distance" tube has been applied to the beam power tube because the distance between screen and anode, in addition to the electron beam space-charge density, helps to determine the requisite potential minimum.

The arrangement of electrodes in the beam power tube permit a considerable increase in the transconductance, g_m, for tubes of specified size and voltage rating, a fact of importance in the selection of tubes for amplifiers designed to operate over a wide frequency range. The beam power tube approaches the ideal power amplifier tube whose plate characteristics would be flat, parallel, and equally spaced over the entire plate diagram.

2-4. Equivalent Circuits of Pentode and Beam Power Amplifiers.

The relation between plate current and electrode voltages for the pentode is indicated, as was done for triode and tetrode by the relation

$$i_b = f(e_b, e_{c_1}, e_{c_2}, e_{c_3}). \qquad (2\text{-}9)$$

With screen (e_{c_2}) and suppressor (e_{c_3}) grid voltages held constant, changes in plate current from the value at quiescence (zero signal voltage) occur for changes in control grid voltage, e_{c_1}, and plate voltage, e_b. The result is identical with that for triode and tetrode, so that an identical equivalent circuit results. It is usually more convenient to use the constant current generator, and frequently the pentode plate resistance is so much higher than external circuit resistances that it may be omitted from the equivalent circuit if the constant current generator is used.

Since beam power tubes are tetrodes, their equivalent circuits are drawn in the same way as for triodes, tetrodes, or pentodes. Therefore, it may be stated that triode, tetrode, pentode, and beam power amplifiers may be represented in the linear region of their dynamic characteristics by exactly the same type of equivalent a-c plate circuit.

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* RCA Receiving Tube Manual, Fig. 9, page 9.

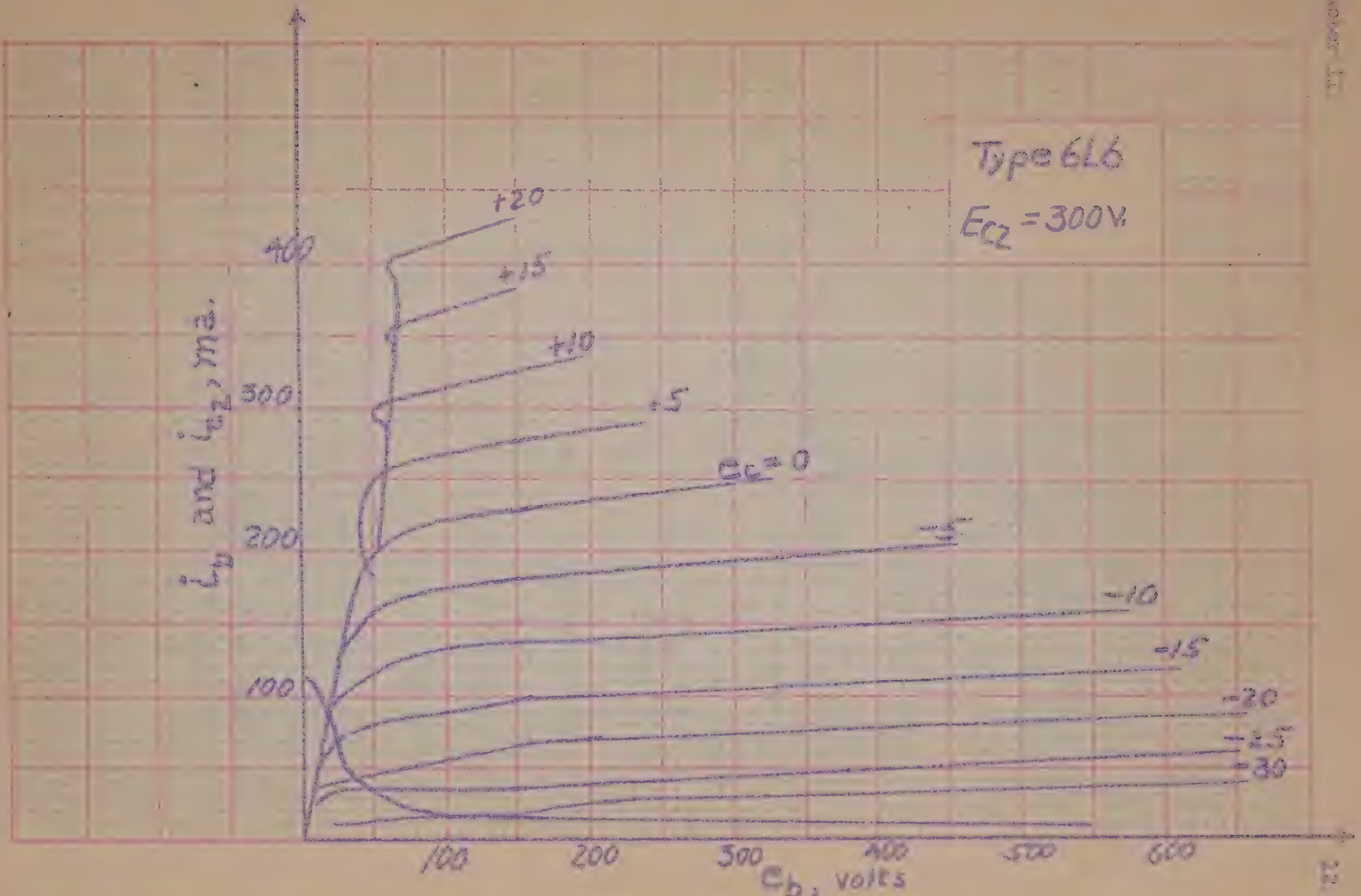
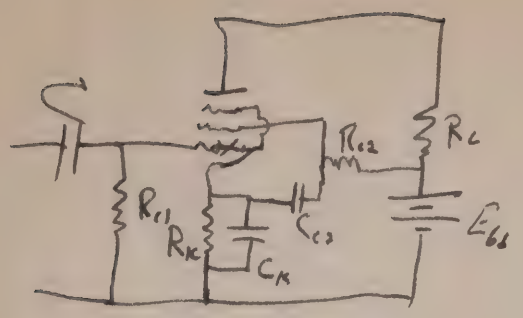

Type 6L6
E_{c2} = 300 v.
+20
+15
+10
+5
e_c = 0
-5
-10
-15
-20
-25
-30
400
300
200
100
i_b and i_{c2}, ma.
100
200
300
400
500
600
e_b, volts

Figure 2-9



$E_b = 300$

$E_{c2} = 150$

$E_{c1} = -2$

$I_b = 10$

$I_{c2} = 2.5$

$g_m = 9000\mu$

$r_p = 750000$

$R_L = 5000$

$f_{min} = 60\,N$

Find $E_{bb}, R_K, R_{c2}, C_K, C_{c2}, R_{c1}, C$

$E_{bb} = 352$

For pentodes, the capacitance between any electrode and each of the other four electrodes in the tube results in a total of ten condensers to be added to the complete equivalent circuit at high frequency. Of the ten capacitances, however, that between control grid and plate is negligible and those between screen and cathode and between suppressor and cathode are short-circuited, as shown in Figure 2-10. Other condensers are in parallel, so that they may be replaced by a single condenser of equivalent capacity. In this manner the circuit of Figure 2-10 reduces to a simpler circuit with a single input and a single output capacitance as in Figure 2-6. The relation of C_{11} and C_{22} to the interelectrode capacitances is left as an exercise.

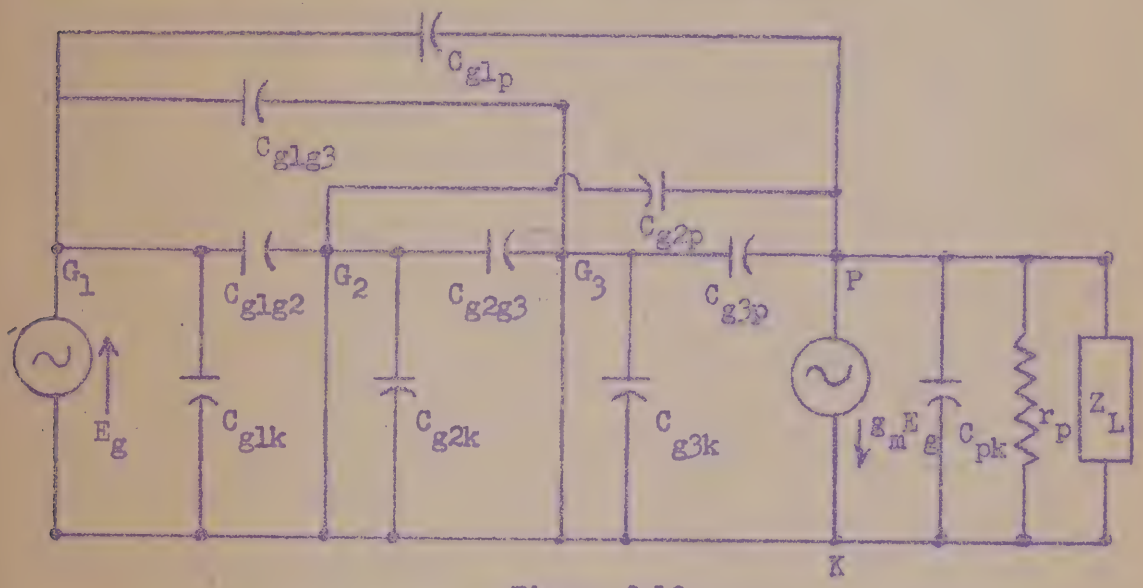


Figure 2-10.

Complete equivalent a-c circuit of a pentode.

2-9. Applications of Pentodes.

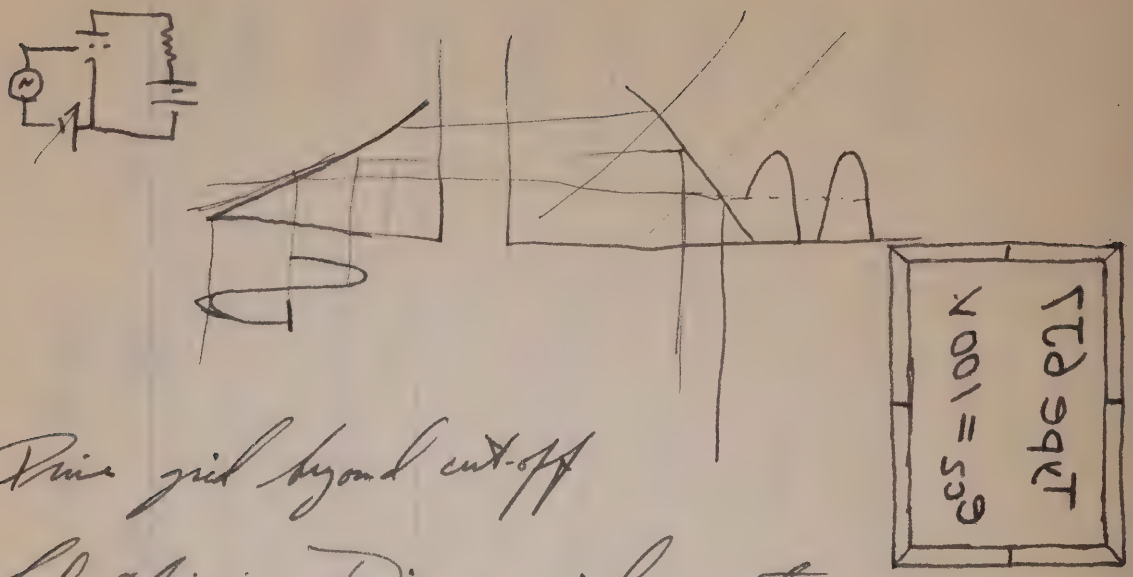
Pentode tubes designed for power amplification have fairly low plate resistances, in the range (for receiving type tubes) of 40,000 to 300,000 ohms. Representative power pentodes are the 6F6 and the 25A6. Voltage amplifier pentodes have much higher values of plate resistance. Representative types of receiving tubes are the 1N5-G, plate resistance 1.5 megohms, input capacitance 3.2 micromicrofarads, output capacitance 11 micromicrofarads, and the 6J7 (see Figure 2-11) of plate resistance 1 megohm. Such tubes are recommended for use in voltage amplifiers and as detectors.

A pentode may be used as a resistance in a circuit to provide a constant current. The type 6J7, for example, if biased to -3 volts, would pass approximately 2 milliamperes for any plate voltage between 100 and 500 volts, as an examination of Figure 2-11 will confirm. This is the basis for a number of applications of the pentode and reference will be made to this property of the tube in later chapters.

Type 6J7
e_{c2} = 100 V
i_b ma.
8
7
6
5
4
3
2
1
e_c = 0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-4.0
-5.0
80
160
240
320
400
e_b volts

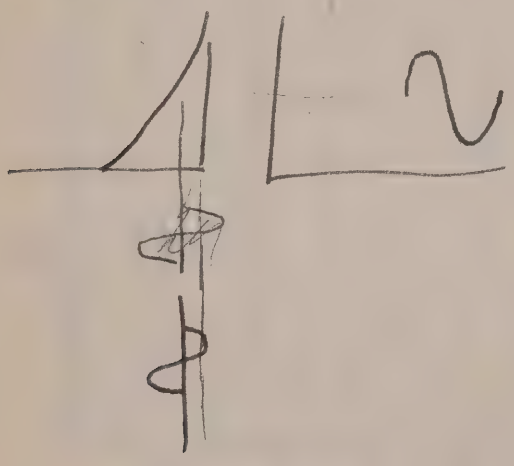
Figure 2-11.

Plate Clipping



Drive grid beyond cut-off

Grid Clipping Drive grid positive



Amplitude distortion

2-10. Distortion.

The term "distortion" has been referred to in earlier sections of this chapter. It is now proposed to define the term and to apply it in a further study of the high-vacuum tube and its characteristics as an amplifier.

The location of an operating point on the dynamic characteristic of a triode, tetrode, or pentode determines the possibility and extent of linear operation as already described in Chapter I. It will be shown in the present chapter that operation into the non-linear region of the dynamic characteristic will result in the introduction, into the output of an amplifier, of frequencies not present in the input. These frequencies are harmonics of fundamental input frequencies, and the presence of such harmonics in the amplifier output is defined as harmonic or amplitude distortion. From the point of view of Fourier analysis, harmonic distortion means that a sinusoidal (single frequency) input voltage will come out of the amplifier as a non-sinusoidal wave, and this wave will contain harmonic frequencies not present in the input.

The problem of determining the harmonic content of the output of an amplifier with sinusoidal single frequency input may be approached by analyzing the dynamic characteristic. With the origin of coordinates at the operating point, the equation of the dynamic characteristic is expressed as

$$i_p = f(e_g) . \qquad (2\text{-}10)$$

Such a relation is possible because the dynamic characteristic is the graphically obtained locus of corresponding values of grid voltage, plate voltage, and plate current. It is drawn on the i_b-e_c mutual characteristics graph, and $i_p = i_b - I_b$, $e_g = e_c - E_c$, as shown by Figure 1-13. By choosing an origin of coordinates at $e_c = E_c$, $i_b = I_b$, the equation of the dynamic characteristic referred to a set of i_p - e_g coordinate axes may be developed by expressing (2-10) as a power series. This use of a series approximation may be very simply explained by the statement that the graph of the actual dynamic characteristic is assumed to be expressible mathematically in terms of sums of powers of the signal voltage, e_g, and that as many terms of the series are used as are necessary to give the desired approximation.

An assumed dynamic characteristic has been drawn for an amplifier as shown in Figure 2-12. Let it be assumed that the curve of Figure 2-12 is represented by the infinite series

$$i_p = C_1 e_g + C_2 e_g^2 + C_3 e_g^3 + \cdots + C_n e_g^n + \cdots, \qquad (2\text{-}11)$$

in which the constants C_i may be determined as are the coefficients in a McLaurin Series or by selecting a number of points on the curve of Figure 2-12 and substituting into (2-11) the coordinates of such points. Simultan-

eous equations involving C_1, C_2, C_3, etc., are then obtained from a partial series, and the equations may be solved for the constants C_i. As many terms must be taken as are needed to approximate the actual curve to the desired accuracy, but of course the number of points taken from the actual curve must equal the number of constants to be computed in order to provide sufficient equations.

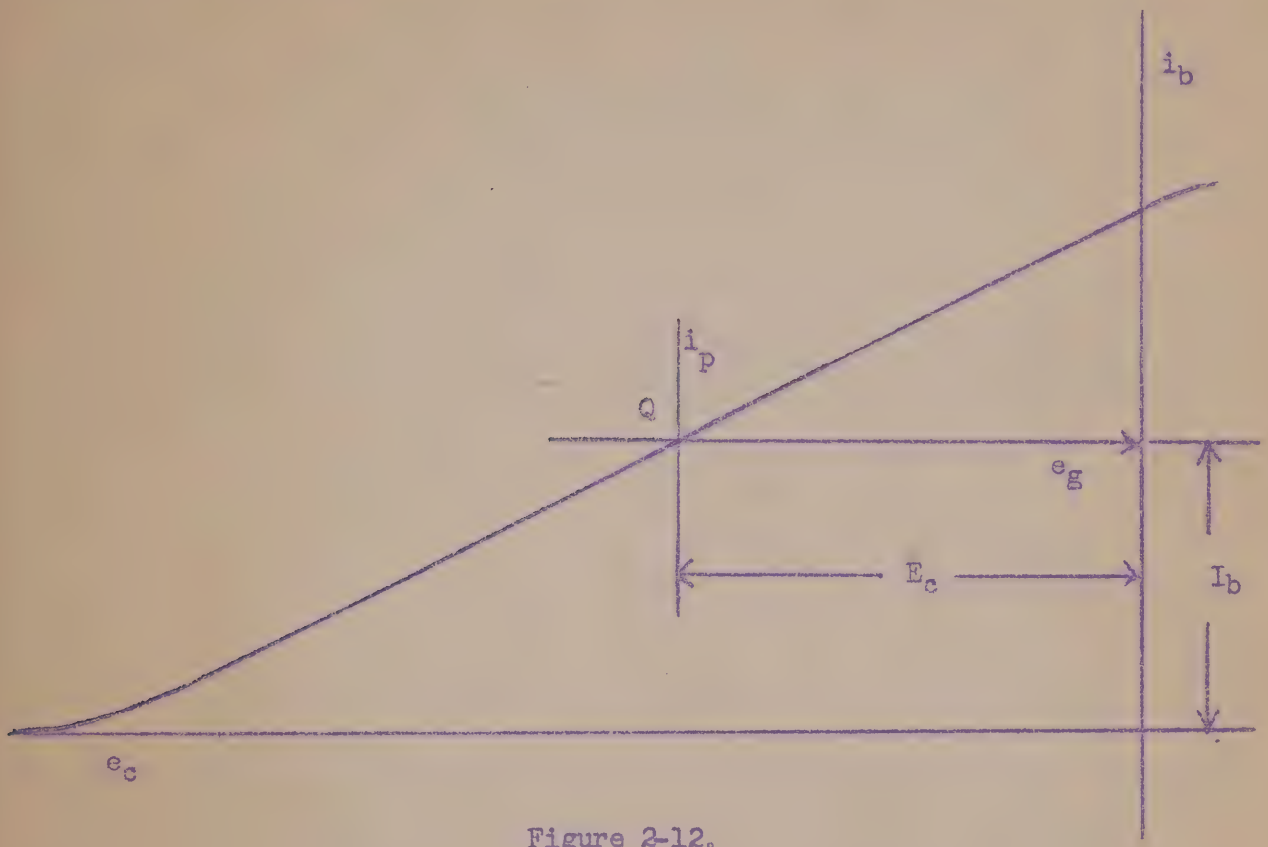


Figure 2-12.

Dynamic characteristic of an amplifier.

If the grid input signal voltage is restricted in amplitude to values for which the curve of Figure 2-12 is a straight line, then (2-11) becomes simply

$$i_p = C_1 e_g , \qquad (2\text{-}12)$$

and according to the linear circuit analysis of Chapter I, Equation (1-32),

$$i_p = \frac{\mu}{r_p + R_L} e_g .$$

Therefore, for linear operation, $C_1 = \frac{\mu}{r_p + R_L}$, which is the slope of the

dynamic characteristic at the operating point. If the input signal voltage is

$$e_g = E_{gm} \cos \omega t,$$

then the a-c component of plate current according to (2-12) is

$$i_p = C_1 E_{gm} \cos \omega t.$$

$$= \frac{\mu E_{gm}}{r_p + R_L} \cos \omega t$$

$$= I_{pm} \cos \omega t.$$

Now, if E_{gm} ($= \sqrt{2}\ E_g$) is increased until a region of the dynamic characteristic is involved where the characteristic is no longer a straight line, let it be assumed that two terms of (2-11) more accurately represent the portion of the characteristic which is used than would a single term. Then

$$i_p = C_1 E_{gm} \cos \omega t + C_2 E_{gm}^2 \cos^2 \omega t$$

$$= C_1 E_{gm} \cos \omega t + C_2 E_{gm}^2 \left(\frac{1 + \cos 2\omega t}{2} \right)$$

$$= \frac{C_2 E_{gm}^2}{2} + C_1 E_{gm} \cos \omega t + C_2 \frac{E_{gm}^2}{2} \cos 2\omega t, \quad (2\text{-}13)$$

and it is to be noted that the output current now contains a d-c component and a second harmonic not present in the input voltage. These quantities are distortion components introduced by the lack of linearity of the dynamic characteristic.

In case the characteristic is adequately represented by 3 terms of (2-11), and again $e_g = E_{gm} \cos \omega t$,

$$i_p = C_1 E_{gm} \cos \omega t + C_2 E_{gm}^2 \cos^2 \omega t + C_3 E_{gm}^3 \cos^3 \omega t$$

But $\cos^3 \omega t = \frac{1}{4} \cos 3\omega t + \frac{3}{4} \cos \omega t$, so that

$$i_p = \frac{C_2 E_{gm}^2}{2} + \left(C_1 E_{gm} + \frac{3}{4} C_3 E_{gm}^3 \right) \cos \omega t$$

$$+ \frac{C_2 E_{gm}^2}{2} \cos 2\omega t + \frac{1}{4} C_3 E_{gm}^3 \cos 3\omega t \quad (2\text{-}14)$$

where it would be noted that a third harmonic component is now present in the output.

The foregoing analysis may be indefinitely extended. It is perhaps sufficient to state that, in general, the dynamic characteristic may be represented to any practical desired degree of accuracy by using enough terms of the series of (2-11). It can be shown that the substitution of $e_g = E_{gm} \cos \omega t$ into (2-11) yields an expression which may be reduced to

$$i_p = B_o + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + \cdots + B_n \cos n\omega t + \cdots , \qquad (2\text{-}15)$$

where the B's represent harmonic amplitudes.

The next problem is that of determining the values of B_o, B_1, B_2, etc., from graphical data. It is desirable that all data used be obtainable from the plate diagram. In the method to be used in the following, all measurements may be made on the plate diagram, although the values may be represented more clearly in connection with the dynamic characteristic.

The total plate current is obtained by adding the quiescent value, I_b, to i_p. Thus (2-15) becomes

$$i_b = I_b + i_p = I_b + B_o + \sum_{n=1}^{\infty} B_n \cos n\omega t. \qquad (2\text{-}16)$$

In practice it is found that the amplitudes of distortion components beyond the fourth harmonic are seldom worth the effort of graphical computation, particularly as harmonic analyzers have been developed and are available for measurement of harmonic amplitudes. Therefore, the analysis and the results given here are included because of use in the interpretation and extension of amplifier theory.

For the case of (2-13) where two terms only of the series are used, values of B_o, B_1, and B_2 may be computed by reading from the graph sheet, Figure 2-13, the values of i_b for particular values of ωt. For this case, (2-16) becomes

$$i_b = I_b + B_o + B_1 \cos \omega t + B_2 \cos 2\omega t. \qquad (2\text{-}17)$$

In Figure 2-13, refer to the curve of grid voltage $e_g = E_{gm} \cos \omega t$. For $\omega t = 0$, $i_b = i_{b_{max}}$; for $\omega t = \pi/2$, $i_b = I_b$; for $\omega t = \pi$, $i_b = i_{b_{min}}$.

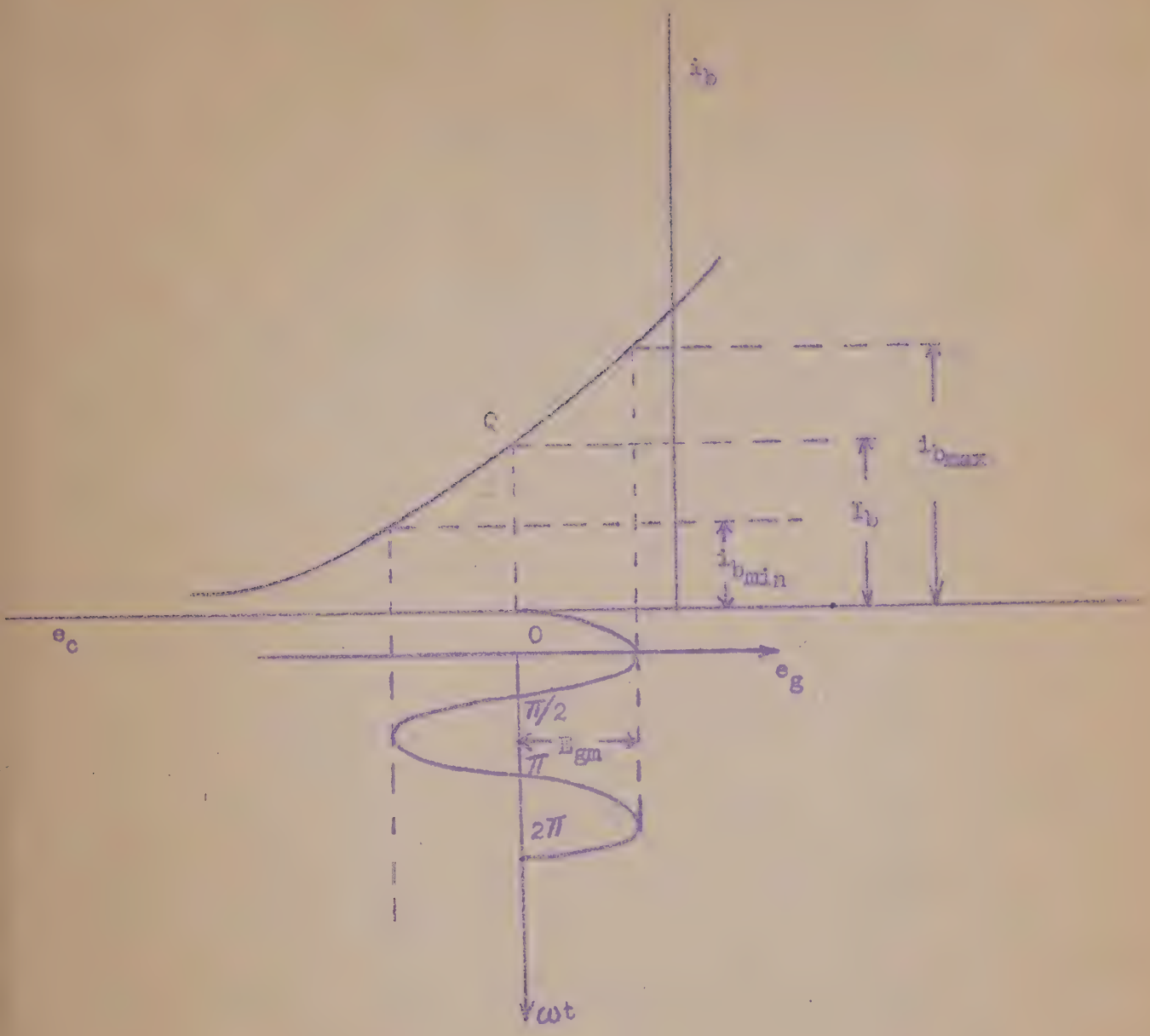


Figure 2-13.

Points for finding second harmonic distortion.

These values may be obtained, also, from (2-17). Thus, for

$$\omega t = 0, \quad i_b = i_{b_{max}} = I_b + B_o + B_1 + B_2$$

$$\omega t = \pi/2, \quad i_b = I_b = I_b + B_o - B_2$$

$$\omega t = \pi, \quad i_b = i_{b_{min}} = I_b + B_o - B_1 + B_2$$

and these equations may be solved for the harmonic component amplitudes in terms of the values of i_b determined from the graph. The results are:

$$B_o = B_2 \qquad (2\text{-}18)$$

$$B_1 = \frac{i_{bmax} - i_{bmin}}{2} \qquad (2\text{-}19)$$

$$B_2 = \frac{i_{bmax} + i_{bmin} - 2\,I_b}{4} \,. \qquad (2\text{-}20)$$

If more components are desired, more terms of (2-16) may be used, and additional values of ωt introduced, following exactly the same procedure used in obtaining (2-18), (2-19), and (2-20). A convenient set of values of ωt for determining five harmonic amplitudes are the values for which e_g is equal to E_{gm}, $1/2\ E_{gm}$, 0, $-1/2\ E_{gm}$, and $-E_{gm}$. The corresponding values of i_b obtained graphically from Figure 2-14 and analytically from (2-16) up to and including B_4 are listed in the following tabulation:

ωt	e_g	i_b (graph)	i_b (from (2-16))
0	E_{gm}	i_{bmax}	$= I_b + B_o + B_1 + B_2 + B_3 + B_4$
$\pi/3$	$\frac{1}{2}\ E_{gm}$	$i_{b1/2}$	$= I_b + B_o + \frac{B_1}{2} - \frac{B_2}{2} - B_3 - \frac{B_4}{2}$
$\pi/2$	0	I_b	$= I_b + B_o - B_2 + B_4$
$2\pi/3$	$-\frac{1}{2}\ E_{gm}$	$i_{b(-1/2)}$	$= I_b + B_o - \frac{1}{2} B_1 - \frac{1}{2} B_2 + B_3 - \frac{1}{2} B_4$
π	$-E_{gm}$	i_{bmin}	$= I_b + B_o - B_1 + B_2 - B_3 + B_4$

(2-21)

The foregoing equations may be solved by determinants or otherwise for the harmonic amplitudes. The final results are:

$$\left.\begin{aligned}
B_o &= \frac{1}{6}\left[i_{bmax} + 2\,i_{b1/2} + 2\,i_{b(-1/2)} + i_{bmin}\right] - I_b \\
B_1 &= \frac{1}{3}\left[i_{bmax} + i_{b1/2} - i_{b(-1/2)} - i_{bmin}\right] \\
B_2 &= \left[\frac{i_{bmax} + i_{bmin} - 2\,I_b}{4}\right] \\
B_3 &= \frac{1}{6}\left[i_{bmax} - 2\,i_{b1/2} + 2\,i_{b(-1/2)} - i_{bmin}\right] \\
B_4 &= \frac{1}{12}\left[i_{bmax} - 4\,i_{b1/2} - 4\,i_{b(-1/2)} + 6\,I_b + i_{bmin}\right]
\end{aligned}\right\} \qquad (2\text{-}22)$$

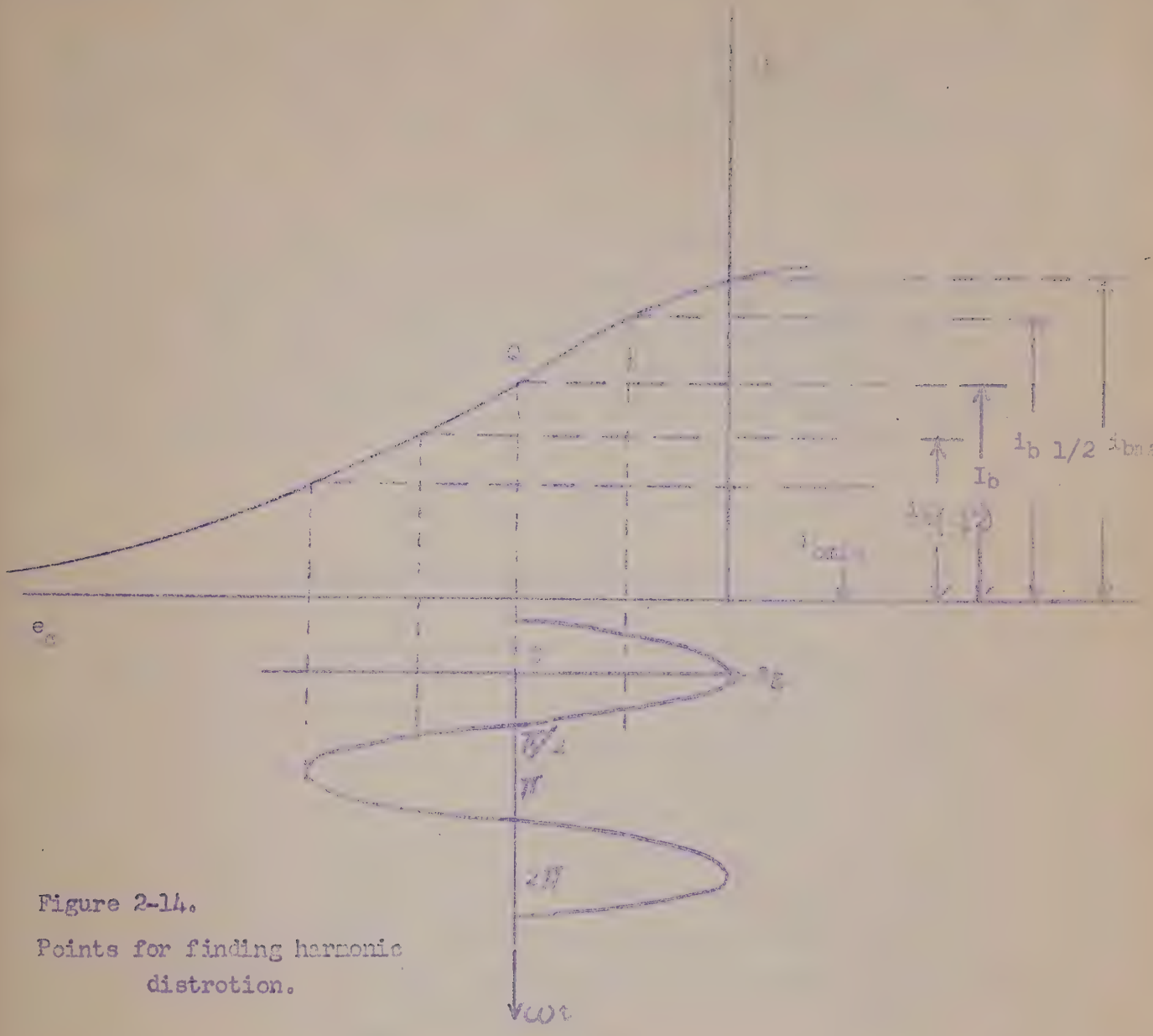


Figure 2-14.

Points for finding harmonic distrotion.

It is possible to determine from the plate diagram or from the dynamic characteristic the values of i_b corresponding to the various selected values of e_g shown in the schedule of (2-21), and thus to compute the harmonic distortion component amplitudes from (2-22). The plate diagram is always used to obtain the necessary values of i_b if the plate characteristics corresponding to $e_g = 0$, $\frac{1}{2} E_{gm}$, E_{gm} are available. Harmonic distortion is frequently computed in percent. For example, the percent 3rd harmonic distortion is $100 \frac{B_3}{B_1}$. The total percent harmonic distortion

is defined as the square root of the sum of the squares of the individual percent harmonic distortions. The second harmonic is ordinarily the largest contribution to the total distortion. In amplifiers for sound reproduction it is conventional practice to design for less than 5% second harmonic.

2-11. Harmonic Distortion in Tetrodes and Pentodes.

As shown in Section 2-10, the criterion for freedom from harmonic distortion of an amplifier tube is the straightness of its dynamic characteristic. Because of the bending of the static characteristics for low plate voltages, pentodes are found to be limited by third harmonic distortion. It is easy to eliminate the second harmonic in pentodes by a proper choice of load resistance. The amplitude of the second harmonic was given by the relation (2-20) as

$$B_2 = \frac{i_{bmax.} + i_{bmin.} - 2\,I_b}{4}\,.$$

For pentodes and beam tubes with their flat plate characteristics for negative values of control grid voltage, $i_{bmin.}$ for a fixed grid swing remains practically constant regardless of the slope of the load line. For example, if a load line were drawn through an operating point at $I_b = 37.5$ ma., $E_b = 250$ volts, $E_{c1} = -15$ volts for the 6F6 of Figure 2-8, and the control grid swings 15 volts on either side of the bias voltage, then $i_{bmin.}$ is approximately 6 ma. for any load resistance, R_L. However, when the grid swing is in the positive direction and reaches zero, $i_{bmax.}$ will vary rapidly with R_L if the load line intersects the characteristic for $E_{c1} = 0$ near the rounded knee of the curve. Therefore, for a fixed value of $i_{bmin.}$ and I_b, it is easy to select a value of load resistance, and hence of slope of the load line for which $B_2 = 0$, which requires that

$$i_{bmax.} = 2\,I_b - i_{bmin.}$$

This particular value of maximum plate current is located somewhere along the tube characteristic corresponding to maximum grid swing. For the 6F6 operating point and grid swing suggested,

$$i_{bmax.} = 75 - 6 = 69 \text{ ma.}$$

The slope of the load line through the 69 ma. point on the $E_c = 0$ characteristic would then be the negative reciprocal of

$$R_L = \frac{\Delta\, e_b}{\Delta\, i_b} \cong \frac{250 - 40}{(69 - 37.5) \times 10^{-3}} = \frac{210{,}000}{31.5} \cong 6700 \text{ ohms.}$$

If a load resistor of this value were used, the second harmonic would be eliminated.

For power amplifiers, other considerations relating to optimum values of load resistance for best power output impose additional requirements. These will be taken up in the chapter on power amplifiers.

2-12. Variable μ Tubes

It is desirable for certain applications to have the amplification factor, μ, or the control grid-plate transconductance, g_m, of the tube depend upon the grid bias. Such a result is achieved by making the pitch of the spiral control grid winding variable along the length of the grid structure, which amounts to a variable, rather than a fixed, grid-wire spacing along the length of the grid. Control of electron flow from the cathode is dependent upon the field intensity at the cathode and thus cut-off of plate current occurs for areas of the cathode nearest the closely spaced region of the grid at less negative grid voltages than is the case for areas opposite the more widely spaced grid wires. Such a tube is called a variable mu, remote cut-off, or super-control tube, and is used in radio receivers where the grid bias is made to depend upon audio amplifier output so that the volume of the amplifier is automatically controlled. Cut-off for a variable mu tube is approached asymptotically as compared with the sharp cut-off of the tube with uniform grid wire spacing. A typical remote cut-off pentode is the 6K7; a sharp cut-off pentode is the 6J7. The student should compare the plate and mutual characteristics of these tubes.

2-13. Other Multi-electrode Tubes.

Many multi-electrode tubes combine two or more electrode assemblies in the same glass or metal envelope. For example, two triodes, or a triode and a pentode may be enclosed in the same bulb. They may or may not use the same cathode, but each has its typical characteristics independent of the other, and so need not be considered as different from similar triode or pentode assemblies in separate envelopes.

Tubes using more than three grids are employed in frequency conversion. Such tubes may have two control grids, two screen grids and a suppressor, and are known as pentagrid mixer amplifiers, as for example, the 6L7. The plate current depends upon both control grid signal voltages in a way which involves the product of functions of each voltage. As the signal voltages may be of different frequencies, a modulation results and new frequencies appear in the output which were not present in either input. The pentagrid converter also uses five grids, including two control grids. One of these is used in an oscillator circuit, and the other obtains signal from a separate source. The two frequencies are effective simultaneously upon the plate current, and frequency conversion results.

2-14. Parallel-feed.

In the simple amplifier circuits shown thus far, the direct plate supply voltage has been fed to the plate of the vacuum tube through the load resistor. The flow of the d-c component of the plate current through the load resistor results in a power loss $I_b^2 R_L$, and in a voltage drop, $I_b R_L$. It is possible to avoid the power loss and to reduce the magnitude of the required plate supply voltage by supplying the direct current and voltage to the plate through an inductance of very low resistance. A blocking condenser is used to prevent direct current from flowing through the load resistance. The reactance of the condenser is negligible compared with the load resistance in the frequency range in which operation is desired. A typical arrangement is shown in Figure 2-15, in which inductance L is chosen of such size that ωL is very large and C is large enough that $1/\omega C$ is very small compared with R_L. Except at very low frequencies neither choke nor condenser need be shown in the equivalent a-c circuit, since the choke is assumed to have such high reactance that it passes negligible alternating current and may be replaced by an open circuit, and since the condenser has such small reactance that it may be replaced by a short circuit. This circuit arrangement is known as parallel feed.

If the resistance of the choke is appreciable, two load lines must be drawn, namely, the d-c or static load line for the resistance of the choke, and the a-c or dynamic load line for the resistance of the load. If the resistance of the choke is negligible, the static load line is vertical and the operating plate voltage, E_b, is equal to the plate supply voltage. The dynamic load line would then be drawn through the operating point with a slope equal to the negative reciprocal of the resistance of the load.

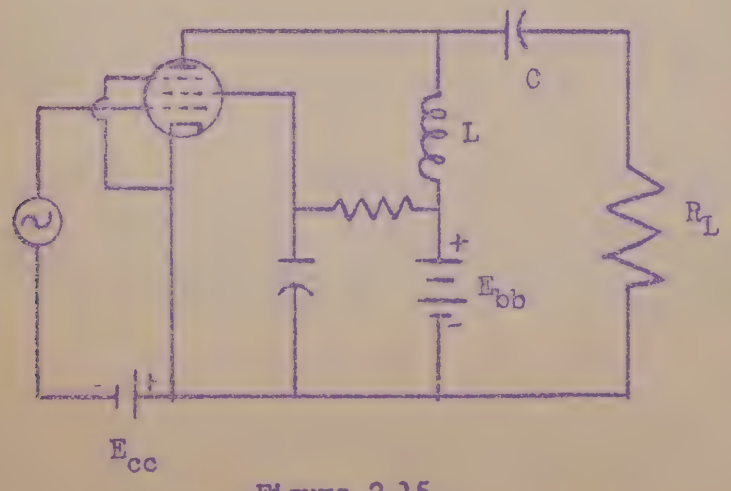


Figure 2-15.
Parallel feed.

2-1. Using a set of Type 24-A plate characteristics, draw a load line to intersect the $E_{c_1} = 0$ characteristic at $i_b = 7$ ma., and the $E_{c_1} = -6$ characteristic at $e_b = 450$ volts. Using parallel feed for the plate circuit, and a grid bias of -2 volts, and neglecting choke resistance and condenser reactance,

(a) determine the plate battery voltage and load resistance needed;

(b) draw the dynamic characteristic corresponding to your load line and show the operating range for a grid swing of 1.5 volts (on either side of the bias). Determine the voltage gain graphically. Determine r_p and μ at the operating point and calculate the gain from the gain formula.

(c) Compute the percent second harmonic distortion for the amplifier.

2-2. Use the definition of plate resistance to sketch the variation of r_p as a function of e_b, obtaining data from the Type 24-A characteristic for $E_{c_1} = -3$ volts. Qualitative, rather than quantitative, results are desired.

2-3. An amplifier with resistance load uses a Type 59 pentode with a load resistance of 5000 ohms; $E_{bb} = 300$ volts, $E_{cc} = -20$ volts.

(a) Draw the circuit diagram for parallel feed, showing all grid and battery connections. Draw the equivalent a-c plate circuit.

(b) Draw the load line on a set of plate characteristics and determine the coordinates of the operating point. Obtain the dynamic characteristic.

(c) If the grid input voltage has a maximum value of 10 volts, determine the voltage gain graphically.

2-4. Referring to Problem 2-3, determine g_m and r_p from the graph sheet at or near the operating point. Determine the slope of the dynamic characteristic at the operating point. Calculate μ and the voltage gain and compare the latter with the value obtained graphically in Problem 2-3.

2-5. Select a load resistance to be used with the amplifier of Problem 2-3 so that the second harmonic amplitude will be zero, and compute the percent third harmonic distortion for maximum grid voltages of

(a) $\sqrt{2}\ E_g = 10$ volts;

(b) $\sqrt{2}\ E_g = 15$ volts.

2-6. Simplify the circuit of Figure 2-10 by combining capacitances in parallel and neglecting C_{glp}. Determine the input and output capacitances in terms of the interelectrode capacitances of the circuit.

2-7. Using the plate characteristics of the 6J7 and 6K7 pentodes construct mutual characteristics on the same sheet at e_b = 200 volts for each tube, and compare the remote and sharp cut-off characteristics. Determine the maximum and minimum values of g_m for each tube in the range of your drawing.

2-8. Draw the connection diagram for a 6J7 pentode amplifier circuit, obtaining the necessary screen supply voltage from a potentiometer across the plate supply. Use a cathode resistor for obtaining grid bias. If the pentode operates at E_b = 250 volts, E_{c_2} = 100 volts, E_{c_1} = -2.5 volts, and the load resistor is 100,000 ohms, find:

(a) the proper size of cathode biasing resistor;

(b) the plate supply voltage.

2-9. If the signal voltage at the grid of the tube in Problem 2-8 is a sine wave of frequency 400 cycles, amplitude 2.5 volts, compute the output voltage and the percent harmonic distortion for all harmonics up to and including the fourth.

2-10. Explain fundamentally why the insertion of grid wires between control grid and anode in a vacuum tube reduces the effective grid-plate capacitance. Assume that the inserted grid is maintained at a-c ground potential.

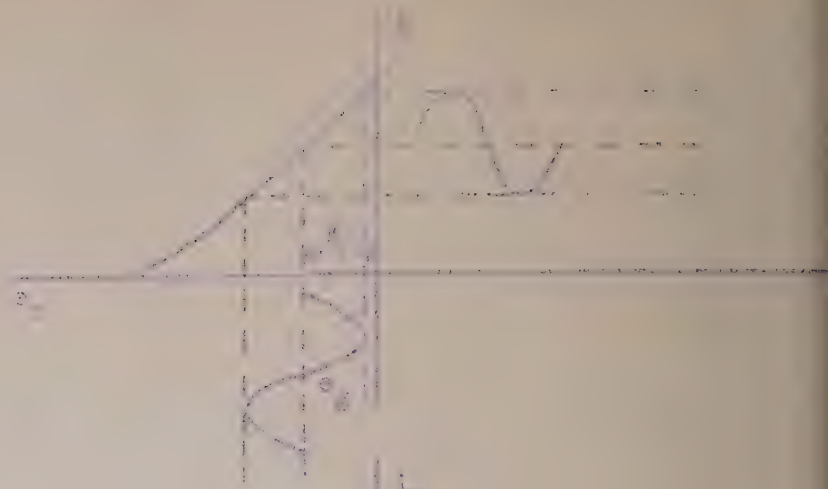


Figure 3-1.
Class A Amplifier Operation

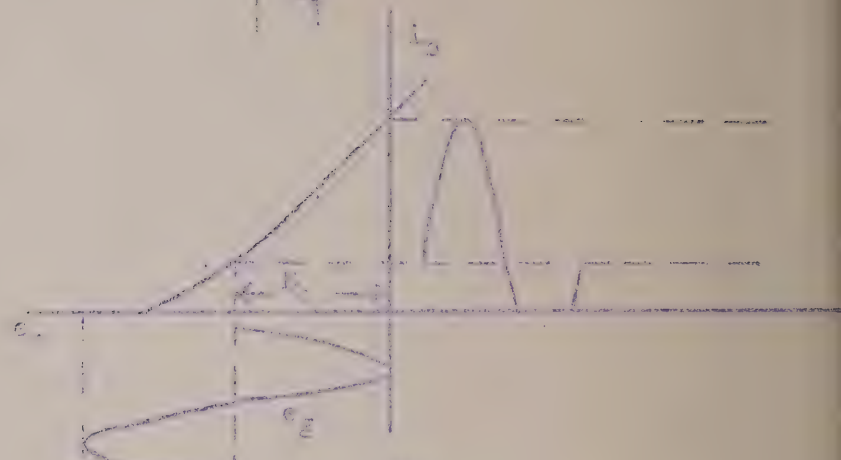


Figure 3-2.
Class AB Operation

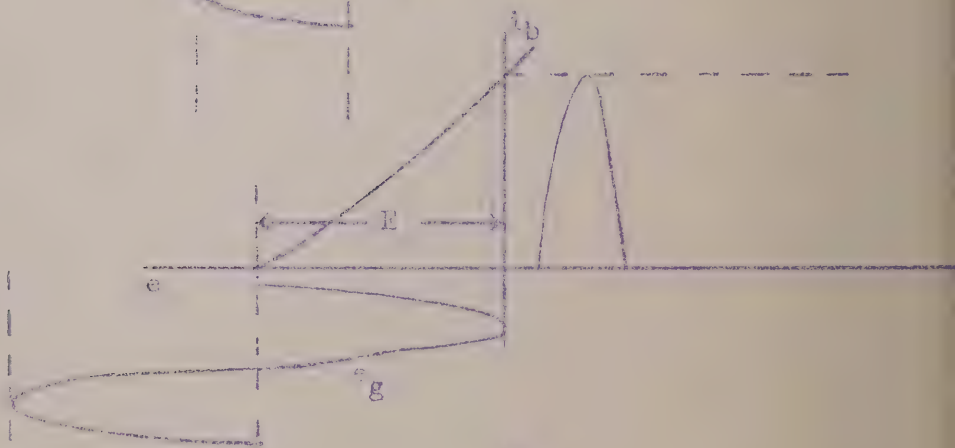


Figure 3-3.
Class B Operation

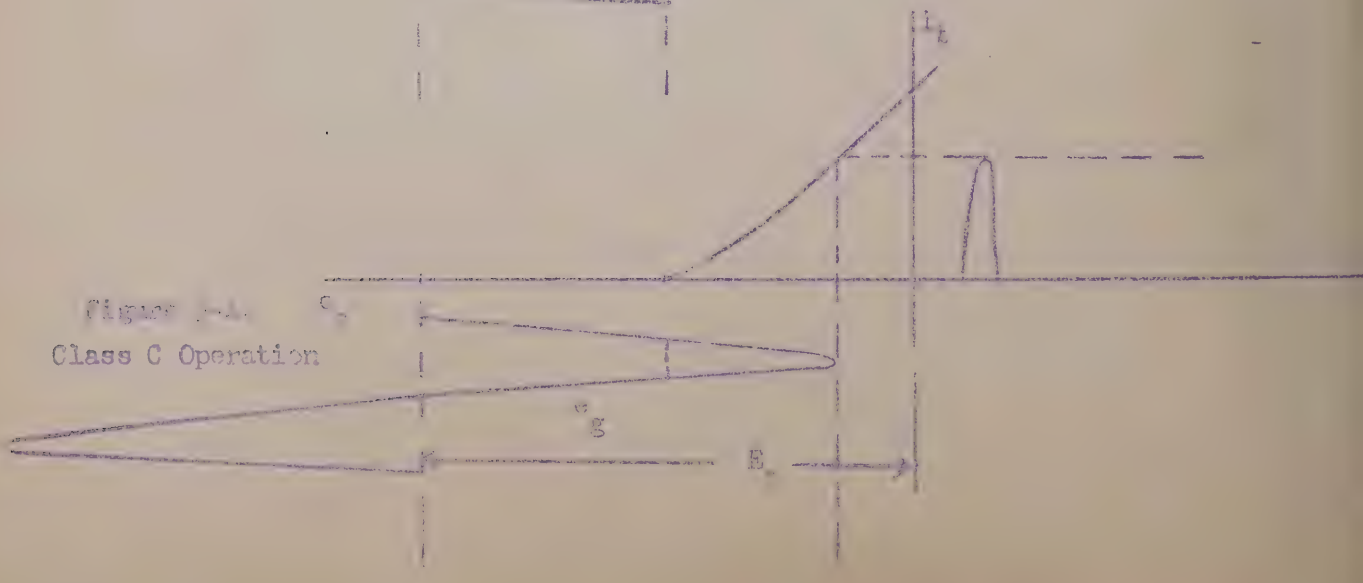


Figure 3-4.
Class C Operation

KNOW

Chapter III.

Audio-Frequency Voltage Amplifiers

The single-tube or one-stage amplifier circuits of Chapters I and II were introduced to provide examples of tube applications in basic circuits. The present chapter will be devoted to a study of methods of coupling tubes in multi-stage amplifiers designed for amplification in the audio frequency range.

3-1. Amplifier Classifications.

Amplifiers may be divided in respect to their use into the classi-

Amplifiers, as used in this text, are four-terminal networks designed to provide an increase in the amplitude of an alternating voltage. This voltage is referred to as the signal and is applied at the input terminals of the four pole. If the emphasis is placed upon the production at the output terminals of a higher output voltage, with the magnitude of output current considered as unimportant, the amplifier is referred to as a voltage amplifier, and certain types of high vacuum tubes are designed for such use. If the major purpose of the amplifier is to increase the amplitude of a current, then a voltage signal proportional to the current and

but the output terminals must be capable of supplying a current of amplitude greater than that of the signal current. Such an amplifier is referred to as a current amplifier. If both current and voltage amplitudes are required to be large at the output terminals, the amplifier is referred to as a power amplifier. It is generally true that vacuum-tube amplifiers are voltage operated devices, that is, they respond to voltage variations at their input terminals. The overall requirement of the amplifier as to output determines the classification as to use as voltage, current, or power amplifiers and dictates the essential circuit properties of tubes and other circuit components which together comprise the 4-pole.

Amplifiers may also be classified on the basis of frequency range as

range over which amplification is desired.

Additional standard classifications as to service have been set

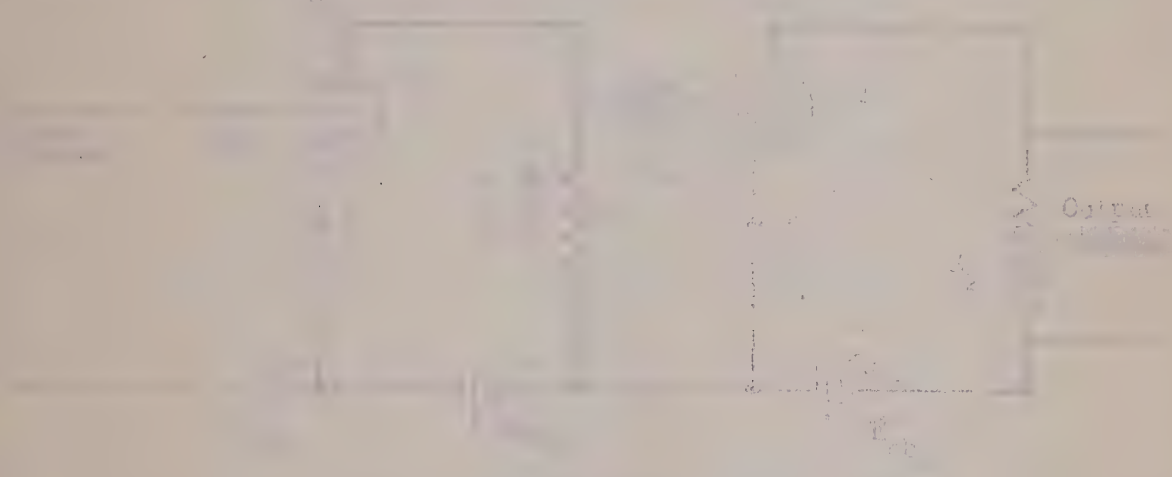
Class B, and Class C. These classifications depend upon the portion of the input cycle during which plate current flows, and this period of plate conduction is determined by the grid bias and the excitation amplitude. Figures 3-1, 3-2, 3-3, and 3-4 illustrate classes A, AB, B, and C respectively.

resulting plate current. It should be particularly noted that Class A

operation implies bias on the linear portion of the characteristic, and plate current flows only during one-half the cycle of grid voltage; for Class C operation the bias is below cut-off and plate current flows only during a period less than half that of the grid cycle. Since audio amplifiers require freedom from distortion, they are almost always Class A. the distortion inherent in classes other than A. Since plate current flows be much greater than in Class A, the B and C amplifiers operate more effectively than the A class. Class AB is intermediate between Classes A and B. A further distinction, depending upon grid swing, is in general use. grid current is expected to flow and the subscript 2 is used. When grid current does not flow, subscript 1 is used. For example, Figure 3-1 represents a Class A_1 amplifier, Figure 3-3 a Class B_1. If E_{gmax} were greater than E_c in Figure 3-3, the amplifier would operate Class B_2.

3-2 Distortion in Amplifiers.

In order to describe a sine or cosine wave, three properties of the wave must be known. These properties are amplitude, frequency, and phase. When a given alternating voltage is fed into an amplifier, it is usually desirable that an output voltage of identical frequency, but increased amplitude will result. As shown in Chapter I, the phase difference between input and output voltage for amplifiers with pure resistance load circuits is 180 degrees. When a number of sinusoidal voltages of different amplitudes, frequencies, and phases are simultaneously fed into the input of an amplifier, as is the case for the amplification of music or speech, it is desirable and necessary that: (1) the amplifier output contain exactly the same frequencies as are present in the input; (2) the ratio of the amplitudes of output to input voltage components be the same for all frequencies, which is equivalent to requiring that the gain be the same for all frequencies; (3) the relative phase differences between the various frequencies be the same in the output as in the input. The departure of the amplifier from exact conformity with these three conditions is known as distortion. The introduction by the amplifier of frequencies not present in the input results from the non-linearity of the dynamic characteristic, as has been discussed in Chapter II, and is known as harmonic distortion. Failure of the amplifier to meet the second requirement of equal gain for all frequencies is known as frequency distortion and is caused by circuit or tube reactance values which change with frequency. If the amplifier circuit introduces only a 180-degree phase shift per stage for all frequencies, requirement (3) is met perfectly, but since resistance values of the interelectrode capacitances, and circuit inductive and capacitive reactances depend upon frequency, output frequencies will not have the same relative phase differences as input frequencies, and phase distortion will result. For speech or music, the ear does not detect small amounts of phase distortion, but it is quite sensitive to harmonic and frequency distortion.



A circuit utilizing a single plate voltage power supply and the potentiometer principle was first suggested by Loftin and White[1]. Such a circuit is illustrated in Figure 3-6, in which R_c is the bias resistor for tube 1. With proper choice of power supply voltage and resistors, the

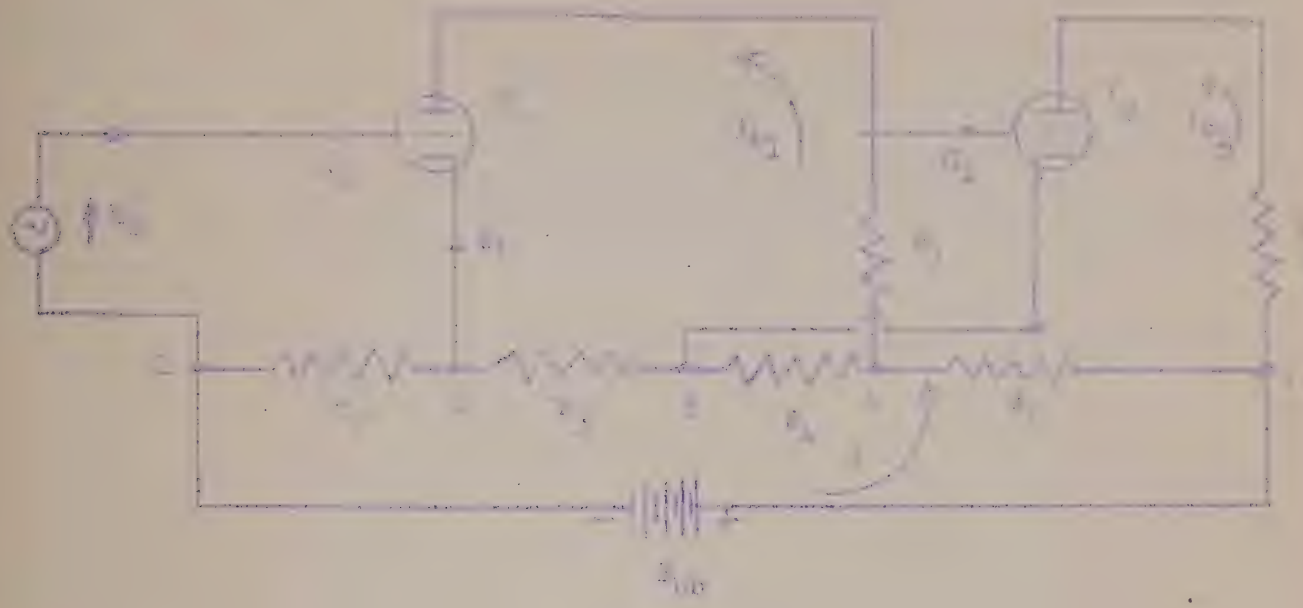


Figure 3-6.

Two-stage resistance-coupled amplifier using single power supply for plate and bias voltages.

voltages required for the circuit of Figure 3-5 can be supplied as shown in Figure 3-6. The direction of the direct current in the power supply circuit is such that K_1 is positive with respect to G_1 by the amount of the drop IR. The battery voltage E_{bb_1}, of Figure 3-5 has been replaced by E_{24} (voltage rise), E_{bb_2} by E_{35}. The bias voltage of the second tube can be obtained from the relation

$$E_{35} = I_{b_1} R_3 + V_{G_2K_2}$$

where $V_{G_2K_2}$ = voltage drop, grid-to-cathode

$= E_{c_2}$, the bias voltage (usually a negative number) or the voltage rise, d-c component, cathode-to-grid.

---

[1] Loftin, R. B. and White, W. T., Proc. IRE 16, 281 (1928)

e. If $R_1 = 50{,}000$ ohms, $I_{b1} = 1.8$ ma., $E_{c2} = -5$ volts. The re-
[illegible] given by

$$E_{34} = 1.8 \times 10^{-3} (50{,}000) - 5$$

$$= 90 - 5 = 85 \text{ volts.}$$

[illegible] be noted that the cathode of tube 2 is at a higher direct potential
[illegible] of tube 1. Either separate heater supplies must be used, or the
[illegible] of cathode to heater for each tube must be capable of withstand-
[illegible] fairly high potential if a common heater source is used for both
[illegible].

Referring again to Figure 3-6, the d-c supply voltage E will be
[illegible] of $E_{12} + E_{23} + E_{34} + E_{15}$, or

$$E_{15} = E_{12} + E_{23} + E_{35}. \qquad (3\text{-}2)$$

[illegible]

$$E_{23} = V_{K_2 G_2}$$

$$= E_{bb_1} - I_{b_1} R_1$$

$$= E_{bb_1} - (I_{b_1} R_1 - V_{K_2 G_2}) \qquad (3\text{-}3)$$

[illegible] on 3-3 may be written as, with $E_{2} \cong E_{bb_1}$, as

$$E_{23} \cong E_{bb_1} - I_{b_1} R_1 + V_{K_2 G_2}$$

$$= E_{b_1} + V_{K_2 G_2}. \qquad (3\text{-}4)$$

$$E_{15} = E_{bb_2} = E_{b_2} + I_{b_2} R_2 \qquad (3\text{-}5)$$

Therefore, the d-c supply voltage is

$$E_{bb} = V_{c1} + E_{b1} + E_{b2} + \ldots$$

or the numerical sum of the bias voltages, the plate voltages, and the d-c voltage drop through the load resistor of the last tube

A 3-stage direct-coupled amplifier circuit is given in Figure [illegible]. The proper values of the bias voltages are obtained by proper selection of a voltage drop in the potential divider circuit. In the third or power stage a pentode is used as a triode. In order to avoid undesirable feed-back when audio frequencies are [illegible] that their reactances in the range of frequencies to be amplified are [illegible].

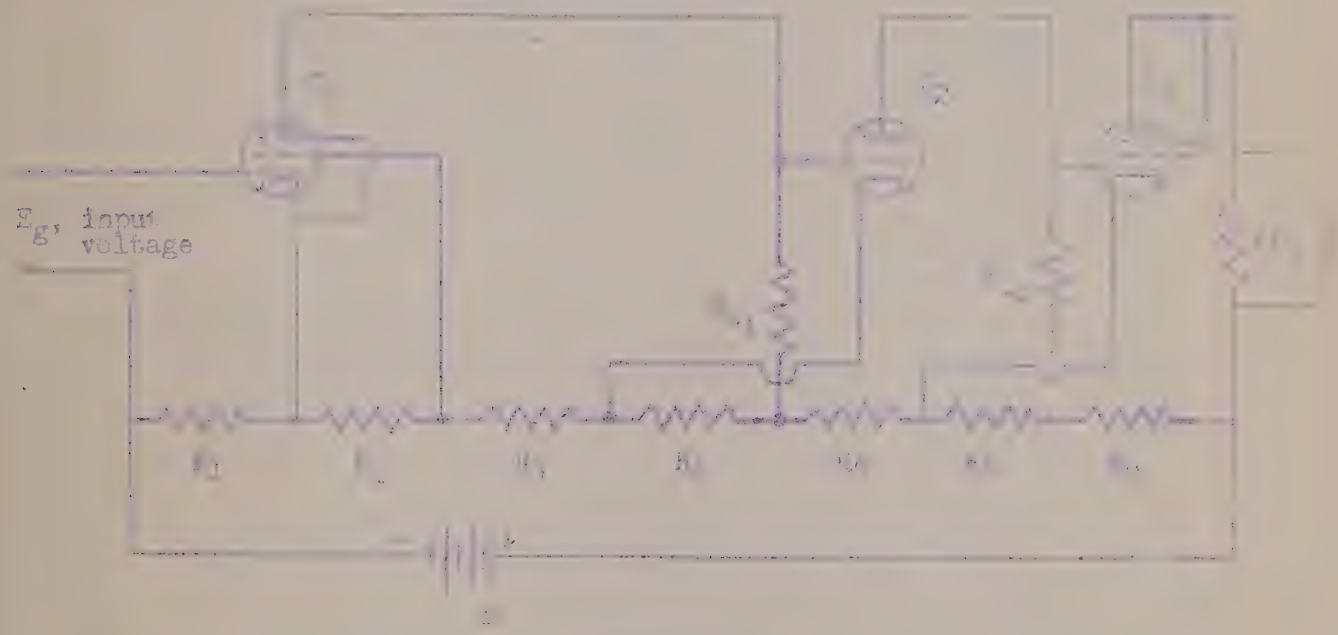
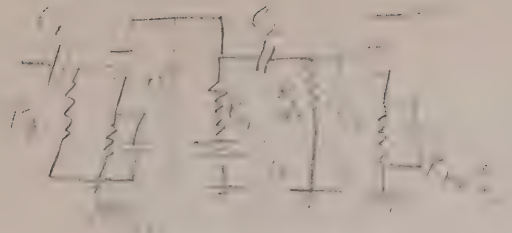
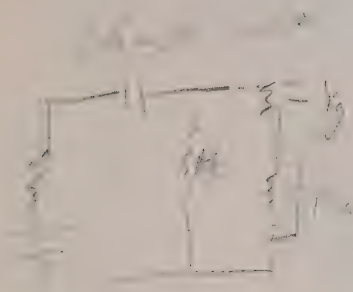


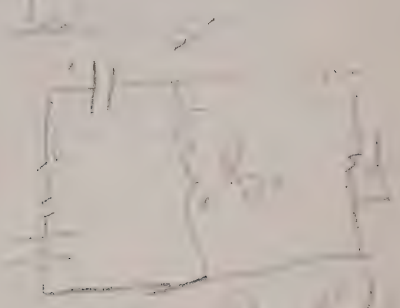
Figure 3-7

Three-stage, direct-coupled amplifier

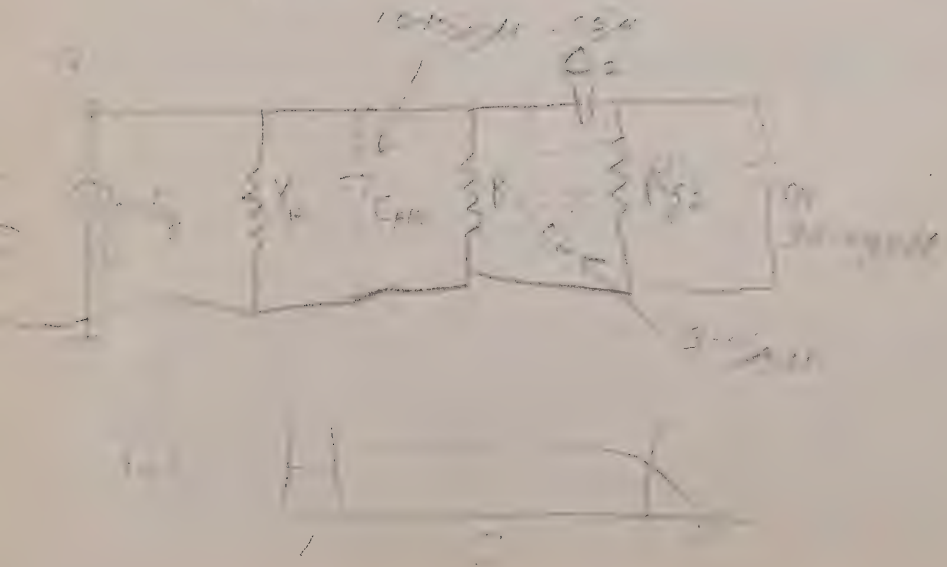
Direct-coupled amplifiers are most useful at low frequencies [illegible] ing range. Such amplifiers also find numerous applications where it is required that a circuit respond to a very small change in voltage, a change which is not necessarily alternating, nor even periodic. Direct-coupled amplifiers have been used in biological studies, for counting purposes, and in experimental psychology in the study of neural response. They have







$$T = (R_1 + r)C_2$$



...s been used in cathode ray oscillographs to amplify the input voltage ... over a very wide range of frequencies.

Certain disadvantages of the d-c (direct-coupled) amplifier ... rent isolation between stages, small changes in tube characteristics or in battery voltages in one stage are amplified in a succeeding stage so that the tube operating point of the later stage may shift out of the linear region of the dynamic characteristic. When the circuit of Figure 3-7 is used to eliminate the large bias and plate supply batteries, there is a ... succeeding stage, sufficient either to overload that stage severely or to ... where in the system changes the current or voltage in the potentiometer, ... for the potentiometer circuit, which calls for a large capacity power sup- ... described in the foregoing paragraph are described in the literature and should be referred to before a serious attempt is made to build such an amplifier.

3-4. Resistance-Capacitance Coupling

... amplifier of Figure 3-8 by the blocking condenser, C. The equivalent circuit of Figure 3-8 is the circuit shown in Figure 3-9, and R_{11} and C_{11} are the resistive and capacitive components of the input impedance of the next stage. (See page 44, Chapter I, Figure 1-28).

... pendently in finding the gain. If, for example, the amplifier stage of ...

$$\frac{\text{...}}{\text{Input signal voltage to first stage}}$$

with proper attention as to the use of voltage drops with respect to the common ground terminal. For the separate stages,

$$\frac{V_{KG_2}}{\text{...}}$$

... references are given in the appendix.

$$\frac{V_{KG_2}}{V_{KG_1}} \cdot \frac{V_{KG_3}}{V_{KG_2}} \cdot \frac{V_{out}}{V_{KG_3}} = A_1 A_2 A_3 \qquad (3\text{-}2)$$

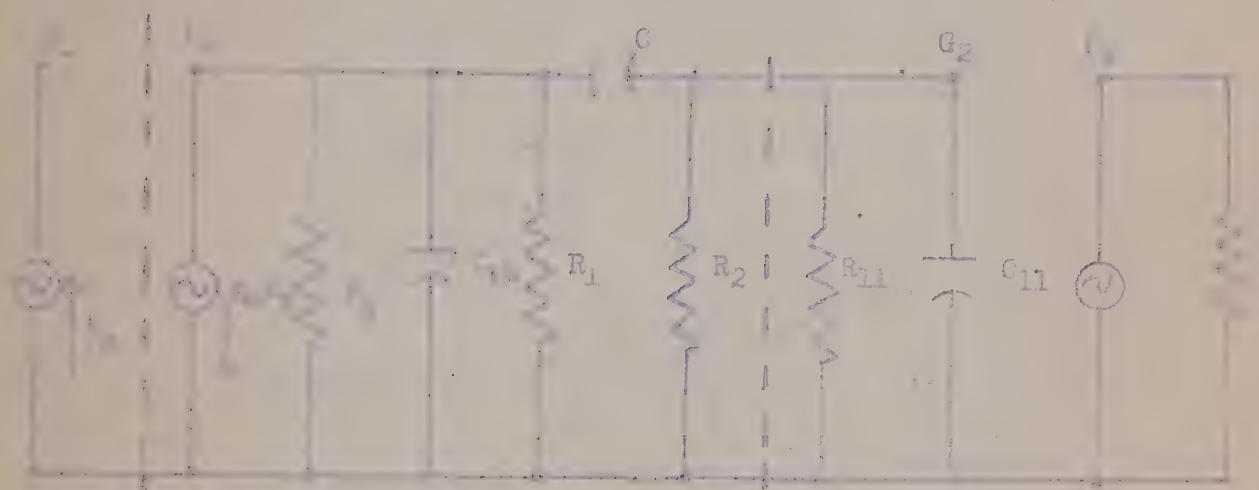


Figure 3-9.
Equivalent a-c circuit of Figure 3-8.

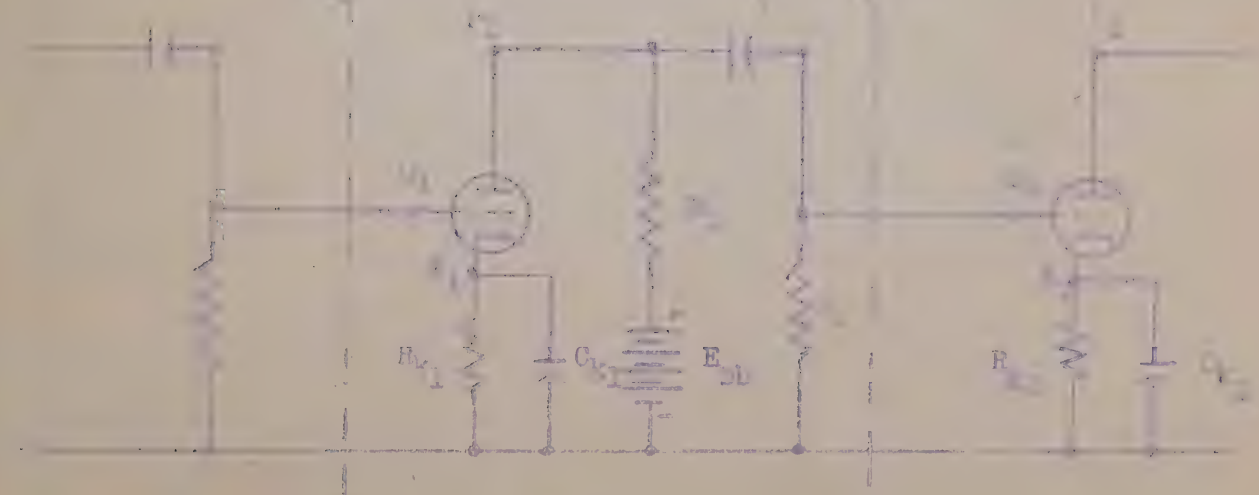
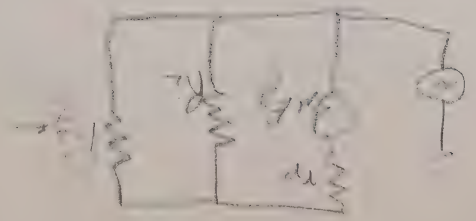


Figure 3-8.



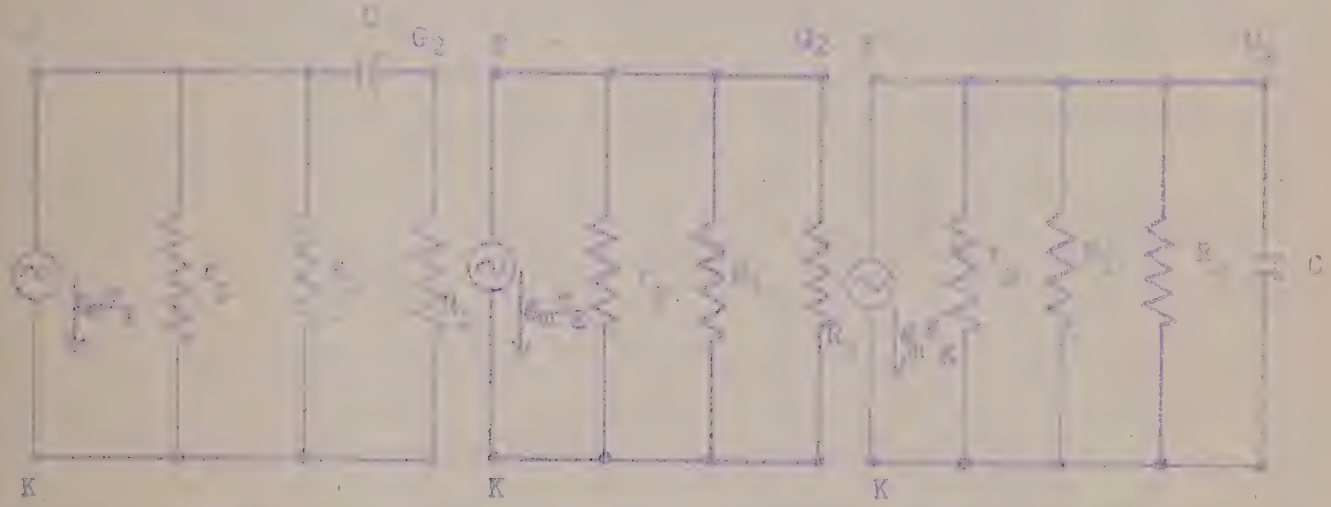
The importance in many applications of the resistance-capacitance method of amplifier coupling requires that the circuit behavior of R-C coupled amplifiers be discussed in some detail. In the circuit of Figure [illegible] provides the proper potential difference E_c between cathode and grid for negative grid bias. The condensers C_k are chosen of sufficiently large capacitance and correspondingly small reactance to by-pass the a-c components of the plate current around the resistors so that on the a-c [illegible] R_k and C_k is comparatively a short circuit and is therefore not shown. The function of capacitor C is to provide d-c isolation for the following stage. To insure adequate d-c isolation, the dielectric used in C must be of high quality material of very high insulation resistance. Any direct current leakage through the condenser from the plate supply will tend to produce a positive bias on the grid of the next stage. If the amplifier is to have minimum frequency distortion, the reactance of condenser C must be negligible. Practically this requirement is met if X_c [illegible] is independent of C, and therefore of frequency so long as the input admittance of the next stage remains negligible. Resistance R_1 [illegible] high gain. The magnitude of R_1 is limited because it must pass the d-c component of [illegible] too large a plate battery supply E_{bb}. From the circuit of Figure 3-8,

$$E_{bb} = I_{b_1} R_1 + E_{c_1} + I_{b_1} R_k$$

The magnitude of R_2 is limited because of the flow of grid current to the grid G_2. Usually R_2 does not exceed one megohm, for even an extremely small grid current would produce an appreciable spurious bias in a resistor of several megohms. Since any charge accumulating on coupling condenser C must leak off through R_2, the product R_2C is limited in magnitude. Too large a product R_2C means that a sudden impulse charging the condenser would affect the grid of the next stage for too long a time, a phenomenon known as "blocking". Such conditions limit the size of R_2 and C.

The constant current generator equivalent circuit has been used in Figure 3-9, and grid conductance has been assumed to be negligible. All circuit elements have been shown, but since the condenser reactances and the magnitude of r_{11} for the next stage (see Chapter I, Equations [1-42] and [1-43]) all depend upon frequency, it is possible to establish frequency ranges within which the circuit may be very much simplified. In the low

frequency range, X_c is greater than one-tenth R_2; also, the reactances of C_{ll} and C_{ll} and the resistance R_{ll} are so large compared with R_2 that C_{ll} and R_{ll} may be considered open circuits and omitted from the circuit diagram. The low frequency circuit is shown in Figure 3-10(a). In the mid-frequency range, Figure 3-10(b), $X_{C_2} \leq 0.1 R_2$ and may be neglected by comparison with R_2 and C_{ll} and R_{ll} may be considered as open circuits because of their relatively large reactances and resistance compared with the other circuit elements that they may be considered as open circuits. In the high frequency range, X_{C_2} is small and may also be neglected; $Y_{ll} = G_{ll} + j\omega C_{ll}$, and it is assumed that $G_{ll} << \omega C_{ll}$, and may be neglected. The circuit is shown in Figure 3-10(c).



(a) Low frequency range. (b) Mid-frequency range. (c) High frequency range

Figure 3-10.

Equivalent circuits for the low, mid-frequency, and high frequency range for an R-C coupled amplifier stage.

Analysis of the gain and phase shift of the R-C coupled amplifier stage of Figure 3-9 is now greatly simplified by use of the equivalent circuits of Figure 3-10. In the mid-frequency range, the current $\dot{I}_g = \dot{E}_g / Z_g$ flows through an equivalent admittance Y_{eq} given by

$$Y_{eq} = Y_p + Y_1 = Y_1 = \frac{1}{Z_{eq}} \qquad (3-8)$$

The output voltage drop of the stage is

$$V_{kg_2} = g_m E_g R_{eq} \qquad (3\text{-}9)$$

and the intermediate frequency voltage gain of the stage is given by the relation

$$A_I = \frac{V_{kg_2}}{E_g} = -g_m R_{eq} = -\frac{g_m}{Y_P + Y_1 + Y_2} \qquad (3\text{-}10)$$

Since A_I involves only pure resistance elements, the intermediate frequency gain is independent of frequency. If the following substitutions are made in (3-10), namely

$$Y_1 + Y_2 = Y_L = \frac{1}{R_L}$$

where $R_L = \dfrac{R_1 R_2}{R_1 + R_2}$ is the equivalent load resistance,

and $g_m = \mu Y_P = \mu / r_p$,

then Equation (3-10) may be written in the familiar form of

$$A_I = \frac{\mu / r_p}{\frac{1}{r_p} + \frac{1}{R_L}} = -\frac{\mu R_L}{r_p + R_L} \qquad (3\text{-}11)$$

In exactly the same manner, the amplifier voltage gain in the high frequency region may be obtained from the circuit of Figure 3-10(b). The high frequency gain, A_H, is then given by

$$A_H = \frac{g_m}{Y_P + Y_1 + Y_2 + Y_{11}} = -\frac{\mu Y_P}{Y_P + Y_1 + Y_2 + Y_{11}} \qquad (3\text{-}12)$$

For convenience in computation, it is desirable to relate A_H to A_I, which may be accomplished by noting that Equations (3-9) and (3-10) may be used in Equation (3-12), so that

$$= \frac{A_I}{1 + R_{eq} Y_{11}} \qquad (3\text{-}13)$$

$Y_{11} = G_{11} + j\omega C_o$ and G_{11} is assumed to be negligible compared with ωC_o. Then $Y_{11} \cong j\omega C_o$ and the quantity $R_{eq} Y_{11}$ may be written as

$$R_{eq} Y_{11} = j\, 2\pi f\, C_o\, R_{eq}$$

If we define the quantity $\frac{1}{2\pi C_o R_{eq}}$ as f_H, then Equation (3-13) may be written as

$$A_H = \frac{A_I}{1 + j\frac{f}{f_H}} \qquad (3\text{-}14)$$

where

$$f_H = \frac{1}{2\pi C_o R_{eq}} = \frac{Y_P + Y_1 + Y_2}{2\pi C_o} \qquad (3\text{-}15)$$

Equation (3-14) may now be expressed in polar form as follows:

$$A_H = \frac{|A_I| \angle 180^\circ}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}\, \angle \theta_H} = \frac{|A_I|}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}\, \angle 180 - \theta_H \qquad (3\text{-}16)$$

where $\theta_H = \tan^{-1} \frac{f}{f_H}$.

If the frequency f increases beyond the intermediate frequency range, the gain decreases and the phase shift angle θ_H increases, according to Equations (3-14) and (3-17). If f becomes equal to f_H,

$$\left|A_H\right| = \frac{|A_I|}{\sqrt{2}} = 0.707 \left|A_I\right|$$

70.7 percent of the constant gain in the intermediate frequency range. and input voltages fails to be 180°. Therefore, θ_H is a measure of the phase distortion in the high frequency range.

For the low frequency gain, the circuit of Figure 3-10(a) pro- nected across the terminals of the constant current generator. If the single resistance equivalent to r_p and R_1 in parallel is

$$R = \frac{r_p R_1}{r_p + R_1} , \qquad (3\text{-}18)$$

then the voltage across R is expressed by

$$V_R = g_m E_g \frac{R (R_2 - j X_c)}{R + R_2 - j X_c}$$

since R is in parallel with the impedance $(R_2 - j X_c)$. Voltage V_R is also the drop across $(R_2 - j X_c)$, and the output voltage drop across R_2 is therefore

$$V_{kg2} = \left(\frac{V_R}{R_2 - j X_c} \right) R_2 . \qquad (3\text{-}19)$$

From Equation (3-19) the low frequency gain is

$$A_L = \frac{V_{kg2}}{- E_g} = \frac{- g_m R R_2}{R + R_2 - j X} \qquad (3\text{-}20)$$

The numerator and denominator of Equation (3-20) may be divided by $(R + R_2)$ in order to introduce R_{eq} and thereby also A_1 into the expression for A_L. The result is

$$A_L = \frac{\frac{- g_m R R_2}{R + R_2}}{\frac{R + R_2 - j X}{R + R_2}} \qquad (3\text{-}21)$$

From Equation (3-8),

Using Equation (3-18)

$$\frac{1}{R_1} = \frac{1}{R} + \frac{1}{R_2}$$

$$R_1 = \frac{R R_2}{R + R_2} \qquad (3-22)$$

If also,

$$\frac{1}{R + R_2} = \frac{1}{2\pi f C (R + R_2)} = \frac{f_1}{f}$$

where $$f_1 = \frac{1}{2\pi C (R + R_2)} = \frac{Y_2 (Y_1 + Y_p)}{2\pi C (Y_2 + Y_1 + Y_p)} \qquad (3-23)$$

then finally Equation (3-21) becomes

$$A_L = \frac{A_I}{1 - j \frac{f_1}{f}} \qquad (3-24)$$

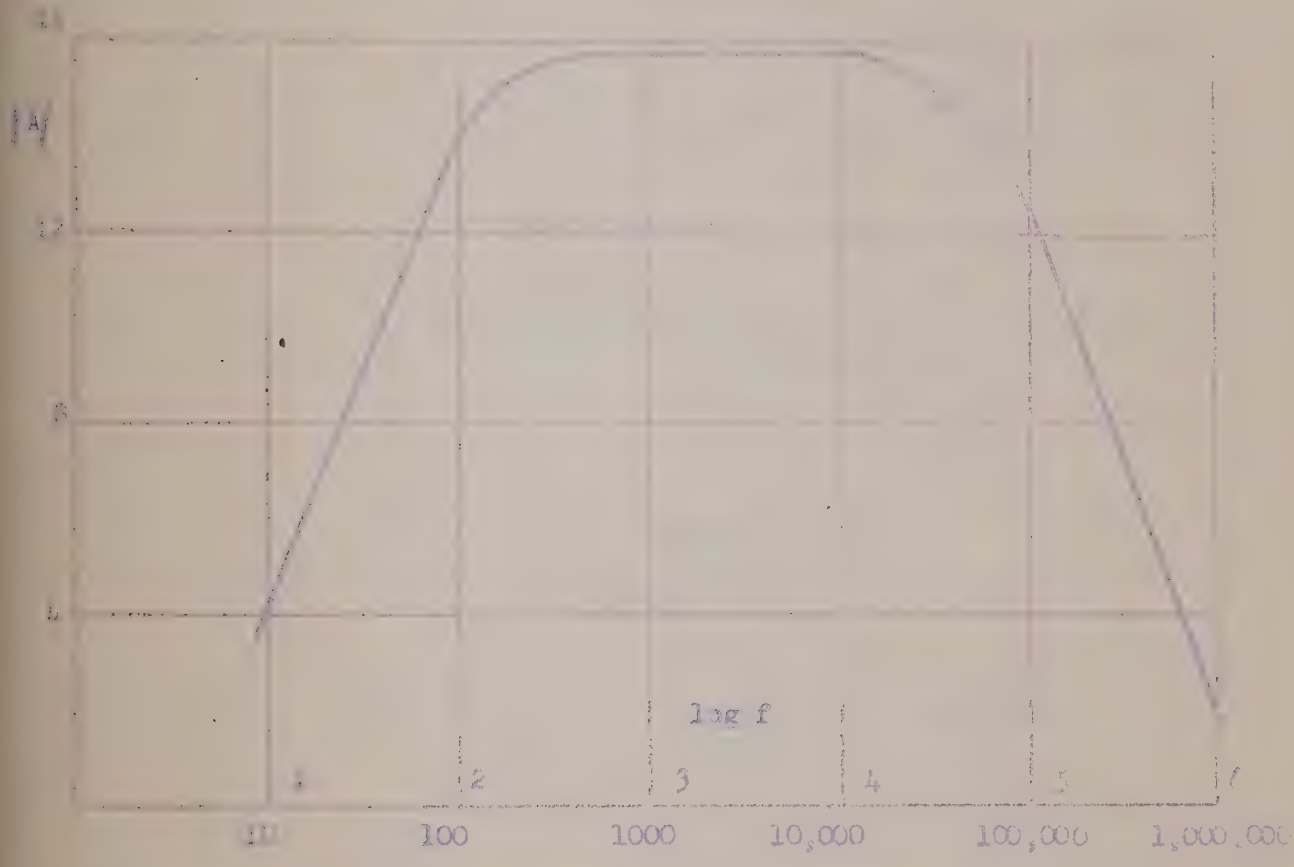
or

$$A_L = \frac{|A_I| \angle 180^\circ}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2} \angle -\theta_L} = \frac{|A_I|}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} \angle 180^\circ + \theta_L \qquad (3-25)$$

where $\theta_L = \tan^{-1} \frac{f_1}{f}$

From Equation (3-23), f_1 is the frequency on the low frequency side of the intermediate range at which the low frequency gain falls to 70.7% of the gain in the intermediate frequency region, and θ_L is the measure of the phase distortion at low frequencies.

The frequencies f_1 and f_H are sometimes referred to as the "half-power" frequencies by analogy with the criterion for sharpness of response in resonance curves. For amplifiers, however, the width of the frequency band $f_H - f_L$ is a measure of the freedom of the amplifier from frequency distortion. A typical R-C coupled amplifier gain-frequency curve is plotted in Figure 3-11. For an amplifier to have the desirable flat region extended over a wide range of frequencies, f_L must be small and f_H large. The requirements are met according to Equation (3-11) and Equation (3-23) by making C large, C_c small, and f_1 and f_2 large (R_1 and R_2 small).



Frequency - cycles per second

Figure 3-11.

Gain-frequency curve for a typical R-C coupled amplifier

Solution: For the 6C5, $g_m = 2000$ micromhos, $r_p = 10{,}000$ ohms,

$$A_I = \frac{-g_m}{Y_p + Y_1 + Y_2} = \frac{-2000 \times 10^{-6}}{10^{-4} + 0.2 \times 10^{-4} + 0.1 \times 10^{-4}}$$

$$= -\frac{20}{1.3} = -15.4$$

$$f_L = \frac{Y_2 (Y_1 + Y_p)}{2\pi C (Y_1 + Y_2 + Y_p)} = \frac{0.1 \times 10^{-4} (1.2 \times 10^{-4})}{6.28 (0.04 \times 10^{-6})(1.3 \times 10^{-4})}$$

$$= 36.8 \text{ cycles/sec.}$$

$$f_H = \frac{Y_p + Y_1 + Y_2}{2\pi C_o} = \frac{1.3 \times 10^{-4}}{6.28 (150 \times 10^{-12})} = 138{,}000 \text{ cycles.}$$

$$\frac{|A_L|}{|A_I|} = \frac{1}{4} = \frac{1}{\sqrt{1 + \left(\frac{36.8}{f}\right)^2}}$$

$$\left(\frac{36.8}{f}\right)^2 = 15, \qquad f = \frac{36.8}{\sqrt{15}} = 9.5 \text{ cycles/sec.}$$

$$\frac{|A_H|}{|A_I|} = \frac{1}{4} = \frac{1}{\sqrt{1 + \left(\frac{f}{138{,}000}\right)^2}}$$

$$\left(\frac{f}{138{,}000}\right)^2 = 15, \qquad f = 138{,}000\sqrt{15} = 535{,}000 \text{ cycles/sec.}$$

Equation (3-16) and Equation (3-17) provide a basis for computing universal phase shift and gain curves in which the phase shift and the gain ratios

$$\left|\frac{A_L}{A_I}\right| \quad \text{and}$$

$$\left|\frac{A_H}{A_I}\right|$$

are plotted against the frequency ratios $\frac{f_L}{f}$ and $\frac{f}{f_H}$. Such curves [illegible] data is given in the following table:

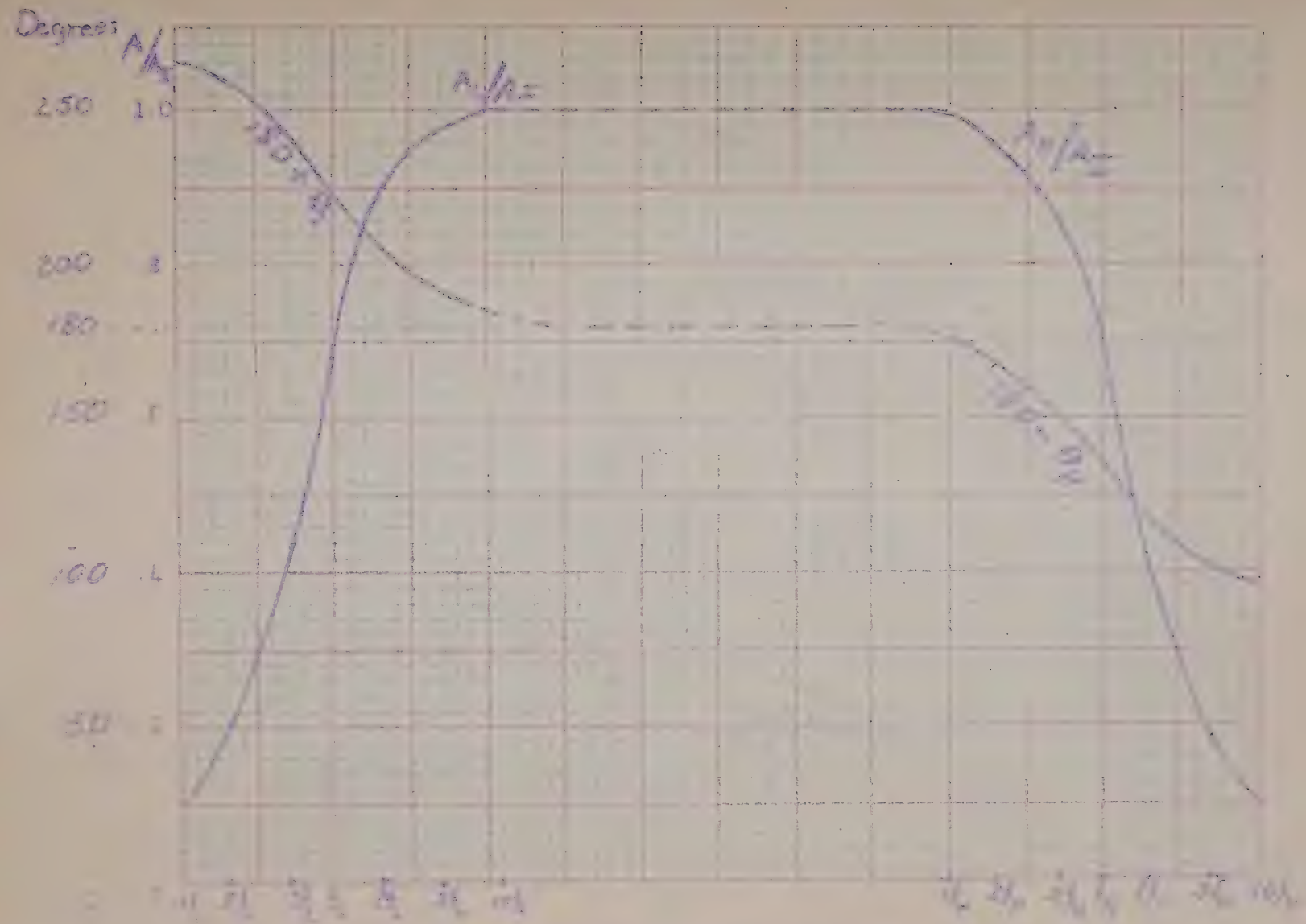
TABULATION OF PHASE SHIFT ANGLES AND GAIN RATIOS FOR ANY R-C COUPLED AMPLIFIER

f_L/f or f/f_H	θ_L or θ_H	f	A/A_I	f
10	84.3	0.1 f_L	0.0995	10 f_H
5	78.7	0.2 f_L	0.196	5 f_H
2	63.4	0.5 f_L	0.446	2 f_H
1	45	1.0 f_L	0.707	1.0 f_H
0.5	26.6	2 f_L	0.893	0.5 f_H
0.2	11.3	5 f_L	0.980	0.2 f_H
0.1	5.8	10 f_L	0.995	0.1 f_H

The values given in the foregoing table are plotted in Figure 3-12. Although the behavior of any R-C coupled amplifier may be predicted by use of the curves of Figure 3-12, it is first necessary to compute f_L and f_H in order to use the curves. Having computed these frequencies, however, it is often as easy and perhaps more accurate to use the gain and phase shift equations (3-16) and (3-20).

3-5. Inductance-Capacitance Coupling.

The method of coupling shown in Figure 3-13 is termed inductance capacitance coupling. In the frequency range over which the gain is substantially constant, X_{C_1} and X_{C_2} are small and may be neglected; X_{L_1} and X_{L_2} are very large, so that the equivalent circuit of [illegible]



Frequency as a function of f_L or f_H

carried out in the same way as for the R-C case. Since the frequency range for which the gain is flat is limited by the inductances as well as capacitances, the range is less extensive than is the case for the R-C amplifier. The chief advantages of the method are that the operating plate battery voltages required are much less as a result of the low direct voltage drop in the inductance, and that the efficiency is somewhat higher. In determining the load line on the plate characteristics, the operating point is approximately the intersection between the vertical line $e_b = E_{bb}$ and the characteristic corresponding to the grid bias used. Through the operating point, the load line may be drawn with slope corresponding to the load resistor R_1 (or R_2), of Figure 3-13.

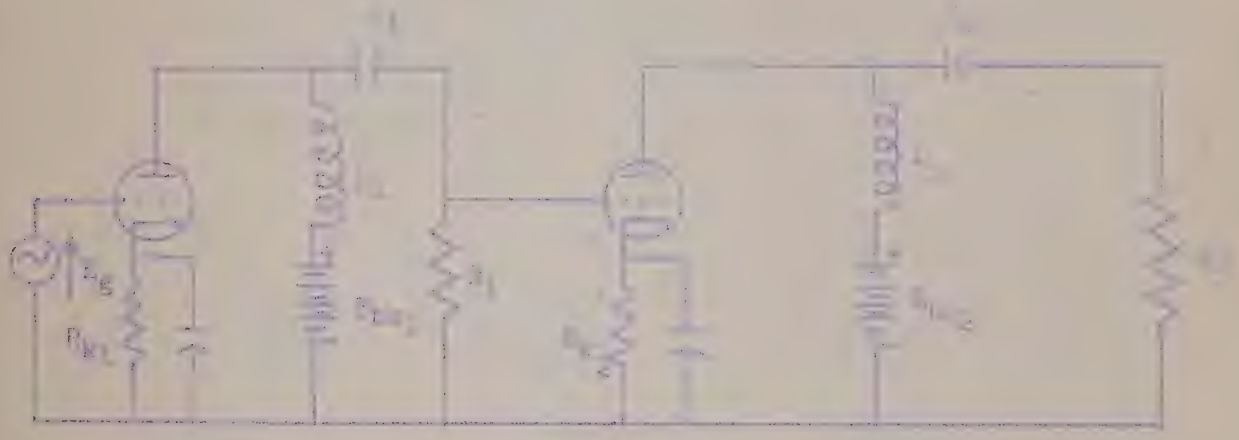


Figure 3-13.

Impedance-capacitance coupled 2-stage amplifier.

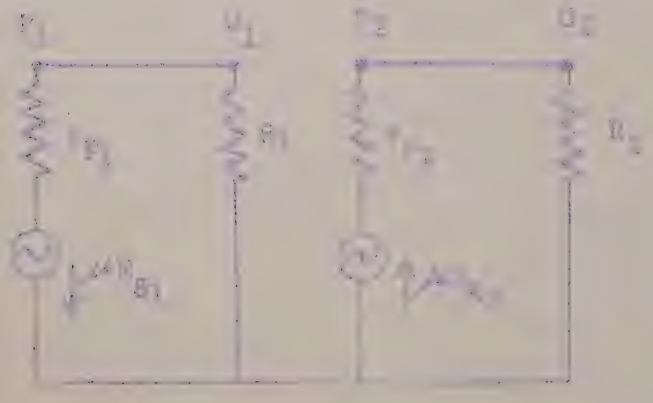


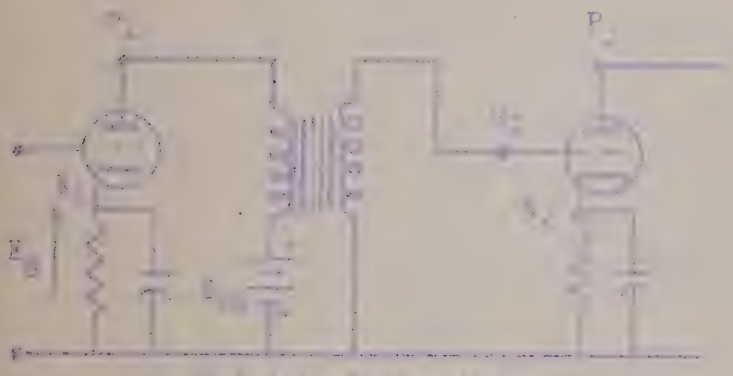
Figure 3-14.

Equivalent A-C plate circuit of the amplifier of Figure 3-13, in Intermediate Frequency Range.

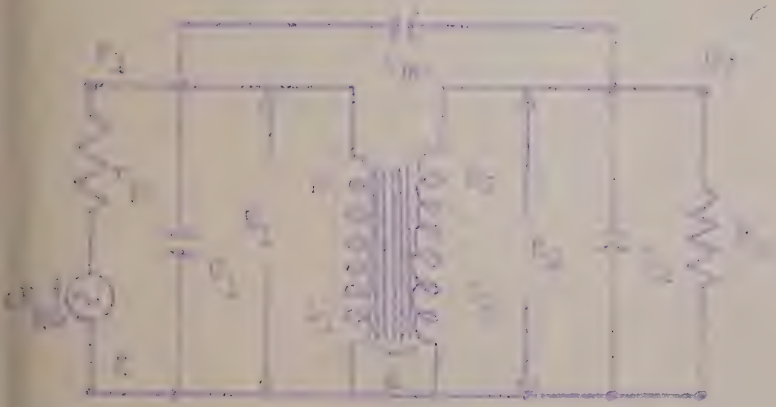
3-6 Transformer-Coupled Amplifiers

The circuit of Figure 3-15 uses a transformer as a coupling device including the following:

(1) each stage has d-c isolation without the use of condensers;
(2) the use of a step-up turns ratio makes possible a voltage gain in excess of unity;
(3) smaller plate supply voltages are required to provide the necessary quiescent operating-tube voltage and current, and the $I_b^2 R$ losses are greatly reduced, as a result of low primary winding resistance;
(4) Coupling to an output stage involving two tubes in push-pull is easily accomplished by using a center-tapped transformer. The push-pull circuit will be discussed later in this chapter.
(5) Phase shift



(a) Actual circuit



(b) Equivalent a-c circuit

Figure 3-15.
Circuits of a transformer-coupled amplifier

R_1 and R_2 are respectively the primary and secondary effective resistances;

L_1 and L_2 are respectively the primary and secondary winding inductances

C_m = capacitance between primary and secondary windings

r_p = plate resistance of tube No. 1

$C_1 = C_{pk} + C_p +$ lead capacitance

C_{pk} = interelectrode plate-cathode capacitance

C_p = primary distributed winding capacitance

$C_2 = C_s + C_{11} +$ lead capacitance

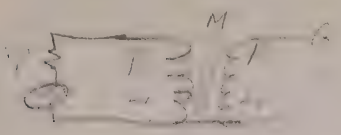
C_s = secondary distributed

C_{11} = input capacitance of next stage

R_{11} = input resistance of next stage

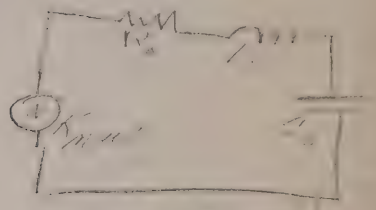
M = mutual inductance between windings

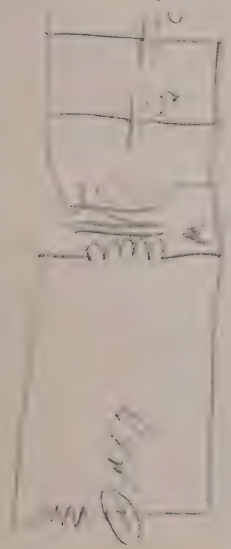
Low Freq. (Hybrid π)



Mid Freq.

High Freq.





be used. Iron core transformers are used over the audio range; the cores of such transformers are usually composed of specially treated high permeability, low-loss iron, and the coefficient of coupling is very nearly equal to unity. For frequencies in the radio range, air core transformers are used. The turns ratio of audio frequency coupling transformers does not usually exceed three because of the introduction of secondary distributed capacitance between turns and because of the desirability of having a reasonably high primary inductance.

The principal disadvantages of audio frequency transformer coupling include the following:

(1) transformers are expensive and bulky;
(2) the frequency range in which there is negligible frequency and phase distortion is relatively narrow;
(3) careful shielding is required (contributing to item one) to prevent coupling due to stray magnetic fields.

Analysis of the equivalent circuit of Figure 3-15(b), like that of the R-C coupled amplifier equivalent circuit, is complicated considerably by the presence of circuit elements whose reactances must be considered practically only in certain frequency ranges. It is, therefore, preferable to make the necessary approximations in dealing with the circuit for the low, intermediate and high frequency ranges rather than to attempt a complete solution in which certain terms would be relatively negligible in the various frequency ranges. The equivalent circuit of Figure 3-15(b) may then be simplified for approximate analysis as follows:

(1) In the audio frequency range over which iron-core coupling transformers work satisfactorily the reactance of condenser C_1 and the resistance R_{11} (see Chapter I, page 44) are so high that these elements may be omitted from the equivalent circuit.

(2) Capacitance C_m contributes a current component to both primary and secondary circuits. These currents, however, become of practical importance only at the high frequency end of the useful frequency range of the amplifier because the reactance of C_m is very high in the low and intermediate frequency ranges. If the transformer is provided with a grounded shield between primary and secondary windings C_m may be reduced to a negligible value at all frequencies. A fairly satisfactory approximation for transformers without shielding is to neglect the contribution of C_m to current in the primary circuit and to replace C_m by an equivalent capacitance connected in parallel with C_2. The equivalent capacitance would be equivalent in the sense that it contributes the same current component to the secondary as does the actual capacitance C_m. The equivalent capacitance C' may be determined as follows: (see equation 3-15)

depends upon the voltage across C_m; this voltage, as may be seen from Figure 3-15(b), is the vector sum (or difference, depending upon relative winding directions of the transformer primary and secondary) of the voltages E_1 and E_2; the assumed relative positive sense of these voltages on Figure 3-15(b) is purely arbitrary; since in magnitude $E_2 = n E_1$, where $n = N_2/N_1$, the secondary-to-primary turns ratio, the current through C_m or through C' is given by

$$I = \frac{(E_1 \pm n E_1)}{-j X_{Cm}} = \frac{n E_1}{-j X_{C'}}$$

or

$$C' = \left(\frac{1 \pm n}{n}\right) C_m$$

The secondary capacitance then becomes

$$C_0 = C_2 + C'$$

The use of C' in the secondary to replace C_m is a good approximation to the extent that the coefficient of coupling, k, may be considered equal to unity. At high enough frequencies, even though $k = 0.98$, the leakage reactances may become appreciable, so that the voltage across C' is not $n E_1$ because of the voltage drop and the phase shift in the leakage reactance. The neglect of a corresponding current component introduced by C_m in the primary is justified because the transformer is an interstage transformer in which the secondary current is negligible. The contribution of C_m to the primary current is negligible compared with the primary a-c component of plate current.

The equivalent circuit of Figure 3-15(b) may now be replaced by that of Figure 3-16. Before giving any definitions of the frequency ranges, the mesh equations of the circuit will be written. From Figure 3-16,

$$E_g = I_1 Z_{11} + I_2 Z_{12}$$

$$0 = I_1 Z_{21} + I_2 Z_{22} \qquad (3\text{-}36)^3$$

where

$$Z_{11} = r_p + R_1 + j\omega L_1$$

$$Z_{22} = R_2 + j\left(\omega L_2 - \frac{1}{\omega C_0}\right)$$

$$Z_{12} = Z_{21} = -j\omega M.$$

---

3 For a discussion of coupled circuits and the use of the double subscript notation see Chapter VII, W. L. Everitt, "Communication Engineering", McGraw-Hill Book Company, 1937.

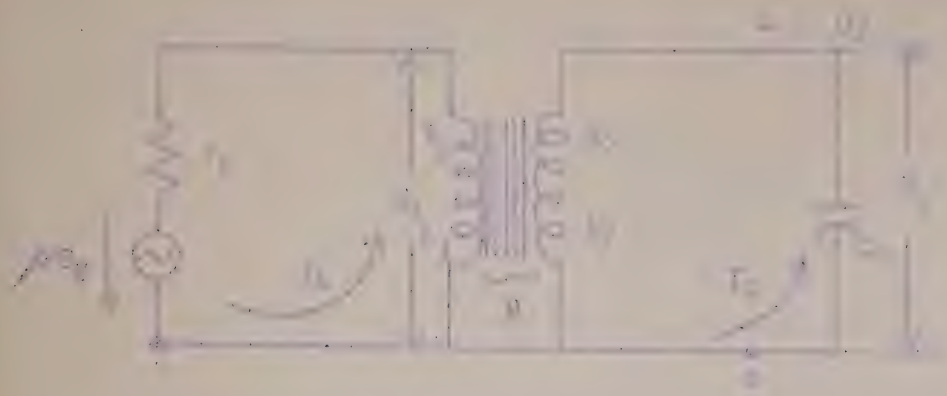


Figure 3-16.
Simplified equivalent circuit of Figure 3-15.

In the low and throughout most of the intermediate frequency range, X_{L_2} is so high that X_{L_2} may be considered infinite. Then I_2 = 0 and

$$I_1 = \frac{\mu E_g}{Z_1}$$

$$E_2 = -I_1 Z_{21} = -j\omega M I_1 = \frac{-\mu E_g (j\omega M)}{r_p + R_1 + j\omega L_1} \qquad (3-27)$$

Now

$$M = k\sqrt{L_1 L_2} \quad , \text{ and } n = \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}}$$

whence

$$M = k L_1 \sqrt{\frac{L_2}{L_1}} = k n L_1 \qquad (3-28)$$

Since k is very nearly unity, M may be replaced by nL_1. Then

$$E_2 = \frac{n \mu E_g (j\omega L_1)}{r_p + R_1 + j\omega L_1} \qquad (3-29)$$

By definition, the amplifier gain is

$$A_L = \frac{V_{kg2}}{-E_g} = \frac{-n\mu (j\omega L_1)}{r_p + R_1 + j\omega L_1}$$

Dividing numerator and denominator by $(j\omega L_1)$,

Equation (3-30) should properly be written with $\pm n\mu$ instead of $n\mu$ in the numerator, since the phase of the secondary voltage depends upon the relative winding directions of primary and secondary. Numerically the gain is given by

$$|A_1| = \frac{n\mu}{\sqrt{1 + \left(\frac{r_p + R_1}{\omega L_1}\right)^2}} \quad . \qquad (3\text{-}31)$$

For very low frequencies, ωL_1 is small and $\left(\frac{r_p + R_1}{\omega L_1}\right)$ large, so that $|A_L|$ is small. The primary inductance L_1 may be in the neighborhood of 20 henries. Resistance R_1 is usually small compared to r_p, and for tubes ordinarily transformer coupled, r_p is in the range 3000 to 15000 ohms. As the frequency increases from low values, $\left(\frac{r_p + R_1}{\omega L_1}\right)^2$ rapidly becomes small compared with unity. The gain, then approaches $n\mu$ as f increases.

To distinguish between low and mid-frequency ranges, the low frequency range will be defined somewhat arbitrarily as the range of frequency below that frequency for which

$$\frac{r_p + R_1}{\omega L_1} = \frac{1}{10} \quad .$$

The intermediate or mid-frequency gain will then be given by

$$A_I = - n\mu \quad . \qquad (3\text{-}32)$$

and is independent of frequency. Therefore the gain-frequency curve in the mid-frequency region will be flat and horizontal.

It is possible to determine the gain in the higher frequency range over which the reactance of C_o may not be neglected by use of the circuit of figure 3-10. If the secondary circuit is opened at terminals a-b, the equivalent circuit looking to the left may be obtained by an application of the mesh equations (3-26) and Thevenin's Theorem. If the driving point impedance as obtained from Equations (3-26) is applied to the secondary, rather than to the primary, the impedance at terminals a-b looking toward the transformer is given by

$$Z_{ab} = R_2 + j X_2 + \frac{(\omega M)^2}{r_p + R_1 + j X_1}$$

$$= R_2 + \frac{(\omega M)^2}{(r_p + R_1)^2 + X_1^2}(r_p + R_1) + j\left[X_2 - \frac{(\omega M)^2}{(r_p + R_1)^2 + X_1^2} X_1\right]$$

$$\frac{(\omega M)^2}{X_1} = \frac{k^2 X_1 X_2}{X_1} = k^2 X_2 = k^2 \omega L_2$$

$$\frac{(\omega M)^2}{X_1} = k^2 \frac{X_2}{X_1} = k^2 \frac{L_2}{L_1} = k^2 n^2 \qquad (3\text{-}36)$$

Then, let

$$R_o = R_2 + k^2 n^2 (r_p + R_1) \qquad (3\text{-}36a)$$

so that Equation (3-34) becomes

$$Z_{ab} = R_o + j\omega L_o \qquad (3\text{-}34a)$$

The equivalent circuit is shown in Figure 3-16a. The voltage E, according

$$= \frac{\mu E_g (j\omega M)}{r_p + R_1 + j X_1} \cong \frac{\mu E_g \omega M}{X_1}$$

$$\omega \geq \frac{10 (r_p + R_1)}{L}$$

or, using (3-35),

$$E = k n \mu E_g \cong n \mu E_g \qquad (3\text{-}37)$$

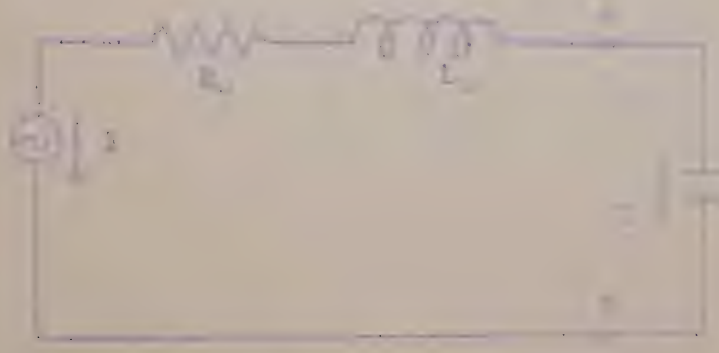


Figure 3-16a

The magnitude of the voltage gain at high frequencies is

$$A_H = \frac{-g_m}{j\omega C_o \left[R_o + j\left(\omega L_o - \frac{1}{\omega C_o}\right)\right]} \qquad (3\text{-}38)$$

Since $A_I = -g_m R_o$, (3-38) may be written as

$$\frac{A_H}{A_I} = \frac{1}{j\omega R_o C_o - \omega^2 L_o C_o + 1} \qquad (3\text{-}39)$$

$$\omega_o^2 = \frac{1}{L_o C_o} \qquad (3\text{-}40)$$

$$Q = \frac{\omega_o L_o}{R_o} \qquad (3\text{-}41)$$

then (3-39) simplifies to

If the ratio $\left|\frac{A_H}{A_I}\right|$ is plotted[4] against $\frac{\omega}{\omega_c}$ for several fixed

in the vicinity of $\omega = \omega_c$, and that the magnitude of the rise of the

tion of the curves of Figure 3-17 will show that the flat region of the gain curve may be extended by choosing $Q_o = 0.75$. At $\omega = \omega_c$, it is evident from (3-42b) that

$$\left|\frac{A_H}{A_I}\right| = Q_o$$

The intermediate frequency range may now be conveniently bounded as follows:

$$\frac{10\,(r_p + R_1)}{L_1} < \omega < \frac{\omega_c}{10} \qquad (3\text{-}44)$$

(3-42b). The result is

$$\left|\frac{A_H}{A_I}\right| = \frac{1}{\sqrt{(1 - 0.01)^2 + \left(\frac{1}{10\,Q_o}\right)^2}}$$

so that, for $Q_o \geq 1$, $\left|A_H\right| \cong \left|A_I\right|$.

In the low frequency range, the gain and phase angle curves were plotted from Equation (3-51) by assigning arbitrary values to the ratio $\left[\frac{\omega\, L_1}{r_p + R_1}\right]$. Partial data for the curves is given in the following tabulation for $Q_o = 1.25$:

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[4] Terman, F. E., "Universal Amplification Charts", Electronics 10, 35, June 1937.

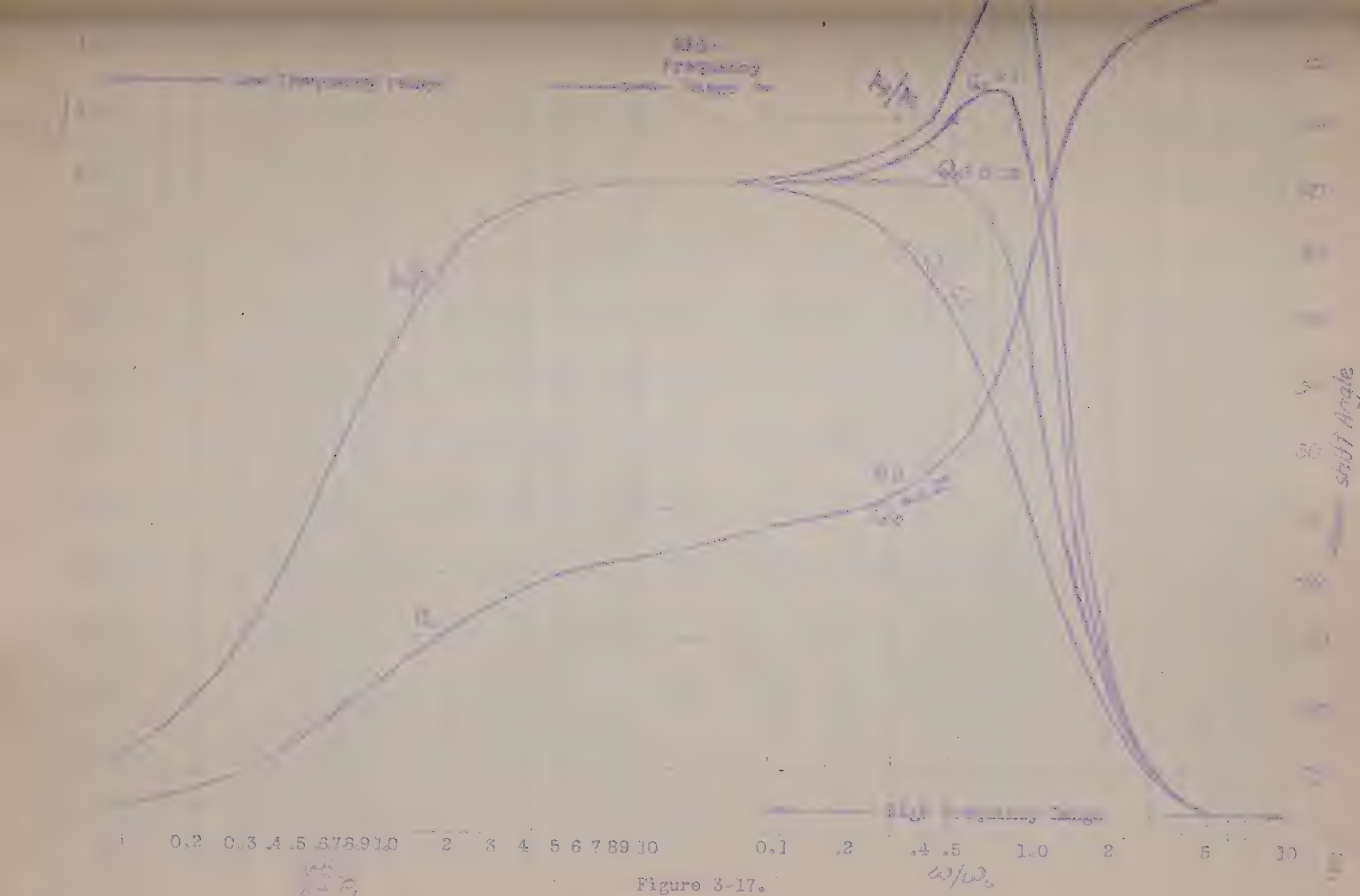

Low Frequency Range
Mid Frequency Range
High Frequency Range
Phase Shift Angle
0.2 0.3 .4 .5 .6 7 8 9 1.0 2 3 4 5 6 7 8 9 10
0.1 .2 .4 .5 1.0 2 5 10

Figure 3-17.

Universal Gain and Phase Shift vs. Frequency Curves for Transformer Coupled Amplifiers.

f/f_{c_2}	$\left(1-\frac{\omega^2}{\omega_{c_2}^2}\right)$	$\frac{\omega}{\omega_{c_2} Q_2}$	θ_H	A_H/A_I
0.1	0.99	0.08	4.64°	1.01
0.2	0.96	0.16	9.1[illegible]°	1.03
0.3	0.91	0.24	14.8°	1.06
0.4	0.84	0.32	20.86°	1.10
0.5	0.75	0.40	28.[illegible]°	1.18
0.6	0.64	0.48	36.8°	1.25
0.7	0.51	0.56	47.7°	1.32
0.8	0.36	0.64	60.6°	1.36
0.9	0.19	0.72	75.2°	1.34
1.0	0.00	0.80	90°	1.25
1.2	-0.44	0.96	114.6°	0.945
1.5	-1.25	1.20	136.2°	0.58
2.0	-3.00	1.60	152°	0.29
3.0	-8.00	2.40	163.3°	0.112
5.0	-24	4.00	170.5°	0.041
10.0	-99	8.00	175°	0.01

It is evident from the equations (3-31) and (3-32) from which the curves of Figure 3-17 were prepared that an extension of the region of uniform gain to low frequencies requires that the ratio $\frac{r_p + R_1}{C_1}$ be decreased. This requires that $r_p + R_1$ should be reduced, L_c increased. An extension of the region of uniform gain to higher frequencies requires an increase in the resonant angular frequency ω_{c_2}, but to increase ω_{c_2} requires a decrease in L_c or in C_{c_2} or both. Since $L_c = L_m(1-k^2)$, a decrease in L_c is obtained by (1) decreasing L_2, or (2) decreasing the leakage inductance by increasing k toward 1. Since C_{c_2} depends upon the distributed capacitance, the conditions for widening the range of uniform gain contradict each other, for an increase in L_2 for the purpose of improving the response at the low end requires more turns and introduces more distributed capacitance for a fixed turns ratio, and results in increasing L_c, rather than in decreasing it as required at the high frequency end. The practical result is that turns ratios for coupling transformers seldom exceed three, and that triodes of reasonably low values of plate resistance are used in transformer coupled amplifiers. The low value of $Q_2 = 0.75$ or 0.8 necessary to suppress the resonant peak may be controlled when ωL_c is large by increasing the resistance $R_c = R_m + k^2 n^2 (r_p + R_1)$. This is accomplished by increasing R_2 since the secondary resistance is not involved in determining the low frequency gain. The secondary resistance may be increased by using high resistivity wire for the secondary winding. A

It should be pointed out that the phase angle θ_L of Figure 3-17 is given by its tangent from the relation (from Equation (3-30))

$$-\theta_L = \tan^{-1}\left[-\frac{r_p + R_1}{\omega L_1}\right] \qquad (3\text{-}45)$$

From Equations (3-30) and (3-32)

$$\left|\frac{A_L}{A_I}\right| = \frac{1}{\sqrt{1+\left(\frac{r_p + R_1}{\omega L_1}\right)^2}} \angle\theta_L \qquad (3\text{-}46)$$

so that θ_L is the angle of lead of A_L with respect to A_I. In the same way, from Equation (3-42b), θ_H is the angle of lag of A_H with respect to A_I. In each case the angles are plotted in Figure 3-17 as computed from their defining equations. The actual angle between output and input voltage depends, as has been stated, upon the relative winding directions of the primary and secondary of the transformer.

The theory of the transformer coupled amplifier will be illustrated by solving the following sample problem:

An amplifier stage exhibits a 6C5 coupled by an interstage transformer to a 6J5. The constants of the 6J5 are $\mu = 20$, $r_p = 10{,}000$; the interstage transformer, $R_1 = 600$ ohms, $r_p = 10{,}000$ ohms, $L_1 = 20$ henries, $N_2/N_1 = n = 3$. Assume that the coefficient of coupling is 0.995. If the equivalent resistance load is $R_L = 40{,}000$ ohms, determine the gain-frequency characteristics of the 6C5 stage.

Solution: $r_p + R_1 = 10{,}600$ ohms; $|A_I| = n\mu = 60$.

For $\omega L_1 = r_p + R_1$, $|A_L| = 0.707\ |A_I|$, or

$$\omega_L = \frac{r_p + R_1}{L_1} = \frac{10{,}600}{20} = 530 \text{ radians per sec. for}$$

$$|A_L| = 0.707\ |A_I|$$

$$f_L = \frac{530}{2\pi} = 84.4 \text{ cycles/sec.}, \text{ and } |A_L| = 60(0.707) = 42.4$$

at $\omega = 530$ rad./sec.

[illegible] Fred. Spec. [illegible] (1928)

The beginning of the intermediate frequency range is determined by

$$\frac{r_p + R_1}{\omega L_1} = 0.1$$

$$\omega L_1 = 106{,}000; \text{ or } \quad f = \frac{5300}{6.28} = 844 \text{ cycles/sec.}$$

$$L_o = L_2 (1 - k^2) \quad \text{where } L_2 = n^2 L_1 = 180 \text{ henries.}$$

Since $k^2 = 0.99$, $L_o = 0.01\ (180) = 1.8$ henries.

From

$$\omega_o = \frac{1}{\sqrt{L_o C_o}} = \frac{1}{\sqrt{1.8 \times 80 \times 10^{-12}}} = 83{,}300 \text{ rad./sec.}$$

$$R_o = R_2 + 0.99(9)\ (10{,}600) = 25{,}000 + 94{,}400 = 119{,}400 \text{ ohms.}$$

From

$$Q_o = \frac{\omega_o L_o}{R_o} = \frac{83{,}300(1.8)}{119{,}400} = 1.25.$$

For

$$\omega = 0.1\ \omega_o \quad , \quad |A_H| = \frac{60}{\sqrt{(0.99)^2 + \left(\frac{1}{12.5}\right)^2}} = \frac{60}{0.99} = 60.6$$

The corresponding frequency is $\frac{8330}{6.28} = 1326$ cycles/sec.

[illegible] sec. and needs to be extended by reducing C_o or L_o. In addition, accord- [illegible] near $f = 0.8\ f_o$ or 10,600 cycles/sec., and at this frequency the gain will be $1.36(60) = 81.6$.

[illegible] be predicted with reasonable accuracy consistent with the approximations used in the foregoing theory.

3-7 Push-Pull Operation.

Transformer coupling as discussed in the foregoing section or [illegible]

[illegible], F. E., "Radio Engineering", 3rd Ed., McGraw-Hill Book Co.

stage audio frequency amplifier. The last stage of a voltage amplifier may be expected to deliver the amplified signal at an a-c power level of several watts, and so is called the power stage. Transformers are used to couple the power stage to the load and are called output transformers to match the output impedance of the amplifier to the impedance of a load,

If a power stage is coupled to a load impedance as shown in Figure 3-18, the amplifier is said to be single-sided. A number of distinct advantages are obtained by using the circuit arrangement of Figure 3-19 in the power stage. The tubes in the circuit of Figure 3-19 are said to be connected in push-pull. The advantages of the push-pull amplifier over the single-sided amplifier are: very much less harmonic distortion, more than twice the power output possible with a single tube, and no transformer core saturation. The reasons for these advantages will be given in the following discussion of the push-pull circuit.

The connection diagram of Figure 3-19 shows that the grids of the two identical tubes driven by the input coupling transformer T_1 will swing in the positive direction alternately, so that the grid-to-cathode voltage of G_1 will be 180° out of phase with that of G_2. If the grid bias

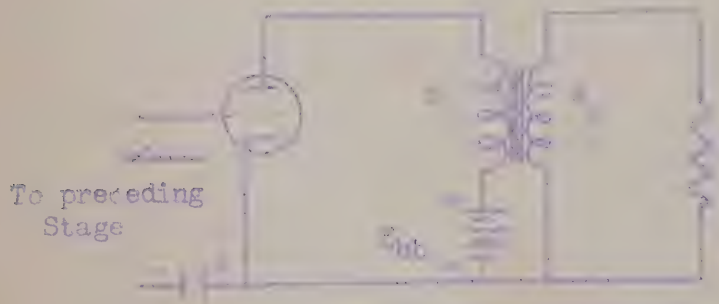


Figure 3-18 - Single-sided output stage.

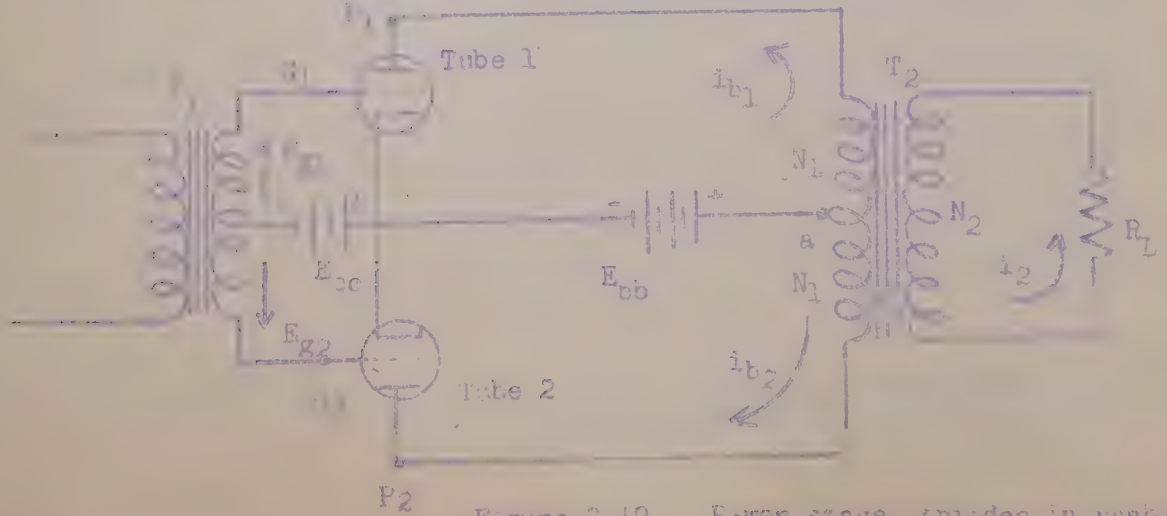


Figure 3-19 - Power stage, triodes in push-pull.

and the excitation of each tube are such as to provide Class A operation, then the plate current of one tube will reach its maximum value at the same instant that the plate current of the other tube will be passing through its minimum value. This may be shown graphically by the use of two identical tube dynamic characteristics, one for each tube, arranged as shown in Figure 3-20, in which the dynamic characteristic of tube 2 has been rotated 180° with respect to the characteristic of tube 1, with the operating points Q_1 and Q_2 in vertical alignment. It will be shown later that the resultant or composite dynamic characteristic of the two tubes is equivalent to the individual dynamic characteristics combined. It should be noticed that the individual plate currents i_{p1} and i_{p2}, Figure 3-20, do not have identical positive and negative peak values, and are not sinusoids because of the curved individual dynamic characteristics, but the composite current wave form is symmetrical, and very closely sinusoidal. Also, the d-c zero signal components I_{b_1} and I_{b_2} are equal.

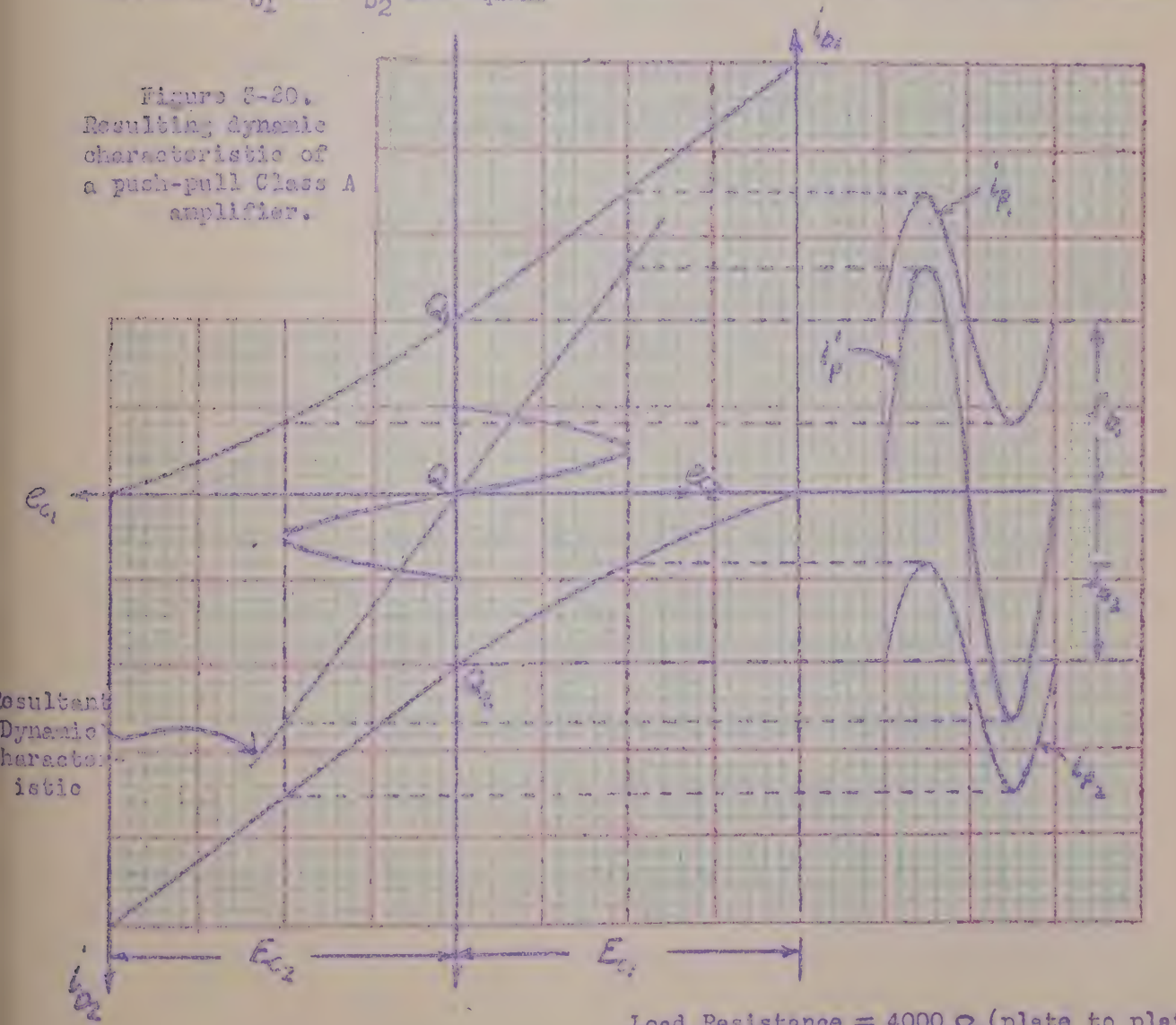


Figure 3-20. Resulting dynamic characteristic of a push-pull Class A amplifier.

Load Resistance = 4000 Ω (plate to plate)
Grid excitation 20^{V} peak

3-8 Harmonic Suppression with the Push-pull Connection

It will be demonstrated in the present section that (1) no even harmonics of signal frequency appear in the output of a push-pull amplifier and (2) that no odd harmonics appear in the plate circuit.

It will be observed that the direct components of plate current I_{b_1} and I_{b_2} flow through the two halves of the primary winding of the output transformer T_2 of Figure 3-19 in opposite directions. As a result, the d-c ampere-turns cancel and there is no magnetic core saturation. If there are N_1 turns in each half of the transformer primary and N_2 turns in the secondary, the instantaneous currents in primary and secondary (for an ideal transformer) are related as follows:

$$N_1 i_{b_1} - N_1 i_{b_2} = N_2 i_2 \qquad (3\text{-}47)$$

where i_2 is the secondary current. The instantaneous value of what may be called the composite primary current is expressed by the instantaneous difference of the total plate currents, or, with $I_{b_1} = I_{b_2}$,

$$i_{b_1} - i_{b_2} = (I_{b_1} + i_{p_1}) - (I_{b_2} + i_{p_2}) = i_{p_1} - i_{p_2} \qquad (3\text{-}48)$$

For identical tubes, the series expansion of Equation (2-11) may be used for each. Then, for either i_{p_1} or i_{p_2} referred to the origin through Q_1 or Q_2,

$$i_{p_{1,2}} = C_1 e_g + C_2 e_g^2 + C_3 e_g^3 + \cdots + C_n e_g^n + \cdots$$

If $e_{g_1} = \sqrt{2}\ E_g \cos \omega t$ is the equation for the input voltage between cathode and grid of tube 1, then $e_{g_2} = -\sqrt{2}\ E_g \cos \omega t$ is the expression for the input voltage for tube 2. The alternating components of plate currents for the two tubes may then be obtained by substituting the respective excitation voltages in the expression for the overall transfer characteristic and simplifying, as in Equation (2-15). The results are

$$i_{p_1} = B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + \cdots + \cdots \qquad (3\text{-}49a)$$

$$i_{p_2} = B_0 - B_1 \cos \omega t + B_2 \cos 2\omega t - B_3 \cos 3\omega t + \cdots + \cdots \qquad (3\text{-}49b)$$

where B_1, B_3, B_5, etc. represent resulting fundamental amplitudes and B_0 is the change in the d-c component. The composite or equivalent instantaneous alternating current in the transformer primary according to Equation (3-47) is

$$i'_p = i_{p_1} - i_{p_2} = 2 B_1 \cos \omega t + 2 B_3 \cos 3 \omega t + \cdots + \cdots \qquad (3\text{-}50)$$

Equation (3-50) shows that even harmonics introduced by non-linearity of the dynamic tube characteristics are eliminated in the transformer primary and secondary by the push-pull connection. Because of elimination of the second harmonic distortion, the grids may be driven positive without exceeding the limits of harmonic distortion in the output, and an output power more than twice that available from a single tube can be obtained.

If the individual tube plate currents represented by the individual tube dynamic characteristics of Figure 3-20 are combined by algebraic addition of ordinates according to Equation (3-48), the individual dynamic characteristics may be combined into a single composite dynamic characteristic for both tubes. As shown in Figure 3-20, the composite dynamic characteristic is a straight line. The sinusoidal grid excitation voltage has been drawn in the usual way with its time axis through Q_1 and Q_2. The composite plate current i'_p may be obtained directly from the composite characteristic or the individual plate currents may be obtained first from the individual dynamic characteristics and then combined algebraically according to Equation (3-48). As previously mentioned, it should be observed that the individual currents i_{p1} and i_{p2} have unequal positive and negative amplitudes, but that the composite current i'_p is quite symmetrical and free from harmonics.

The composite characteristic of two identical tubes in push-pull is more easily obtained than the individual tube dynamic characteristics. Since the two plates of the amplifier tubes in push-pull are connected together through a transformer, a change in the voltage of either plate will affect the voltage of the other plate. The result is that the individual tube load lines and dynamic characteristics are not independent and are not straight lines. This will be shown in the analysis[7] to follow in Section 3-11.

The current in the mid-branch connection through the plate supply battery (of Figure 3-19) is the sum of the separate tube currents, or, using (3-49),

$$i_{b1} + i_{b2} = 2\,(I_b + B_o + B_2 \cos 2\omega t + B_4 \cos 4\omega t + \ldots + \ldots) \qquad (3\text{-}51)$$

According to Equation (3-51), the fundamental and odd harmonic a-c components of plate current add to zero in this part of the circuit. This fact has significance because of implications as to the use of a by-pass condenser across a biasing resistor inserted in the plate return, and because of a simplification of the a-c equivalent a-c circuit. With regard to biasing, since the principal use of a by-pass condenser is to by-pass the fundamental signal frequency around the biasing resistor, the condenser may be omitted from the Class A push-pull amplifier circuit. Application to the equivalent a-c circuit will be discussed in Section 3-9.

Audio Amplifier", Proc. IRE 21, 591 (1933)

3-9. Push-Pull Equivalent Circuits

The alternating current equivalent anode circuit has been drawn only for linear operation, and its use is justified only for negligible harmonic distortion. Therefore, if the external distortion is negligible, as for Class A operation, then since (Equation 3-52) the fundamental does not exist in the connection K to a, Figure 3-19, this connection may be omitted from the equivalent a-c circuit of Figure 3-21. A further simplification in the circuit is possible by using the relation between E_{g1} and E_{g2}. Since $E_{g1} = E_g$, and $E_{g2} = E_g \angle 180°$, the two equivalent generators will be in series with voltages adding in the plate circuit and may be represented as one generator as shown in Figure 3-22. The load resistance R_L when referred to the primary circuit becomes

$$\left(\frac{2N_1}{N_2}\right)^2 R_L, \text{ where } \left(\frac{2N_1}{N_2}\right) \text{ is the primary-to-secondary turns ratio, since the plate}$$

currents have been lumped as $2 r_p$.

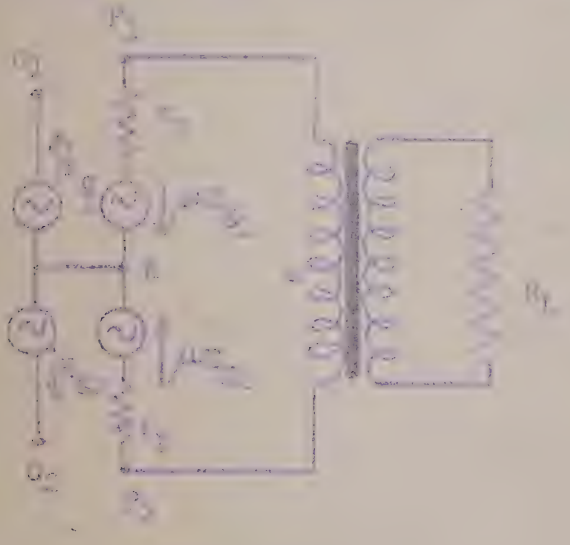


Figure 3-21.

Equivalent a-c circuit of a push-pull, Class A amplifier.

Figure 3-22

Simplified equivalent a-c plate circuit of Figure 3-21

A note on the composite current is appropriate at this point. According to (3-52), the resultant current in the absence of harmonic distortion is given by

$$i_p = B_1 \cos\omega t \qquad (3\text{-}52)$$

where $i_p = B_1 \cos\omega t$ is the fundamental a-c component

yields the composite current. For effective values and according to

$$\left.\begin{aligned} \\ \\ \end{aligned}\right\} \qquad (3\text{-}53)$$

where I_p is the effective value of the a-c component of plate current for a single tube of the push-pull pair.

Now the power delivered to R_L may be expressed in two ways, as follows:

(a) According to Equation (3-53), for an ideal output transformer (unity coefficient of coupling) the power output is

$$P_o = I_2^2 R_L = \left(\frac{N_1}{N_2} I'_p\right)^2 R_L \qquad (3\text{-}54)$$

where I_2 is the rms value of the current i_2, and I'_p the rms value of composite current i'_p.

(b) According to the equivalent circuit, Figure 3-22

$$P_o = \left[\frac{2\mu E_g}{2 r_p + \left(\frac{2 N_1}{N_2}\right)^2 R_L}\right]^2 4\left(\frac{N_1}{N_2}\right)^2 R_L$$

or

$$P_o = \left[\frac{\mu E_g}{r_p/2 + \left(\frac{N_1}{N_2}\right)^2 R_L}\right]^2 \left(\frac{N_2}{N_1}\right)^2 R_L \qquad (3\text{-}55)$$

For an ideal output transformer, both (3-54) and (3-55) represent the power output. Therefore equating the two expressions for P_o, the composite

so that the composite equivalent a-c circuit involves an equivalent generator of voltage μE_g and internal resistance $r_{p/2}$ connected to a resistance $\left(\frac{N_1}{N_2}\right)^2 R_L$. Equation (3-56) shows that this load is in parallel so far as the composite current is concerned. The resistance $\left(\frac{N_1}{N_2}\right)^2$ in the primary as seen by the generator. The equivalent circuit shown in Figure 3-23 is the desired one, since it gives the composite current, and the correct value of the output current I_2.

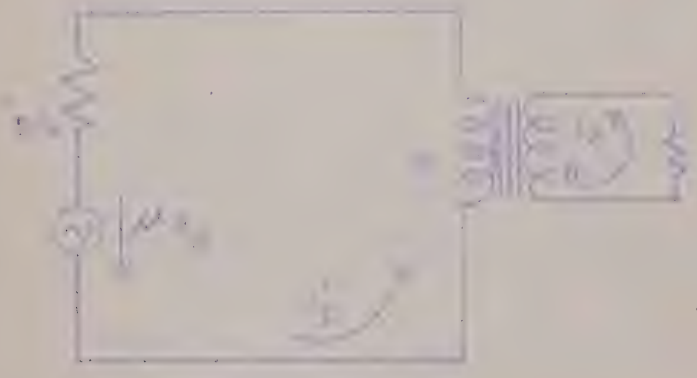


Figure 3-23.
Composite equivalent circuit.

3-10. Push-Pull Amplifier Input Circuits.

If R-C coupling is used, special means must be employed to provide the [illegible]. Such a method frequently used is shown in Figure 3-25 in which

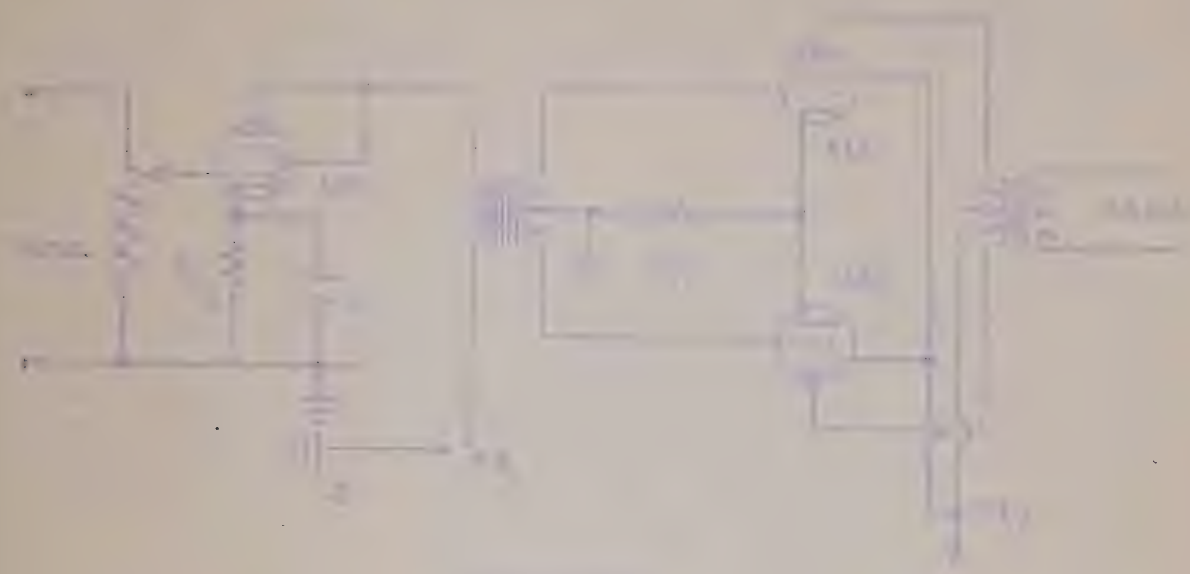


Figure 3-24.

Two-stage amplifier using transformer coupling

an amplifier tube and a phase inverter tube together provide the first stage of amplification and the input to the push-pull stage. The output voltage of the first stage amplifier tube is the voltage across R_1 and R_2 in parallel; the output voltage of the phase inverter appears across R_3

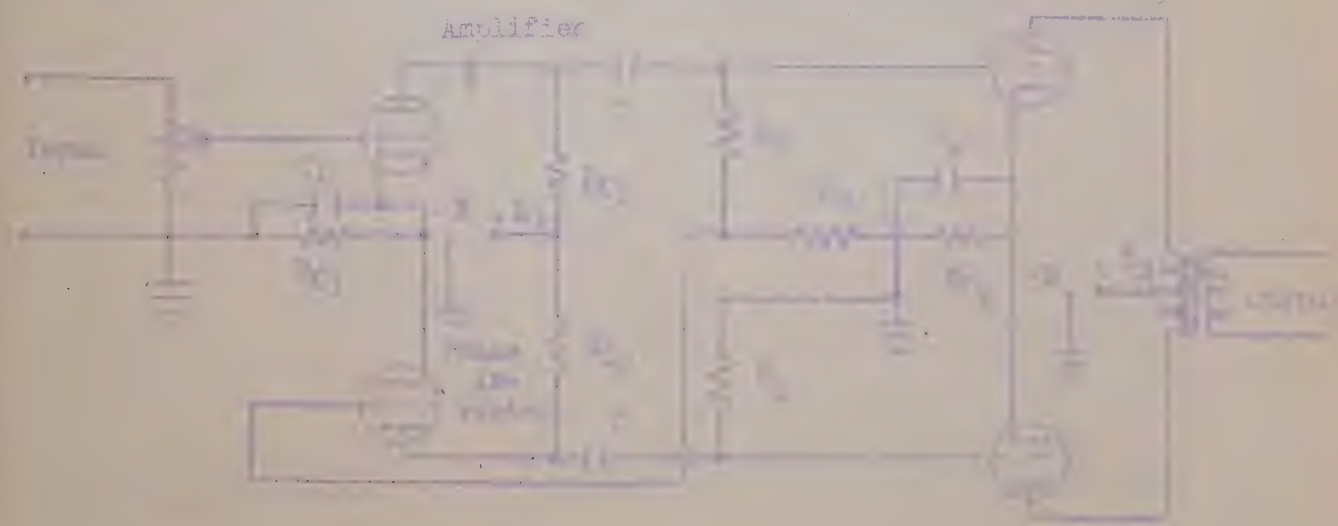


Figure 3-25.

Two-stage R-C coupled amplifier using a phase inverter

and is the input voltage to the lower tube of the push-pull stage. If the resistors R_1, R_2, and R_3 are properly chosen, the voltage across

with the voltage across R_2. Both tubes of the first stage may conveniently be built in the same glass envelope and use a common cathode. Examples of this type of dual purpose tube are the twin triodes 6N7 and 12SC7.

3-11. Composite Characteristics.

Because of the transformer primary which connects the plates of two tubes operating push-pull, an increase in the voltage at the plate of one tube will be accompanied by an equal decrease in the plate voltage of the other tube, just as for the grid-to-cathode voltages at the input. These relations may be stated as follows: at any instant, with a common bias value E_c, and an instantaneous grid voltage e_g, referred to ground, the instantaneous total grid and plate voltages for tubes 1 and 2 are given by

$$\begin{aligned} e_{c_1} &= E_c - e_g \\ e_{c_2} &= E_c + e_g \\ e_{b_1} &= E_b - e_p \\ e_{b_2} &= E_b - e_p \end{aligned} \tag{3-57}$$

corresponding to any value of the instantaneous grid excitation voltage e_g if the plate characteristics of the tube are available. For example, if a value of e_g is specified for a push-pull amplifier operating with a given grid bias E_c and plate voltage E_b, the individual plate characteristics involved are those corresponding to the values of e_{c_1} and e_{c_2}, Equation (3-57). The composite current is $i_p' = i_{b_1} - i_{b_2}$. The e_{c_1} characteristic relates corresponding values of i_{b_1} and e_{b_1}; the e_{c_2} characteristic relates corresponding values of i_{b_2} and e_{b_2}; the composite current is the difference of the ordinates of the two characteristics. If two identical graph sheets of the plate characteristics are arranged as shown in Figure 3-28 with the operating points Q_1 and Q_2 in vertical alignment, the difference of the e_{c_1} and e_{c_2} characteristics corresponding to a given grid excitation voltage e_g may easily be examined graphically with the aid of dividers. For example, in Figure 3-28, $E_c = -40$ volts;

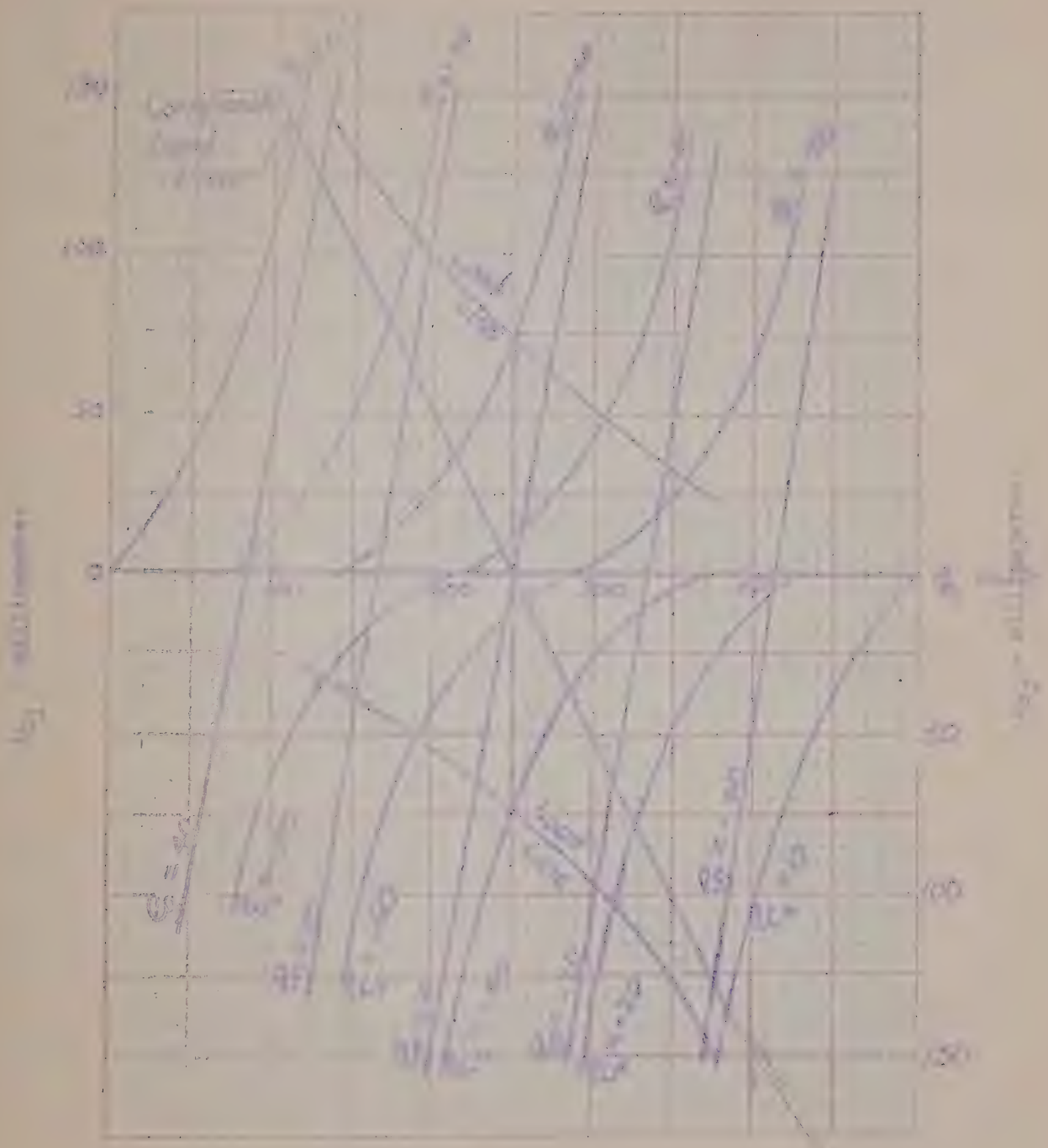


Figure 3-26

Individual and composite static characteristics and load lines for type 2A3 tubes in push-pull Class A operation

if $e_g = +40$ volts, $e_{c1} = E_c + e_g = 0$, $e_{c2} = E_c - e_g = -80$ volts.

ordinates of the $e_{c_2} = -80$ characteristic of tube 2. The resulting or

the composite current to the plate voltages of the tubes. For triodes operating Class A, the push-pull composite characteristics are found to be uniformly spaced, nearly parallel straight lines.

The method of obtaining points for drawing the composite characteristic is shown more clearly in Figure 3-27. For $e_g = 10$ volts, $e_{c1} =$

tained by subtracting the distance QK_2 from QK_1; also, for point B,

$$MB = ML_2 - ML_1 \quad .$$

If $e_g = 0$, the two $e_c = -40$ characteristics are combined, and the resulting composite characteristic passes through the composite operating point Q.

It should be noted that point B of Figure 3-27 may be obtained without the use of the second or inverted sheet of plate characteristics

used. For example, if $OR = QM$, then $RP = ML_2$, and

$$MB = RP - ML_1 \quad .$$

This corresponds to the third and fourth equations of (3-57). Referring again to Figure 3-27, the voltage E_b corresponds to distance OQ or to OQ_1, distance OM corresponds to a negative numerical value of e_{b1}, for which $e_{b1} = OM$ and $e_{b2} = O'M$.

Referring again to the push-pull amplifier circuit of Figure 3-19, the voltage across N_1 turns of the transformer primary is

$$v = i'_p \left(\frac{N_1}{N_2}\right)^2 R_L \quad . \qquad (3\text{-}58)$$

If i_p, the composite plate current is increasing in tube 1, e_b is a voltage drop and

$$e_{b1} = E_b -$$

$$e_{b2} = E_b +$$

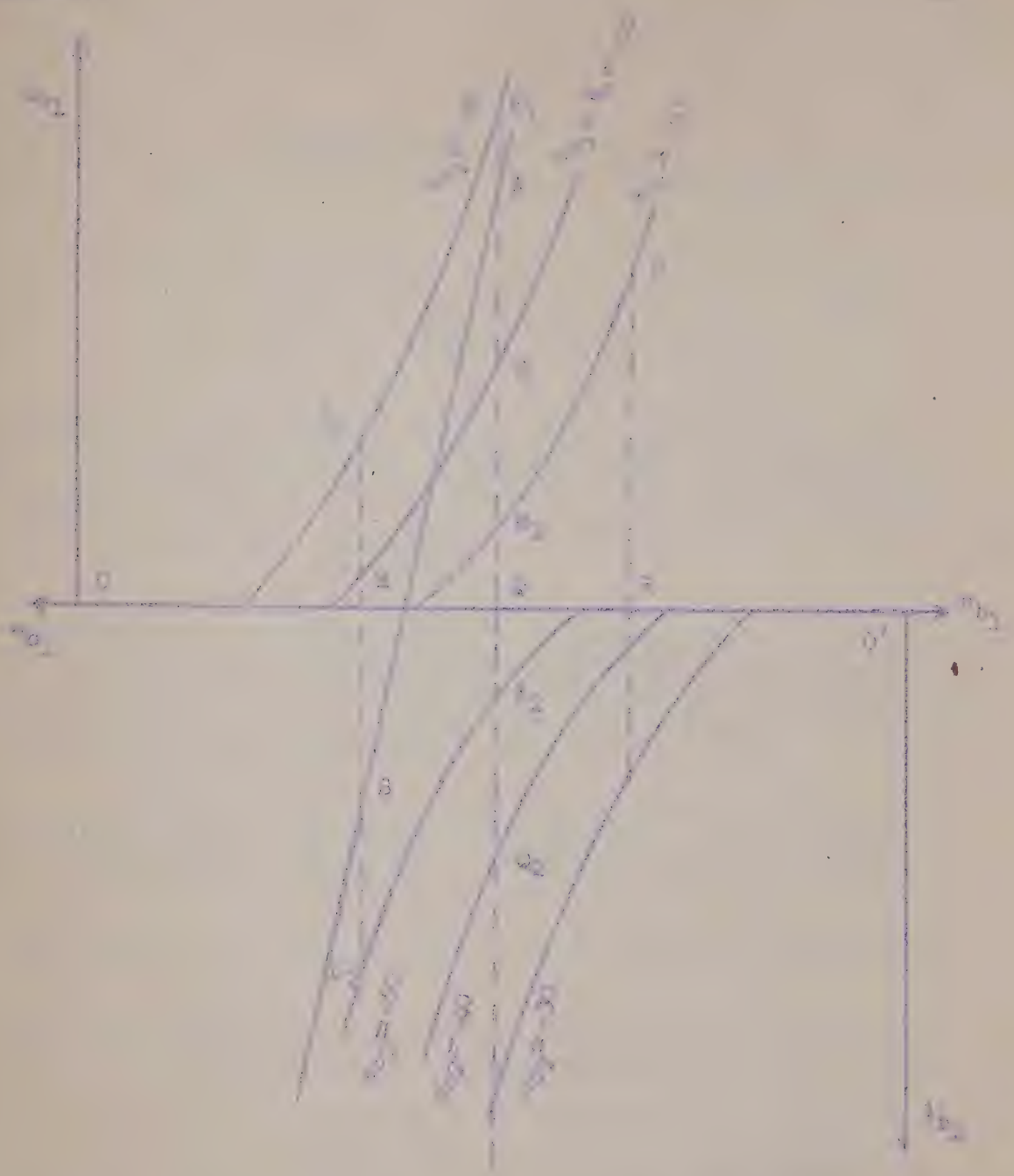


Figure 3-27.

Method of determining the composite static characteristic for tubes connected in push-pull.

These relations correspond exactly to the third and fourth equations of (3-57) where

$$e_{b_1} = E_b - i_p' \left(\frac{N_1}{N_2}\right)^2 R_L \qquad (3\text{-}59)$$

is the equation of the composite load line referred to the grid-cathode coordinates e_{b_1} and i_{b_1} (or i_p'). The slope of the composite load line is then

$$\frac{d\, i_p}{d\, e_{b_1}} = -\frac{1}{\left(\frac{N_1}{N_2}\right)^2 R_L} \qquad (3\text{-}60)$$

The composite load line is drawn through the composite operating point 2, Figure 3-26, with the slope given by Equation (3-60). The intersections of the composite load line with the composite static characteristics may be transferred to the mutual diagram in the usual way to give the corresponding composite dynamic characteristic.

The individual tube-circuit load lines have been obtained from the composite load line and are shown on the diagram, Figure 3-26. Points for the individual load lines are located by projecting vertically from intersections of the composite load line with composite characteristics to corresponding points on the individual tube characteristics. It is evident that the individual load lines are of little practical value.

A numerical example will illustrate the agreement between values of the composite current obtained from the equivalent circuit of Figure 3-25 and from the graphical solution. Assume that the tubes of Figure 3-19 are 2A3's biased to operate at $E_c = -40$ volts, with $E_b = 250$ volts. Let $\left(\frac{2N_1}{N_2}\right)^2 R_L = 1000$ ohms, the plate-to-plate resistance, and the signal voltage e_g (grid-to-cathode) $= 20 \sin \omega t$. Then at a time (ωt) when $e_g = +20$ volts, the instantaneous total grid voltage of tube 1 is $e_{c_1} = -40 + 20 = -20$ volts; of tube 2, $e_{c_2} = -40 - 20 = -60$ volts. The composite characteristic for $e_g = +20$ has been drawn in Figure 3-26, and the composite load line has a slope corresponding to $\left(\frac{N_1}{N_2}\right)^2 R_L = 1000$ ohms.

From the intersection of load line and characteristic, $i_p' = 60$ ma. From the equivalent circuit, Figure 3-25, using $\mu = 4.2$, $r_p = 800$ ohms,

$$i_p = \frac{4.2\,(20)}{400 + 1000} = \frac{84}{1400} = 0.06 \text{ amp}$$

or 60 ma. The corresponding values of e_{b_1} and e_{b_2} are: e_{b_1} = [illegible]
and e_{b_1} = 250 + 0.6 (1000) = 190 using Equation (3-58); e_{b_2} = 310 (again)
and e_{b_2} = 250 + 60 = 310 using Equation (3-59).

Triodes, pentodes and beam power tubes are also operated push-pull in Class AB or Class B. The composite static characteristics for triodes operated Class AB are almost straight lines. The dynamic characteristics for typical Class B operation of triodes are shown in Figure 3-28.

The graphical method of determining the composite static characteristics and the load line applies also to push-pull tubes operated in Class AB or in Class B. Push-pull amplifiers using beam power tubes frequently are operated Class AB with low distortion. A further discussion of these matters will be given after power relations and efficiency in amplifiers have been examined.

Tube Type 6N7
R_L = 6000 ohms, plate-to-plate
e_{c_1} = -5 V.
e_{bb} = 300 V.
e_{g_2} = 20 V.
I_{p_0} = 47 ma (approx)

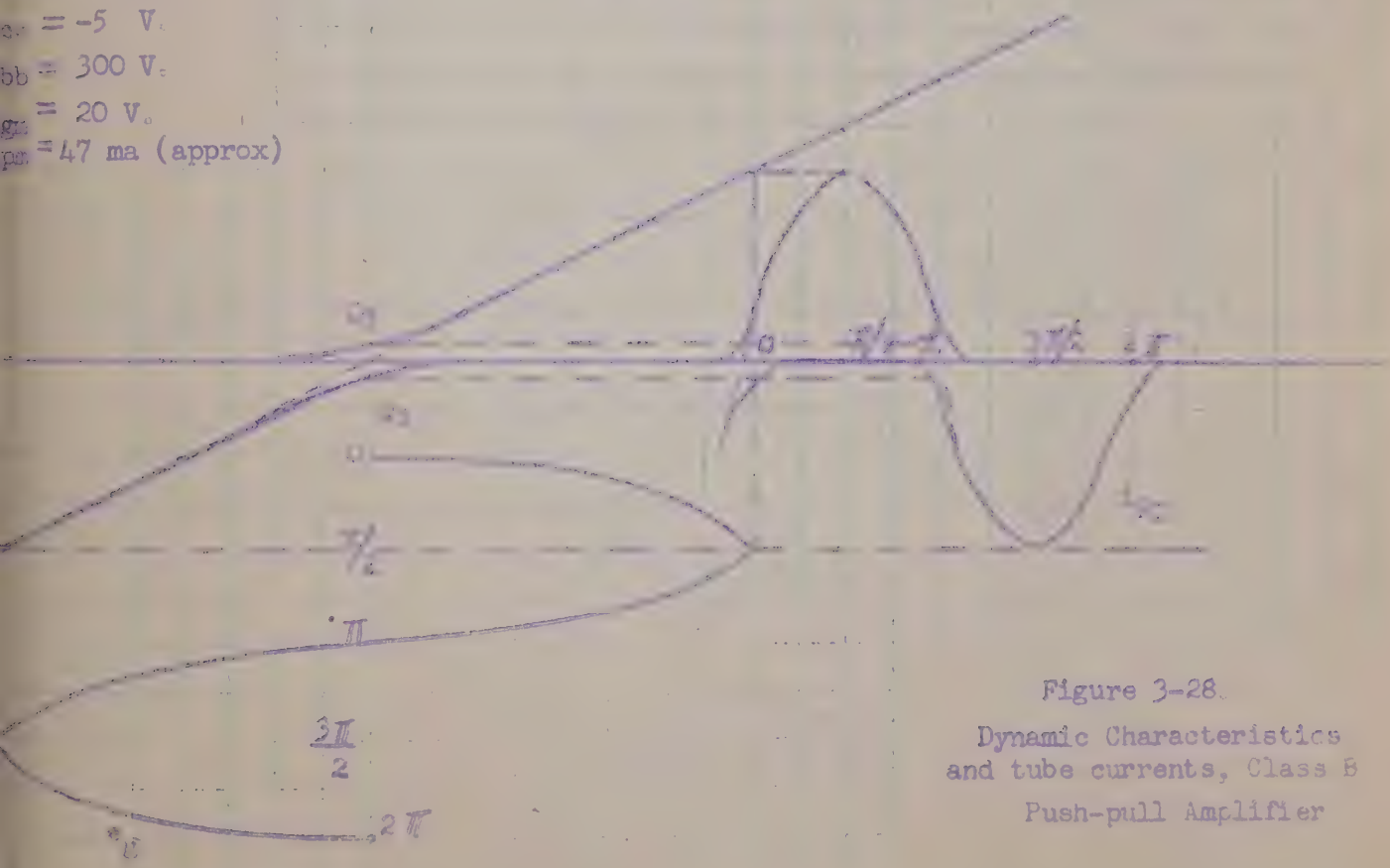


Figure 3-28.
Dynamic Characteristics and tube currents, Class B Push-pull Amplifier

3-12. The Use of Inverse Feedback in Voltage Amplifiers.

As used in relation to amplifier circuits the term "feedback" means the return to the input circuit of a portion of the signal voltage of any stage, of a portion of the output voltage of that stage or of a later stage of amplification. If the voltage fed back is less than 90° out of phase with the signal voltage, the resulting input voltage will be increased and the feedback is said to be regenerative, direct, or positive. If the voltage fed back is more than 90° out of phase with the signal voltage, the feedback is said to be degenerative, inverse, or negative and the amplification will be decreased. If the feedback is sufficiently direct the amplifier gain may increase until the amplifier becomes an oscillator. If the feedback is inverse, however, a number of effects occur in the amplifier circuit, as follows:

(a) the gain is reduced;

(b) the harmonic, frequency, and phase distortions are reduced;

(c) the stability of the amplifier is improved;

(d) the effective output impedance of the amplifier may be controlled.

A short discussion of the effect of inverse feedback on gain, distortion and stability will be given by analyzing simple examples. In Figure 3-61, a triode amplifier is shown with a connection providing both inverse feedback and cathode bias. The input signal voltage, E_g, is in series with the voltage across resistor R_f. The voltage fed back (current times resistance) as a fraction of the output voltage is given by

$$\frac{I_p R_f}{I_p R_L} = \frac{V_f}{V_o} \qquad (3\text{-}61)$$

and is dependent upon the relative magnitudes of R_f and R_L. The voltage rise from the cathode to the grid is

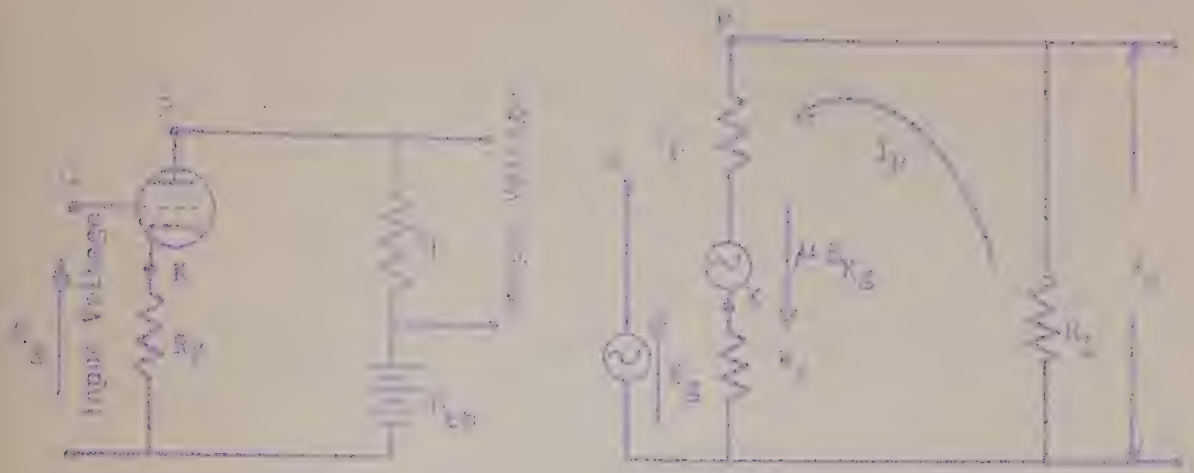
$$E_{gc} = E_g - I_p R_f$$

The plate current is then obtained from

$$I_p = \frac{\mu (E_g - I_p R_f)}{r_p + (\mu + 1) R_f + R_L}$$

which is

$$I_p = \frac{\mu E_g}{r_p + (\mu + 1) R_f + R_L} \qquad (3\text{-}62)$$



(a) Actual Circuit (b) Equivalent a-c Circuit

Figure 3-29.
Inverse feedback single stage amplifier.

It should be noted in Equation (3-62) that one effect of this type of feedback is to increase the effective plate resistance of the amplifier. The output voltage drop V_o, is

$$V_o = I_p R_L$$

The gain, with feedback, is

$$A_f = \frac{V_o}{-E_g} = - \frac{\mu R_L}{r_p + (\mu + 1) R_f + R_L} \qquad (3\text{-}63)$$

If the grid (through voltage source E_g) were connected directly to K (suitable fixed bias being provided) there would be no feedback, and the gain would be

$$A = \frac{I_p R_L}{-E_{Kg}} = - \frac{\mu R_L}{r_p + R_f + R_L} \qquad (3\text{-}64)$$

The relation between A_f and A may now be obtained by dividing numerator and denominator of the right side of Equation (3-63) by $r_p + R_f + R_L$. The result is

$$A_f = \frac{- \dfrac{\mu R_L}{r_p + R_f + R_L}}{1 + \dfrac{\mu R_f}{r_p + R_f + R_L}}$$

The quantity $\frac{\mu R_L}{r_p + R_f + R_L}$ may be written as $\frac{\mu R_L R_f}{(r_p + R_f + R_L) R_f}$

$= A \frac{R_f}{R_L}$, using Equation (3-64), so that, finally,

$$A_f = \frac{A}{1 + A \frac{R_f}{R_L}} \quad . \tag{3-65}$$

If, for example, the tube in Figure 3-29 has $r_p = 10{,}000$, $\mu = 20$, then if $R_L = 35{,}000$ ohms and $R_f = 5000$ ohms,

$$A = - \frac{20 (35{,}000)}{50{,}000} = -14 \; ;$$

$$\frac{R_f}{R_L} = \frac{5000}{35{,}000} = \frac{1}{7} \; ;$$

then

$$A_f = \frac{-14}{1 - (-14)\left(\frac{1}{7}\right)} = - \frac{14}{3} = -4.66$$

The percentage decrease in gain due to feedback of this example is then $\frac{9.34}{14} \times 100 = 66.6\%$. Such a decrease is not desirable, but a resulting improvement in freedom from distortion and in stability serves to compensate for the reduction in gain. In actual practice it may be desirable to provide for more gain than may be required and then discard the unnecessary gain by the use of inverse feedback to utilize the desirable features of the latter.[3]

Before discussing the reduction of distortion by the use of inverse feedback, one additional statement concerning Equation (3-65) should be made. If a fraction β, called the feedback fraction, is defined as the ratio of the voltage fed back to the output voltage, then for the circuit of Figure 3-29 $\beta = R_f / R_L$ and Equation (3-65) becomes

$$A_f = \frac{A}{1 - \beta A} \tag{3-66}$$

Equation (3-66) is a general relationship, applicable to any feedback arrangement, as may be shown by considering Figure 3-30. The phase relation of the output voltage drop, E_2, to the input voltage rise, E_g, depends upon the number of amplifier stages and the phase shift in each stage. However, if the feedback is to be inverse or degative, the gain must be expressed as

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[3] Black, H. S., "Stabilized Feedback Amplifiers", Electrical Engineering 53, 114 (1934)

that the actual amplifier input voltage rise, E_i, must be less than E_g, requiring that the voltage fed back be out of phase with E_g by more than 90°. It is the function of the feedback network to provide the proper magnitude and phase of the voltage fed back. The feedback network is in effect a potential divider to provide a fraction of the output voltage in series [illegible] voltage law, and the double subscript notation,

$$E_{12} = V_{GK} + V_{34} ;$$

or $E_g = E_i + \beta V_o$. Thus,

$$E_i = E_g - \beta V_o . \qquad (3\text{-}67)$$

If the gain of the amplifier without feedback is A, then

$$V_o = -A E_i = -A (E_g - \beta V_o)$$

or

$$V_o = \frac{-A E_g}{1 - \beta A} .$$

The gain of the amplifier with feedback is, by definition,

$$A_f = \frac{V_o}{-E_g} = \frac{A}{1 - \beta A} \qquad (3\text{-}68)$$

which is identical with Equation (3-6a) and applies to any feedback amplifier. A more rigorous definition of inverse feedback may now be given in accordance with Equation (3-68). If $|1 - \beta A| > 1$, the gain decreases and the feedback is inverse. If $|1 - \beta A| < 1$, the feedback is direct or regenerative and the gain increases. If the feedback is regenerative and large enough to replace the signal voltage, E_g, the latter may be removed and the amplifier becomes an oscillator. It must be remembered in the use of Equation (3-68) that, in general, A, β, and A_f are complex or vector quantities.

To consider the effect of inverse feedback upon harmonic distortion, again refer to Figure 3-30. In general,

$$V_o = -A E_i = -A (E_g - \beta V_o).$$

With, for example, a second harmonic distortion voltage of V_d volts without feedback, let the total output voltage be V_o, the total input voltage

Feedback Network

Figure 3-30.

Block diagram showing inverse feedback in an amplifier.

If voltage is introduced by the amplifier itself,

$$V_o = - A E_i + V_d$$

$$= - A (E_g - \beta V_o) + V_d \qquad (3\text{-}69)$$

If Equation (3-69) is solved for V_o, the result is

$$V_o = - \frac{A E_g}{1 - A\beta} + \frac{V_d}{1 - A\beta}$$

$$= - A_f E_g + \frac{V_d}{1 - A\beta} \qquad (3\text{-}70)$$

Equation (3-70) shows that the distortion voltage V_d will be reduced in a result of inverse feedback in the same proportion as is the gain. For example, refer to the circuit previously used where $A = -44$, $A_f = -4.84$, $\beta = 1/11$, $1 - A\beta = 5$. If the voltage E_g is adjusted so that the fundamental voltage with feedback is the same as without feedback, then the value of signal required for each case can be determined from the relation

$$V_o = - A E_{g1} = - A_f E_{g2} \qquad (3\text{-}71)$$

Equation (3-70) will be

$$V_o = -A_f E_{g_2} + \frac{V_d}{1 - A\beta}$$

where $(-A_f E_{g_2}) = 4.66\ (1.5) = 7$ volts, fundamental, and $\frac{V_d}{1 - A\beta} = \frac{0.70}{3} =$ 0.233 volt, second harmonic.* The percent second harmonic distortion has thus been reduced by feedback from 10% to $\frac{23.3}{7} = 3.33\%$. A similar reduction of noise or hum voltages results from the use of inverse feedback.

In considering the effect of inverse feedback upon frequency and phase distortion, the easiest point of view may be based upon a rearrangement of equation (3-66). If both numerator and denominator of Equation (3-66) are divided by $(-\beta A)$ the result is

$$A_f = \frac{-\frac{1}{\beta}}{1 - \frac{1}{\beta A}} \qquad (3\text{-}72)$$

It is generally true that the feedback fraction, β, can be made independent of frequency over a wide range by using pure resistance circuit elements. If then $|\beta A|$ is large so that $\left|\frac{1}{\beta A}\right| \ll 1$, the gain approaches $\left(-\frac{1}{\beta}\right)$ which does not depend upon frequency so that both frequency and phase distortions are eliminated. Although the large value of βA results involves a very great sacrifice in gain, the increase in effective bandwidth and the possibility of eliminating the dependence of amplifier properties upon the individual tube characteristics frequently justify the use of a large value of βA.

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* The total output voltage is not 7.233 volts. If the rms fundamental voltage is equal to 7, and the rms second harmonic voltage is equal to 0.233, the rms output voltage is $\sqrt{7^2 + (0.233)^2}$.

Although it is difficult except for specific circuits to show the reduction of frequency distortion by inverse feedback, experimental gain-frequency curves show it very clearly. Terman[10] has published experimental characteristics of an amplifier for which the gain-frequency curve without feedback was flat from about 400 to 2000 cycles per second. With feedback, the gain-frequency curve of the same amplifier operating with the same plate voltage was flat through a frequency range extending from 30 to 30,000 cycles per second.

The reduction of phase distortion as a result of the use of inverse feedback is shown by the vector diagram of Figure 3-31, for a typical R-C coupled amplifier. In this amplifier a fraction of the output voltage has been fed back in series with the input signal voltage, E_g. The ac-

reactance of the coupling condenser, C_1, is 50,000 ohms. If the output

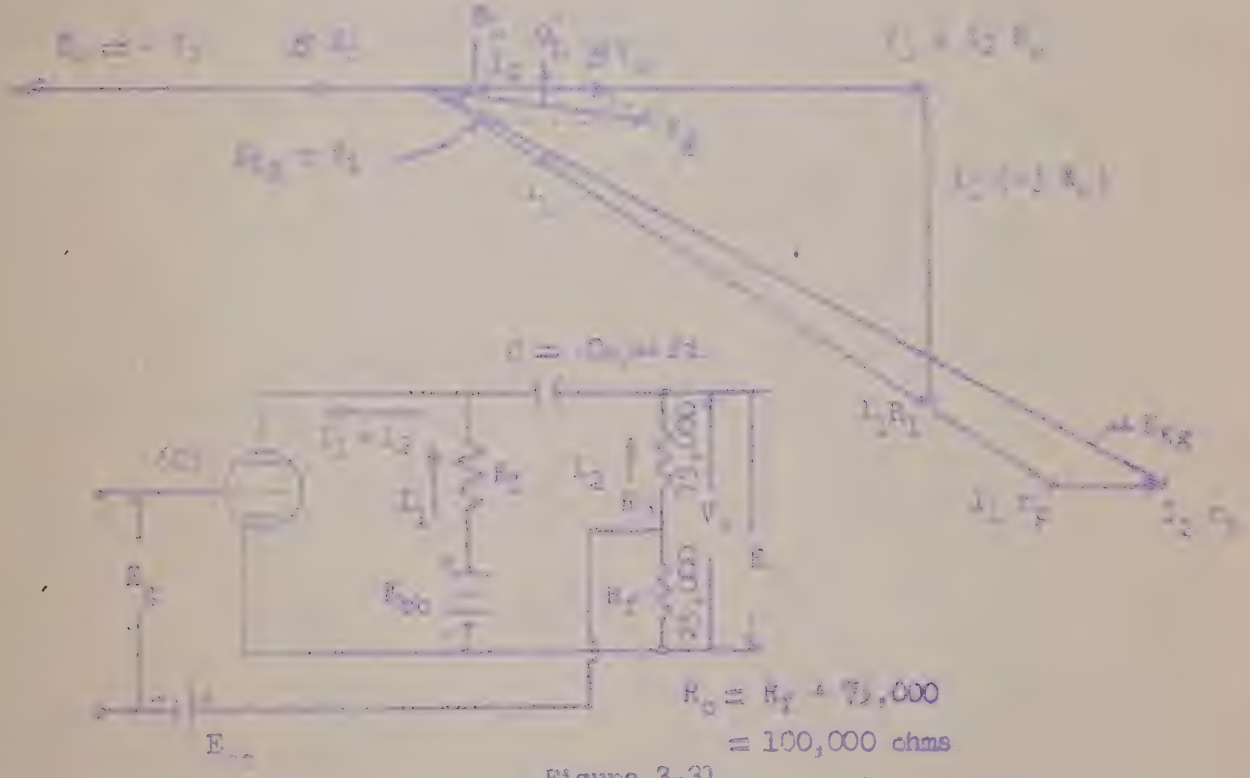

Figure 3-31.

Feedback amplifier circuit and vector diagram.

voltage were maintained constant, and there were no feedback, the required rise of input voltage E_g would be equal to and coincide with the voltage $E_{fg} = E_1$. The phase shift angle would then be θ_L, and the phase shift

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10 Terman, F. E., "Feedback Amplifiers", Electronics, 10, p. 12, Jan. 1937

sary to increase E_g greatly in order to obtain the same output voltage with feedback as without, as the vector diagram shows.

In connection with Figure 3-31, the following problem solution will illustrate the principles involved in the use of inverse feedback. Let it be required to determine the gain of the amplifier of Figure 3-31 with and without feedback, using

(a) ordinary circuit theory,

(b) the general feedback theory.

In addition, the complete vector diagram of the amplifier is required. Let the frequency be 79.6 cycles per second ($\omega = 500$).

Solution: (The equivalent circuit is shown in Figure 3-32)

(a) For convenience, arbitrarily choose $I_2 = 1$ ma. and the output voltage drop as the reference vector. Then

$$V_o = I_2 R_o = 10^{-3} (100{,}000) = 100 \angle 0^\circ \text{ volts}$$

$$V_c = I_2 (-j X_c) = 10^{-3} (-j\, 50{,}000) = -j\, 50 \text{ volts.}$$

The sum of V_o and V_c is equal to the voltage across R_1. Then

$$I_1 = \frac{100 - j\, 50}{50{,}000} = (2 - j\, 1) \cdot 10^{-3} \text{ amp.}$$

$$= 2.24 \angle -26.6 \text{ ma.}$$

$$I_1 R_1 = I_2 (R_o - j X_c) = 112 \angle -26.6^\circ \text{ volts.}$$

By Kirchhoff's voltage law,

$$\mu E_{Kg} = I_1 R_1 + (I_1 + I_2) r_p$$

$$= I_1 (R_1 + r_p) + I_2 r_p$$

$$= 2.24 \cdot 10^{-3} \angle -26.6 \; (60{,}000) + 1 \cdot 10^{-3} (10{,}000)$$

$$= 134.4 \angle -26.6 + 10 = 120 - j\, 60.2 + 10 = 130 - j\, 60.2$$

$$= 143 \angle -24.8^\circ$$

Then

$$E_{Kg} = E_1 = \frac{143}{20} \angle -24.8$$

$$= 7.15 \angle -24.8 = 6.5 - j\,3.0 \text{ volts}$$

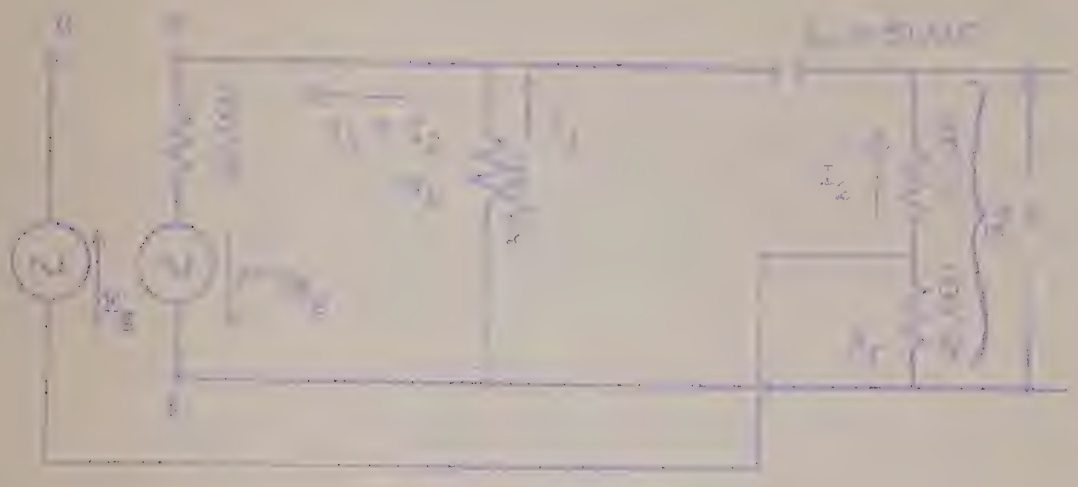

Figure 3-32

Equivalent circuit of Figure 3-31.

This is the actual reactance tube voltage. To find the applied input voltage, E_g, note that the voltage rise from K to G is

$$E_{Kg} = -I_g R_f + E_g$$

whence

$$E_g = E_{Kg} + I_g R_f$$

$$= 6.5 - j\,3.0 + 1 \times 10^{-3}\,(25{,}000)$$

$$= 31.5 - j\,3.0 = 31.8 \angle -5.46^\circ$$

These values have been plotted approximately to scale on the vector diagram of Figure 3-31. The vector gain is

$$A_f = \frac{V_o}{-E_g} = -\frac{100 \angle 0^\circ}{31.5 \angle -5.46}$$

$$= -3.17 \angle 5.46^\circ .$$

Without feedback,

$$A = \frac{V_o}{-E_{Kg}} = -\frac{100 \angle 0^\circ}{7.15 \angle 24.8} = -14 \angle 24.8$$

(b) Using the relation $A_f = \frac{A}{1 - A\beta}$;

$$= \frac{\quad}{1 + 14 \angle 24.8 (0.25 \angle 0)}$$

$$= \frac{\quad}{1 + 3.18 + j\,1.47} \qquad \frac{\quad}{4.42 \angle 19.3}$$

$$= 0.17 \angle -19.3^{\circ}$$

The gain-frequency curve for the amplifier of Figure 3-31 is flat within 1 db from 50 to 100,000 cycles per second, with phase shift angles varying from +4.7 to -46°. Without feedback, however, the gain-frequency curve is flat only in the range 400 to 13,800 cycles per second. The phase shift angle for the amplifier without feedback is (+ 36.4°) at 50 cycles and (-36°) at 100,000 cycles.

3-3). Inverse Feedback and Uniformity of Amplifier Operation.

It has already been mentioned in relation to Equation (3-72) that for $|\beta A|$ large and $\frac{1}{|\beta A|} \ll 1$, the amplifier gain becomes dependent only on the feedback circuit. For this condition it may thus be considered that the amplifier behavior will not change because of small changes in tube characteristics, or in power supply voltages. This is one of the most important advantages of inverse feedback because it provides assurance that the behavior of the amplifier will remain uniform over long periods of time. For vacuum tube voltmeter circuits employing inverse feedback amplifiers, it has meant the possibility of maintaining accurate instrument calibration unaffected by tube changes or replacements.

Even though βA is not large enough that $A_f \approx \frac{1}{\beta}$, large changes in A result only in small changes in A_f. For example, a change ΔA in A produces a change ΔA_f in A_f. Application of differential calculus to Equation (3-72) shows that the fractional change in A_f is

$$\frac{d A_f}{A_f} = \frac{\frac{d A}{A}}{(1 + A\beta)} \qquad (3\text{-}73)$$

If A changes by 10% as a result of change in tube characteristics, voltage variations, and so on, then $\frac{d A}{A} = 0.1$. For the circuit of Figure 3-31 in the mid-frequency range, $|A| = 15.4$, $\beta = \frac{1}{20}$, $1 + A\beta = 1 + 15.4 \times \frac{1}{20} = 1.85$. Then

$$\frac{d A_f}{A_f} = \frac{0.1}{1.85} = 0.054$$

or 5.4% change in gain resulting from a change of 10% in the basic amplifier itself.

3-14. Inverse Feedback and Stability

As already mentioned in the discussion of Equation (3-66), feedback is degenerative or inverse if $|1 - A\beta| > 1$, regenerative if $|1 - A\beta| < 1$. In the latter case the gain with feedback is increased with respect to the gain without feedback, so that there may be a tendency for the amplifier A to become unstable. It is important to examine the complex quantity $(1 - A\beta)$ throughout the frequency range of the amplifier to see if there are any frequencies for which $(1 - A\beta)$ has a numerical value less than unity, for in such a frequency range the feedback would be regenerative and the amplifier might behave as an oscillator. Since the complex gain, A, and corresponding angle θ_A, or θ_β are available from universal amplification curves or from gain equations of the type given in Sections 3-4 and 3-5 for R-C and transformer coupled amplifiers, it is an easy matter to plot $A\beta$ at once for either of these amplifiers if β is independent of frequency. The resulting graph will then show immediately the range of frequencies, if any, for which the feedback is regenerative. For example, referring to the R-C coupled amplifier of Figure 3-31, $\beta = 1/4$. In the mid-frequency range, $A_I = -45.4 \angle 180°$. The values of $\frac{|A_L|}{|A_I|}$ and $\frac{|A_H|}{|A_I|}$ and the corresponding phase shift angles are given in tabular form in Section 3-4, and graphically in Figure 3-12. From the tabulation on page 47 and the curves of Figure 3-12 for $f_L = 36.8$ and $f_H = 138{,}000$ for this amplifier (see page 48) the following values of $A\beta$ are computed:

f	$A\beta$	
3.68	0.383	$\angle 180 + 84.3$
7.36	0.755	$\angle 180 + 78.7$
18.4	1.72	$\angle 180 + 53.4$
36.8	2.72	$\angle 180 + 45$
73.6	3.44	$\angle 180 + 26.6$
184	3.77	$\angle 180 + 11.3$
368	3.83	$\angle 180 + 5.8$
Midfreq. range	3.85	$\angle 180°$
13,800	3.83	$\angle 180 - 5.8$
27,600	3.77	$\angle 180 - 11.3$
69,000	3.44	$\angle 180 - 26.6$
138,000	2.72	$\angle 180 - 45$
276,000	1.72	$\angle 180 - 63.4$
690,000	0.755	$\angle 180 - 78.7$
1,380,000	0.383	$\angle 180 - 84.3$

drawn so that the vector difference, $(1 - A\beta)$, may be shown graphically. A circle of unit radius is drawn around the point $(1 + j\,0)$. Since the vector $(1 - A\beta)$ is drawn from any point on the locus of the vector $A\beta$ to the end of the unit vector $1 \angle 0°$, the numerical value of $(1 - A\beta)$ can never be less than unity unless the polar plot of $A\beta$ crosses inside the circle. It may then be concluded that the feedback amplifier of Figure 3-31 has degenerative feedback at all frequencies and does not oscillate.

Figure 3-33

Polar plot of $A\beta$ for an R-C coupled amplifier.

If a fraction β of the output voltage of a single stage transformer coupled amplifier is fed back into the input, the polar plot of the sensitivity may be obtained from the gain equations or from the gain-frequency curves of Section 3-3, Figure 3-17. With a polar plot for a transformer coupled stage has been drawn in Figure 3-34, for $\frac{f}{f_c} = 1.71$ (a low frequency, the gain approaches zero and the phase shift angle approaches 180°. As the frequency increases, the phase shift angle becomes less and the gain greater, approaching zero phase shift angle and unity gain in the midfrequency range. As the high frequency region the phase shift and gain both increase because of reactance effects, and at still higher frequencies (at which point the curve of $1-\beta$ crosses over to the axis), as does the value of $A\beta$ within the unit circle the feedback is regenerative. If the value of $A\beta$ passes through the point (-1, j0), the gain becomes theoretically infinite and the amplifier becomes an oscillator. Nyquist[11] has developed a general criterion

Figure 3-34

Polar plot of $A\beta$ for a transformer-coupled amplifier.

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[11] Nyquist, H., "Regeneration Theory", B.S.T.J., 11, 126-147, (1932)

of feedback amplifier stability which may be stated as follows: if the polar plot of $A\beta$ includes the point $(1 + j0)$, the amplifier will oscillate, otherwise not. In order not to include the point $(1 + j0)$, the numerical value of $A\beta$ must be very much reduced where the phase shift is large, i.e., the gain must be controlled, i.e., reduced beyond danger of oscillation, over a frequency range many times greater than the useful range.

3-15. Output Impedance and Inverse Feedback.

It was mentioned in Section 3-12 that the effective terminal impedance at the output terminals of an amplifier may be controlled by the use of inverse feedback. In this connection, it is important to note that there are two distinct types of inverse feedback, and that the examples of inverse feedback circuits thus far discussed are examples of the two kinds of feedback. In the first example given, the circuit of Figure 3-29, the voltage fed back is proportional to the current flowing in the plate circuit of the amplifier. If the load impedance R_L (or more generally, Z_L) increases, the plate current I_p decreases, and the feedback voltage $I_p R_C$ decreases, but this means that the gain of the amplifier increases which will tend to increase I_p and so will tend to maintain I_p constant. This is known as current feedback, and has the effect of causing the amplifier to approach the behavior of a constant current source if the input voltage E_g remains constant. As shown in Equation (3-62), the effective plate resistance or internal impedance of the amplifier is increased by current feedback, and this is generally true. If the load resistance R_L of Figure 3-29 is replaced by a generator of voltage E, and the input voltage source E_g is replaced by a short circuit, output of the equivalent circuit for the impedance, Z_o, looking back into the amplifier at the terminals of the generator E shows that $Z_o = r_p + R_C (1 + \mu)$, which may be described as the output terminal impedance of the amplifier with the input short.

In Figure 3-31, the feedback voltage depends upon the output voltage and is a fraction of it. If the load impedance is assumed to be an impedance Z_L in parallel with R_o, then the output voltage and also the feedback voltage would tend to increase or decrease with Z_L, but an increasing feedback voltage results in decreased gain, so that the effect of this type of feedback is to maintain the output voltage constant. This is known as voltage feedback and amplifiers so equipped approach the behavior of a constant voltage source for constant input voltage.

The resistance R_o of Figure 3-31 and Figure 3-32 may be thought of as the feedback network. As such, it is merely a potentiometer connected across the load impedance, Z_L (Z_L is not shown in Figure 3-31 or Figure 3-32). If a generator of voltage E is connected across R_o in the

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[12] Bode, H. W., "Relations between Attenuation and Phase in Feedback-Amplifier Design", B S T J., 19, 421-454, (1940).

the generator E. The circuit has been redrawn in Figure 3-35. For convenience, the resistance of the coupling condenser has been neglected, and the input voltage source has been replaced by a short circuit. The impedance $Z_2 = R_2/\mu_2$. In the intermediate frequency range and without feedback (grid resistor R_1 only) the impedance Z_o' as seen at the terminals a-b is given by

$$Z_o' = \frac{\qquad\qquad}{\frac{1}{r_p} + \frac{1}{R_1} + \frac{1}{R_2}}$$

With feedback, however, for I_2 given

$$I_2 = \frac{E}{Z_1} = \frac{\qquad\qquad}{\beta E_g + \frac{E}{Z_o'}}$$

$$= \frac{Z_o'}{1 + \beta g_m Z_o'} \qquad (3\text{-}74)$$

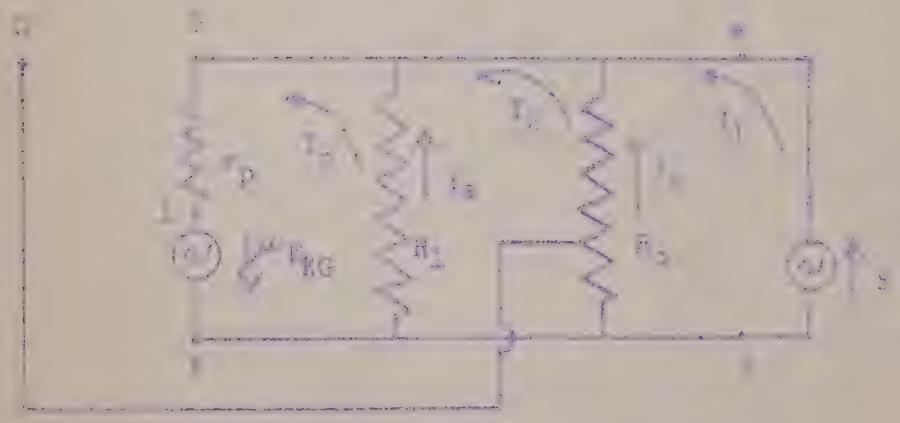
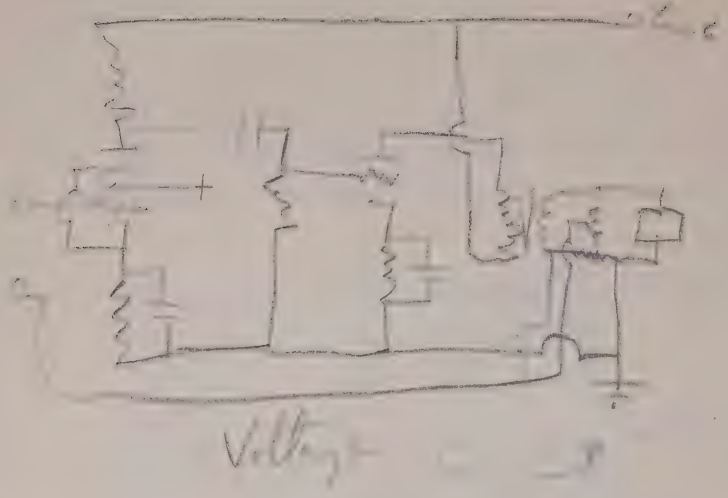
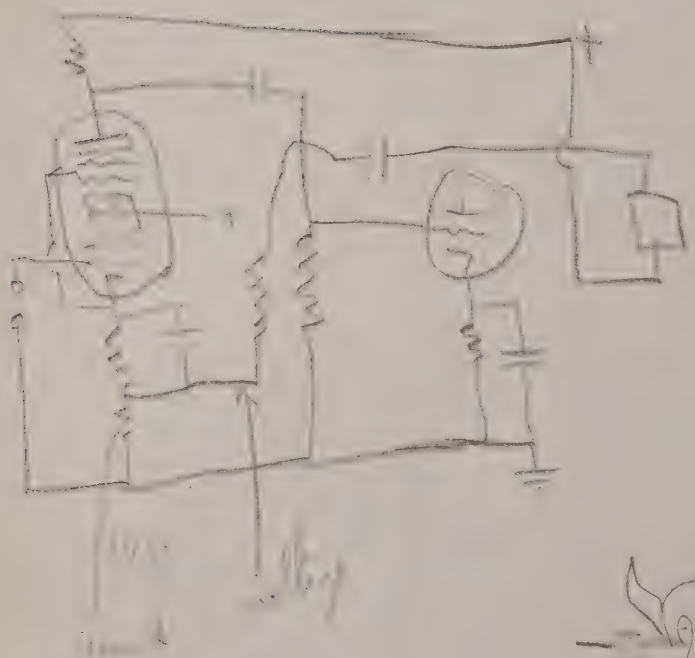


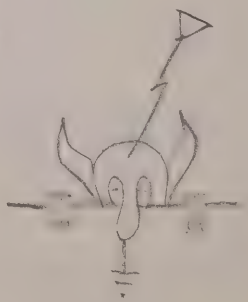
Figure 3-35.
Equivalent circuit of a voltage feedback amplifier in which a generator E replaces the load

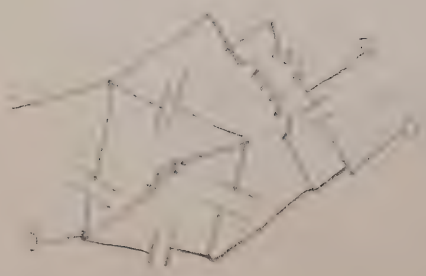
From Equation (3-10), $A = - g_m Z_o'$ so that

$$Z_o = \frac{Z_o'}{1 - \beta A} \qquad (3\text{-}75)$$









where $\beta = -\frac{R_f}{R_o}$ (It can be shown that the relation of Equation (3-25)

is generally true for voltage feedback. The effective terminal impedance of the amplifier is therefore reduced by inverse voltage feedback. By using combinations of current and voltage feedback, the amplifier may be made to present almost any desired impedance to the load.

3-16 Other Inverse Feedback Circuits.

Only two feedback circuits have thus far been discussed. These circuits are quite basic and illustrate the general principles and properties of inverse current and voltage feedback. However, because many of the beneficial results derived from the use of inverse feedback increase with the gain A, it is desirable to increase A rather than β, and to use not only high gain stages, but several stages of amplification. Triodes have been used in the feedback circuits already discussed, but tetrodes and pentodes are preferable because high intrinsic gain is desirable. The same principles apply to triodes, tetrodes, or pentodes. Many practical circuits have been described in the literature.[14,15] A few other examples of typical one and two-stage circuits will be given here.

A single stage circuit is shown in Figure 3-36 in which both inverse current and voltage feedback are employed. Resistances R_1 and R_2, and R_o constitute the feedback network. Condenser C_1 is a blocking condenser and should have negligible reactance in the frequency range of the amplifier.

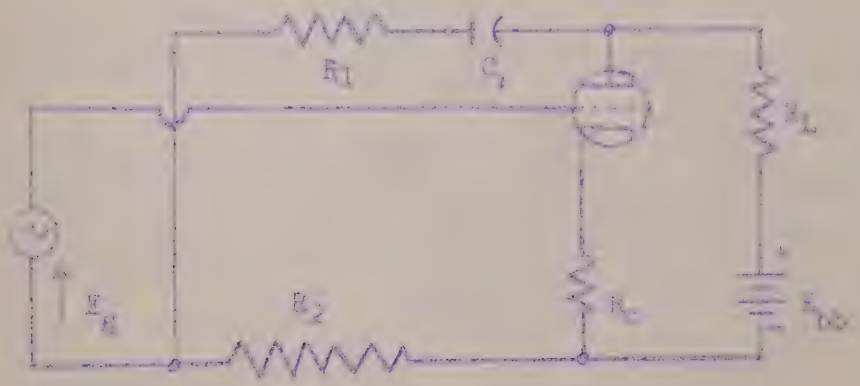


Figure 3-36.

Inverse feedback amplifier.

A feedback arrangement for the push-pull connection is shown in Figure 3-37, and the inverse voltage feedback network is seen to be identical with that of Figure 3-36. No current feedback is utilized because of the

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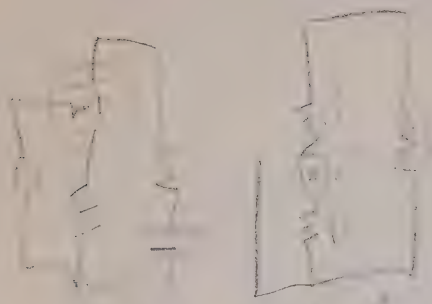
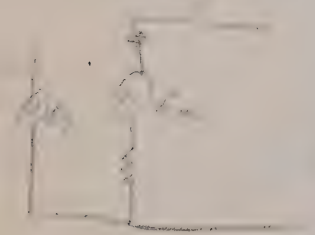
13 Mayer, HF, "Control of the Effective Internal Impedance of Amplifiers by Means of Feedback", Proc. IRE, 27, 213-217, (1939).

14 Terman, F.E., "Feedback Amplifier Design", Electronics, 10, 12-15, 48-50, (1937).

15 Peg, J.R., and Russell, J.B., "Practical Feedback Amplifiers", Electronics, 10, 12, April 1937.

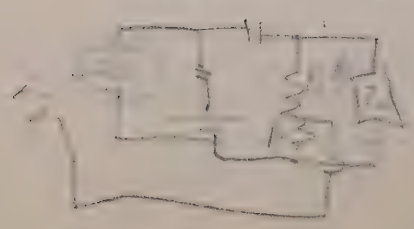


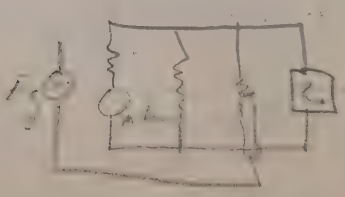


$$E = IR_p - \mu I R_g + IR_k$$

$$E = IR_p + \mu I R_g + IR_k$$

$$= I(R_p + \mu R_g + R_k)$$





by-pass condenser across the biasing resistor, in addition to the fact that the fundamental and odd harmonics are not present in that part of the circuit for the push-pull connection, as shown in Section 3-8. The blocking condenser, C_1, may be omitted if the connection through R_1 is made to the load side of the output transformer. As a result of the output voltage stabilizing property of [illegible], this tendency of the gain to fall off at low frequencies because of low transformer primary reactance is counteracted.

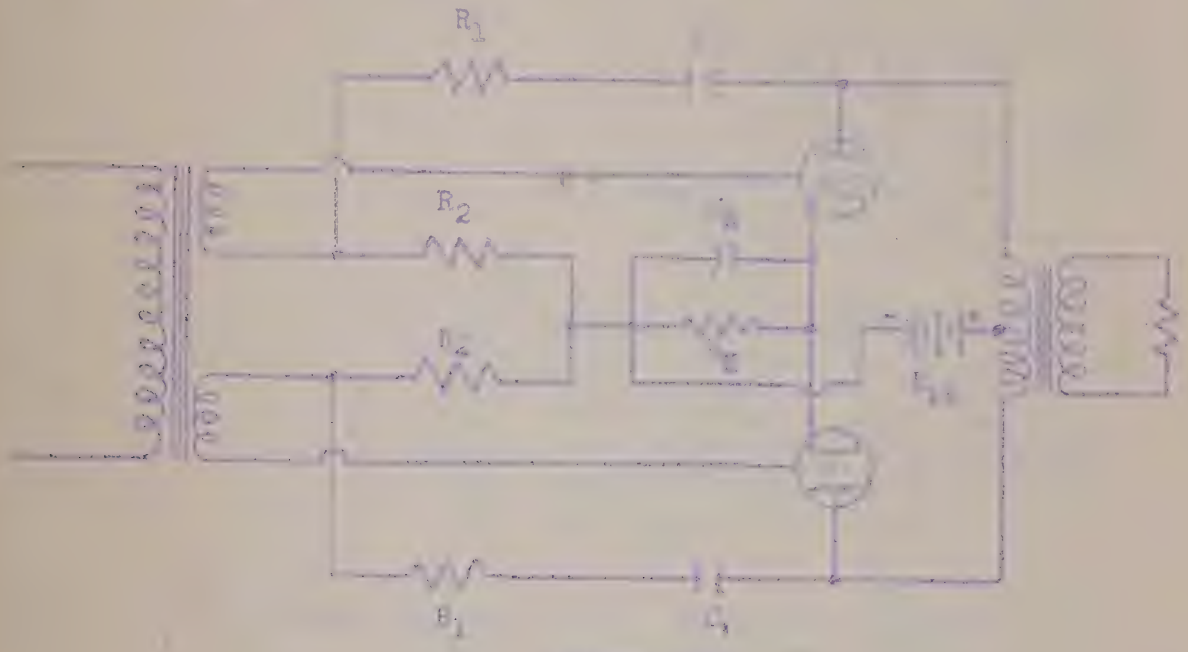
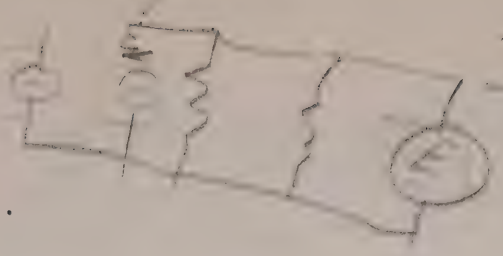


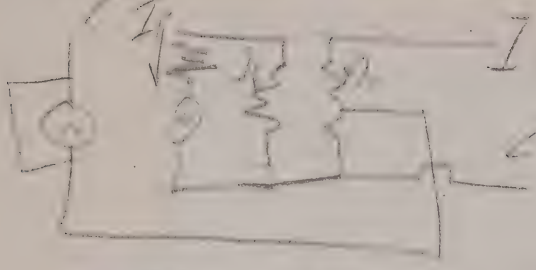
Figure 3-37

Voltage inverse feedback for a push-pull amplifier stage.

A two-stage feedback amplifier arrangement is shown in Figure 3-38. The first stage is R-C coupled to the second stage. R_1 is a dropping resistor to provide the proper screen grid voltage for the pentode. The biasing resistors R_{K1} and R_{K2} are well by-passed. The load impedance is R_{L2}, and C_2 is, as in Figure 3-37, a blocking condenser. Resistors R_2 and R_3 have the same voltage dividing function as in the circuit of Figure 3-37. Differences in the feedback connections of Figures 3-38 and 3-31 should be examined and explained. Feedback from the second to the first stage of transformer coupled amplifiers is derived according to principles identical with those illustrated in Figure 3-31.

It has been shown that the phase shift angles of individual R-C coupled amplifier stages approach 90° at low or at high frequencies. For three stage amplifiers, the overall phase shift angle may then approach





Tank to
main instant
valley gain

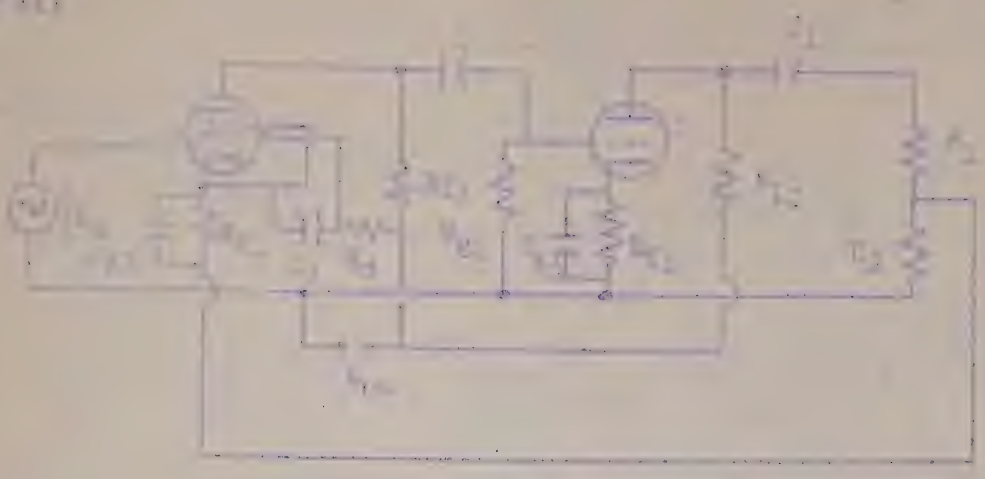


Figure 3-38.
Two-stage inverse voltage feedback.

270° at the frequency extremes. This means that the polar plot of $A\beta$ will cross into the unit circle, so that, to avoid oscillation, the gain must be greatly reduced at high or at low frequencies. For transformer coupled amplifiers, the phase-shift at high frequencies may approach 180° for a single stage (See Figure 3-2). Thus great care must be exercised in the design of multi-stage, transformer-coupled amplifiers.

3-17. The Cathode Follower Stage.

The amplifier circuit of Figure 3-39 will be recognized as a feedback amplifier in which the feedback and the output voltages are identical. This circuit has found many important applications which depend upon the utilization of one or more of the following list of properties:

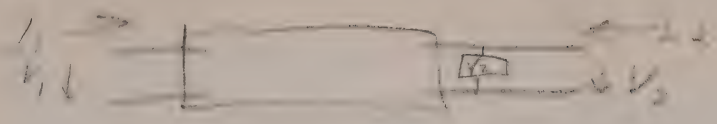
(1) Gain, A_f, approximately equal but always less than unity.

(2) Higher effective input impedance than that of an ordinary amplifier stage.

(3) Low output terminal impedance which may be made a pure resistance of value less than 200 ohms.

Analysis of the equivalent circuit of the cathode follower is straight forward and has been left to the problems. The equivalent circuit including the interelectrode capacitances has been drawn in Figure 3-40. Points 1 and P are at the same a-c potential (grounded plate), and as a result, the input signal voltage applied between terminals G and 1 is shunted by the grid-plate capacitance. The input and output voltage drops may both conveniently be referred to the ground terminal. The voltage gain then may be expressed as (see Figures 3-39 and 3-40).

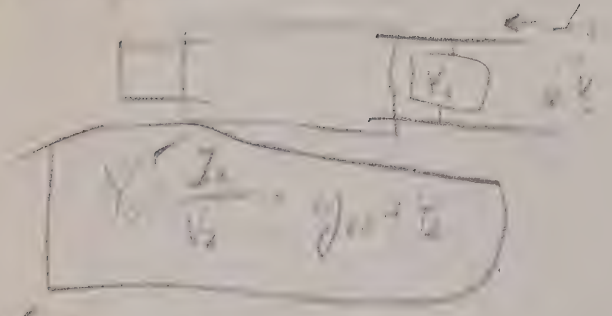
Voltage Feedback (Shunt)



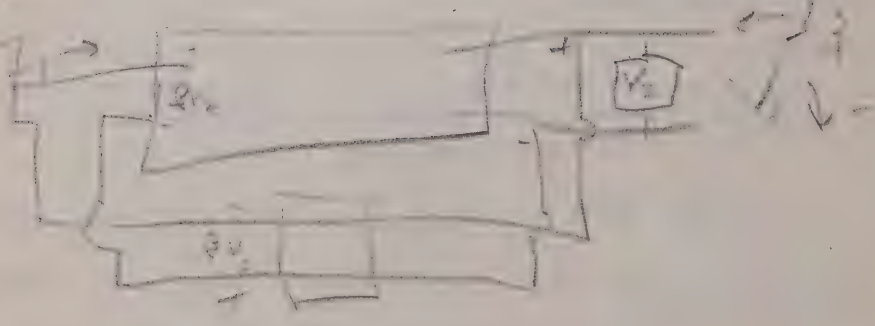
$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

method (2) (Z parameters)



$Y_0 = \frac{I_1}{V_2} = Y_{11} + Y_{12}$



$V_1 = \beta V_2$

$I_2 = \beta V_2 Y_{21} + V_2 (Y_{22} + Y_2)$

$Y_0 = \frac{I_1}{V_2} = \beta Y_{21} + Y_{22} + Y_2$

$Y_0 = Y_0' \left(1 + \frac{\beta Y_{21}}{Y_2 + Y_{22}}\right) = Y_0' (1 + \beta A)$

$Z_0 = \frac{Z_0'}{1 + \beta A}$

$= \frac{Y_{21}}{\beta Y_{21} + Y_{22} + Y_2}$

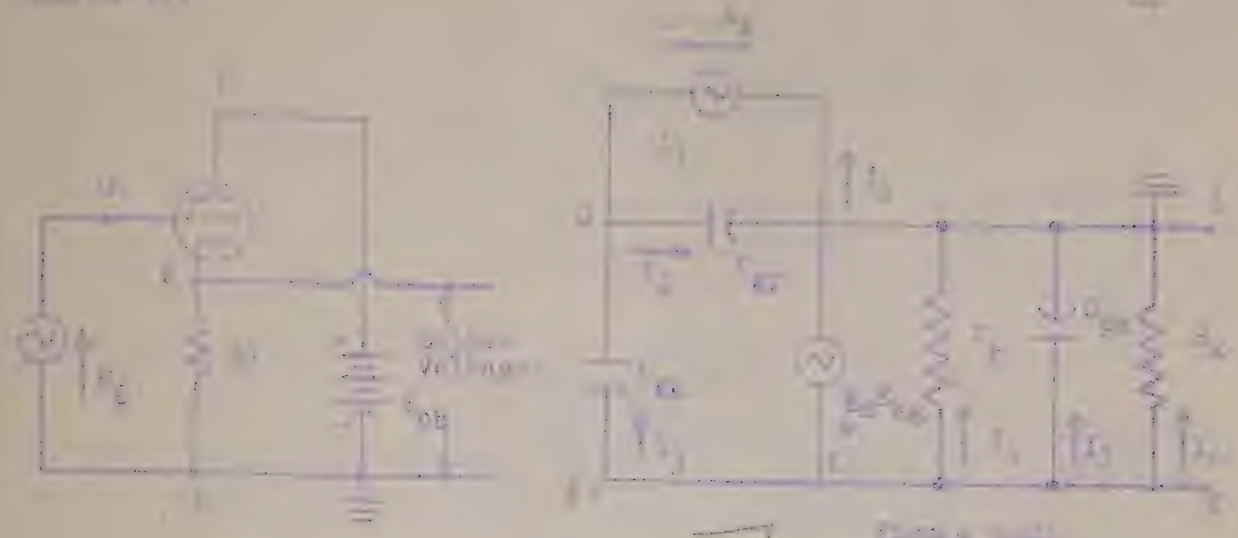
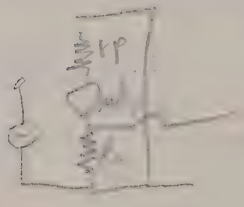


Figure 3-39.



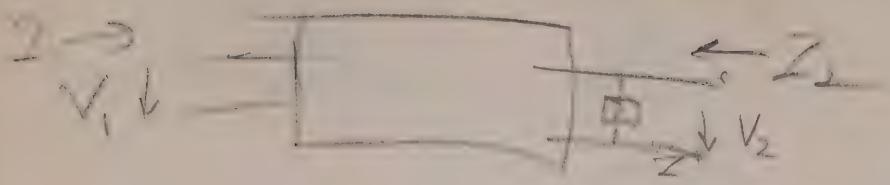
$$A_f = \frac{g_m + j\omega C_{gK}}{g_m + \frac{1}{r_p} + \frac{1}{R_K} + j\omega(C_{gK} + C_{gp})} \qquad (3\text{-}77)$$

which, at low enough frequency, reduces to

$$A_f = \frac{\mu R_K}{r_p + (\mu + 1) R_K} \qquad (3\text{-}78)$$

The input admittance of the cathode follower may be derived from Figure 3-40 by determining the ratio of I_1 to E_g. The result is

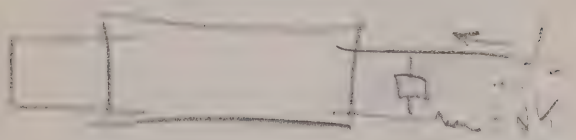
$$Y_{11} = \frac{I_1}{E_g} = j\omega \left[C_{gp} + (1 - A_f) C_{gK} \right] , \qquad (3\text{-}79)$$



$I_1 = V_1 y_{11} + (V_2 - I_2 Z) y_{12}$

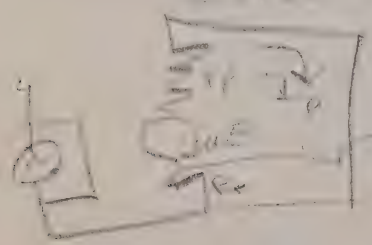
$I_2 = V_2 y_{21} + (V_2 - I_2 Z)(y_{22} + y_L)$

Without Feedback



$I_2' = (V_2 - I_2' Z)(Y_L + Y_{22})$

$$Z_0 = Z' - AZ$$



$$E = \left(I - I_{(p)}\right) R_K \qquad I_{(p)} = \frac{E + \mu E}{r_p}$$

$$I R_K = \frac{(E + \mu E)}{r_p}$$

$$E\left(1 + \frac{(1+\mu)R_K}{r_p}\right) = I R_K$$

$$Z_0 = \frac{R_K}{1 + \frac{(1+\mu)R_K}{r_p}}$$

$$= \frac{r_p R_K}{r_p + (1+\mu)R_K}$$

$$Z_0 = \frac{V}{I} = \frac{r_p}{\mu} = \frac{1}{g_m}$$

in which it should be noticed that, with A_f positive and real, the input admittance is a capacitance of value slightly greater than C_{gp}. For $A_f = 0.9$ as in the example already used,

$$Y_{11} = j\omega(C_{gp} + 0.1\, C_{gK})$$

The input impedance, $\frac{1}{Y_{11}}$, is evidently much higher than for an ordinary triode amplifier stage.

The effective output admittance may be found, also, from analysis of Figure 3-40. The method of approach is to connect an a-c voltage source at the output terminals 1 - K, at the same time replacing the voltage source E_g at G - P by a short circuit. If the applied voltage source is E, and the current delivered at terminals 1 - K is I, then the output admittance with input short is

$$Y_o = \frac{I}{E} = \frac{\mu_{eq}}{r_p} + \frac{1}{R_K} + j\omega(C_{gK} + C_{pK}) \qquad (3\text{-}80)$$

In the frequency range where C_{gK} and C_{pK} may be neglected,

$$Z_o = \frac{1}{Y_o} = \frac{r_p}{(\mu + 1) + r_p/R_K} \qquad (3\text{-}81)$$

If R_K is made very much greater than r_p, and if $\mu >> 1$,

$$Z_o \cong \frac{r_p}{\mu} = \frac{1}{g_m} \qquad (3\text{-}82)$$

The effective output impedance with input short is therefore very small and may be in the range 200 to 1000 ohms.

These properties of the cathode follower have given it a number of interesting applications. It is frequently used as the first stage of a multistage amplifier when a high input impedance is essential over a wide frequency range and where the first stage must handle a wide range in values of input voltage and not be overloaded. Such an application is the amplifier for the vertical plates of a cathode ray oscilloscope. The cathode follower is used also to reduce frequency and phase distortions by placing it between two high gain amplifier stages. Still another application is that of impedance matching where it may be used instead of a transformer to match a high impedance circuit to a much lower one.

If T_1 in Figure 3-6 is a 6C5 and T_2 is a 2A3, determine the resistors so that the tubes will have the following operating points:

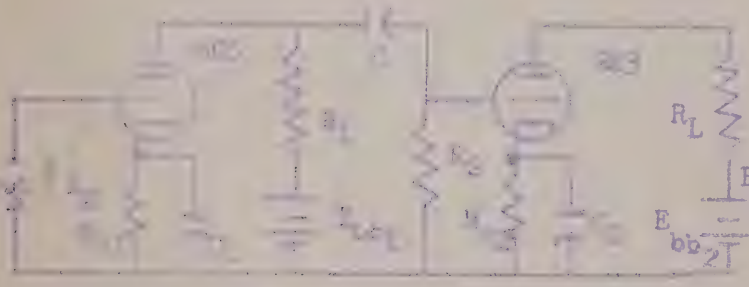
6C5 : $E_b = 200$ volts, $E_c = -6$ volts;

2A3 : $E_b = 250$ volts, $E_c = -40$ volts.

Use $R_1 = 20{,}000$ ohms, $R_2 = 500$ ohms, and $R_5 = 1000$ ohms. Find the bleeder current I and the required power supply voltage

Using the amplifier of Problem 3-1 assuming that R_c, R_3, R_4, and R_5 are all adequately by-passed by large size condensers. Calculate the overall voltage gain.

If the signal voltage $E_g = 0.1$ volt, compute the effective value of the load current (a-c) in R_2. Draw a complete vector diagram for the circuit and determine the phase relation between input voltage drop ($-E_g$) and the output voltage drop in R_2.



$R_1 = 50{,}000$ ohms

$R_2 = 250{,}000$ ohms

$C = 0.2\ \mu$fd.

$E_{bb1} = 400$ volts $= E_{bb2}$

$C_1 = C_2 = 10\ \mu$fd.

$R_L = 2500$ ohms.

(a) Determine R_{c_1} and also R_{c_2} so that the 6C5 will be biased at -8 volts, and the 2A3 at -43.5 volts.

(b) Determine the frequencies at which the gain of the first stage will fall to 70.7% of the intermediate frequency gain

(c) Neglecting the amplitude distortion of the first stage, what is the maximum input voltage E_g for a second harmonic distortion [illegible] in the second stage? [illegible]

A 6K7 pentode operating at $E_b = 250$ volts, $E_{c_1} = -3$ volts, $E_{c_2} = 100$ volts, is R-C coupled to a 6F6 power pentode (Fig. 2-a) operating at $E_b = 250$ volts, $E_{c_1} = -16.5$ volts, $E_{c_2} = 250$ volts. The coupling resistance is $R_c = 250{,}000$ ohms, $R_g = 250{,}000$ ohms, and $C = 0.2\ \mu fd$. The 6K7 is transformer coupled to a 500 ohm load. The output transformer has negligible leakage reactance and its resistance, and a primary to secondary turns ratio of 2:1.

(a) Draw the amplifier circuit using cathode biasing and screen dropping resistors so that a single power supply may be used. Compute required values of all resistors.

(b) Draw the a-c equivalent circuit and determine the amplifier gain at $f = 796$ cycles per second. What circuit elements may be expected to limit the region of uniform gain at high frequencies?

(c) Find the load current (a-c) if the input signal voltage is 0.2 volt rms.

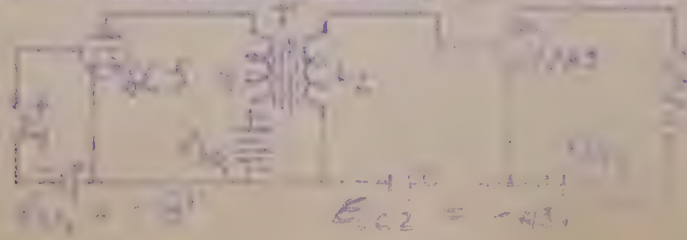
A resistance-capacitance coupled class A voltage amplifier of two stages uses a 6J7 (pentode) in the first stage and a 6D6 in the second stage. The load resistance for the 6J7 is 250,000 ohms, and the grid leak is 1 megohm. Output resistance for the 6D6 is 20,000 ohms. Fixed bias is used.

(a) Draw the equivalent a-c circuit for the complete amplifier. Neglect interelectrode capacitances.

(b) What should be the size of the coupling condenser, C, such that the gain-frequency curve will have 0.707 of its mid-frequency value at $f = 20$ cycles per second?

(c) If $C = 0.005\ \mu fd$, what is the overall gain of the amplifier in the mid-frequency range?

Given $n = \frac{N_2}{N_1} = 3$, $L_1 = 100$ henries, $L_2 = 900$ henries, $R_1 = 100$ ohms, $R_2 = 1000$ ohms, $E_{bb_1} = 250$ volts, $E_{bb_2} = 400$ volts. If the coefficient of coupling, K, is 0.988, and $R_L = 2500$ ohms, compute a gain-frequency curve for the range 20 cycles to 40 kilocycles per second. Assume that the total capacitance, C_2, of Figure 3-H is



Two 6L6 tubes are connected in push-pull and operated Class A_1 with $E_b = 250$ volts, $E_{c1} = -15$ volts, $E_{c2} = 250$ volts. Obtain the composite plate and dynamic characteristics of the two tubes if the effective plate-to-plate load resistance is 5000 ohms. Find the fundamental component of plate current flowing in the transformer primary by graphical means and by use of the equivalent circuit. Use an input voltage of $e_g = 15 \cos \omega t$.

(a) What is the equivalent plate-to-plate load resistance of two tubes connected in push-pull and operated Class A if the output transformer has a turns ratio, primary to secondary, of 3 to 1 and $R_L = 4000$ ohms?

(b) If each tube is said to be "working" into the equivalent external resistance between its plate and point "a" of Figure 3-19, compute this resistance for conditions as in part (a).

Two 2A3 tubes are operated in push-pull Class AB_1 with $E_{c1} = -60$ volts, $E_b = 300$ volts. Determine graphically the fundamental component of plate current, if the effective plate-to-plate load resistance is 2000 ohms. Prepare a set of composite plate characteristics for $E_{c2} = 0$ volts.

(a) Draw the a-c equivalent circuit of Figure 3-25 for the mid-frequency range.

(b) If the amplifier and power inverter tubes of Figure 3-25, this stage, are identical and $\mu = 20$, $r_p = 10{,}000$ for each, compute R_2 (if $R_{L1} = R_{L2} = 50{,}000$, $R_3 = 200{,}000$ ohms) so that the input voltages to the push-pull stage will be equal.

Analyze the circuit of Figure 3-29 for the effect of inverse feedback on frequency and phase distortions if R_L is replaced by an inductive load $Z_L = R_L + j X_L$ where $R_L = 30{,}000$ ohms, and $x_L = 2500$ ohms at $f = 5000$ cycles, and $R_f = 5000$ ohms. Note that β is no longer independent of frequency. For the tube, use a type 6J5.

Derive the mid-frequency gain formula for the amplifier of Figure 3-31 with and without the use of the feedback equation.

Plot the curve of gain versus frequency for the inverse feedback amplifier of Figure 3-31, and compare with the curve obtained without feedback. How much has inverse feedback extended the region of flatness of the gain-frequency curve?

3-14. Derive equation (3-75) from Figure 3-36.

3-15. Obtain an expression for the voltage gain of the amplifier of Figure 3-36 in the intermediate frequency range.

3-16. [illegible] pull amplifier of Figure 3-37, if the secondary voltage of the top half of the input transformer is E_g volts.

3-17. Referring to the amplifier of Figure 3-38, determine the overall amplifier gain in the intermediate frequency range if: the pentode is a 6J7 with operating values $E_b = 250$ volts, $E_{c_1} = -3$ volts, $E_{c_2} = 100$ volts, $I_b = 2$ ma, $I_{c_2} = 0.5$ ma; the triode is a 6C5 with $E_b = 250$ volts, $E_c = -8$, $I_b = 8$ ma; $R_{b_1} = 100{,}000$ ohms; $R_g = 250{,}000$ ohms; $R_{b_2} = 25{,}000$ ohms; $C = 0.04\,\mu f$; $C_{k_1} = 10\,\mu f$; $C_{k_2} = 2.5\,\mu f$; $C_1 = 0.4\,\mu f$; $C_d = 0.1\,\mu f$; $R_1 = R_2 = 50{,}000$ ohms. In addition,

(a) Find suitable values of R_{k_1}, R_{k_2}, and E_{bb}.

(b) Draw the complete intermediate frequency equivalent circuit.

(c) If the total capacity shunting R_{g_1} is $80\,\mu\mu f$, compute and plot a gain-frequency curve over the range 40 to 40,000 cycles.

(d) Compute a gain-frequency curve over the same range if $R_2 = 0$, $R_1 = 100{,}000$ ohms.

3-18. A [illegible]-coupled amplifier circuit is connected as [illegible] the circuit of Problem 3-3. Feedback may be provided by means of a resistor R_3 connected directly from plate-to-plate (between P_1 and P_2) of the two triodes. Neglecting the reactance of the coupling condenser, C, draw the complete a-c equivalent circuit of the amplifier and show that the gain of the second stage will be zero for $R_3 = \frac{1}{g_{m_2}}$, where g_{m_2} is the grid-plate transductance of the second tube. Derive the following expression for the overall gain of the amplifier:

$$A = - \frac{A_2 \, g_{m_1}}{Y_0 - A_2 \, Y_3}$$

where A_2 is the gain of the second stage, g_{m_1} = transconductance of the first tube, and

$$Y_0 = \frac{1}{r_{b_1}} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$Y_3 = \frac{1}{\quad}$$

$R_3 = 50{,}000$ ohms of Problem 3-18 using the circuit values given in Problem 3-3. Neglect all condenser reactances, and compute the mid-frequency gain of the amplifier

3-20. Compute the gain and input impedance of the cathode-follower amplifier of Figure 3-39 at $f = 79.6$, 796, 79,600, 796,000, and 1,593,000 cps if the tube is a 6C5 for which $r_p = 10{,}000$ ohms, $\mu = 20$ and $R_k = 10{,}000$ ohms.

Draw the equivalent circuit of the cathode follower, Figure 3-39, and connect a generator of voltage E across the output terminals. Assuming the voltage E_g to be zero, derive Equation (3-80).

Determine R_k, Figure 3-39, so that the cathode follower output impedance (with input short) will be 400 ohms at 1000 cycles. Assume that the tube is a 6C5, $\mu = 20$, $r_p = 10{,}000$ ohms.

Chapter IV

The Audio Amplifier Power Stage

The work of the preceding chapters has been concerned primarily with the increase of the voltage level in amplification. The present chapter will discuss the problems encountered in the design of the output or power stage of an audio frequency voltage amplifier. The power stage is required to provide, as nearly as possible, the optimum coupling to the load so that the requisite power will be delivered to the load with minimum distortion. Operation may be either as a class A operation with a single tube or in Class AB or Class B operation in push-pull.

4-1. Power Relations in Amplifiers

Power may conceivably be delivered to an amplifier from two possible sources, namely, the points of grid and plate bias, and the power sources which supply the cathode heating, the screen and control grid bias voltages, and the plate voltage. In Class A amplifiers where linear operation is desired, the grid does not swing positive, grid current is very small, and grid input power is negligible in the audio range. Grid and screen bias and plate voltages are usually supplied from a common power source usually derived from alternating current through a rectifier and filter system. Cathode heater or filament power may be derived from a low voltage winding on the power transformer supplying the rectifier.

Power is delivered by an amplifier to its load resistance, and the useful power output of the amplifier consists entirely of the a-c component. In an intermediate stage of a voltage amplifier, the power output of the stage might be defined as the a-c power developed in the resistor or equivalent resistor in which the input voltage for the succeeding stage is developed. For example, the useful output power of an R-C coupled stage of a voltage amplifier would, by definition, be developed in the grid leak resistor. Since the power so developed serves merely to heat the resistor, and does not contribute to the power input of the succeeding stage, it is evident that intermediate stage power output for R-C coupled voltage amplifiers, however, terminals to which may be called a power stage; this supplies both voltage and power to an equivalent load resistor, which may be, for example, a relay or a loud speaker. For maximum power transfer, such load resistances usually require matching networks or output transformers to match the power tube to the load. Optimum values of load resistance will be considered in a later section.

The following analysis of a power amplifier with a resistance load serves the purpose of illustrating fundamental definitions. The circuit of the amplifier is shown in Figure 4-1. Linear Class A_1 operation is assumed. The power input is entirely d-c if grid input power may be neglected. The power input, then, is

$$P_i = E_{bb} I_{b(dc)}$$

where I_b = the average (or quiescent) value of plate current. The losses are as follows: the d-c heat loss in the load resistor: $I_b^2 R_L$, and the power input to heat the cathode. Since the last is quite constant and independent of operating conditions, it is customary to omit it in considering plate circuit losses and power output. The energy dissipation at the anode is called the plate dissipation. It is the result of the [illegible] The temperature of the anode rises until it is able to radiate and conduct the heat of electron bombardment away as fast as it is generated. If the plate dissipation in watts is P_a, and the power output in watts is P_o, then the law of conservation of energy requires that

$$P_i = \text{output} + \text{losses}$$

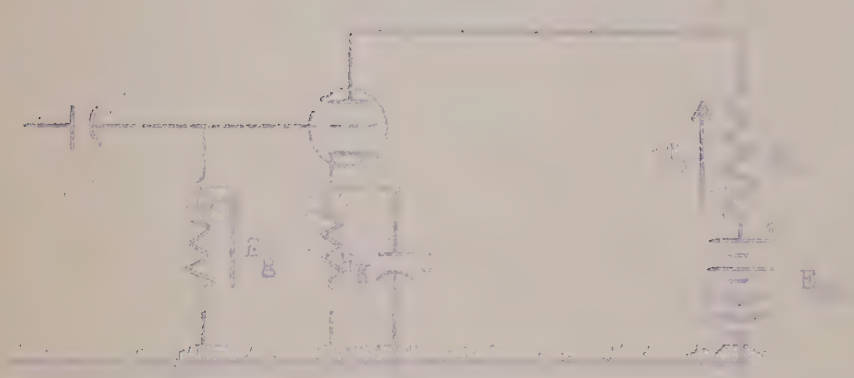


Figure 4-1.
Series fed power amplifier.

If the effective a-c component of plate current is I_p, then

$$P_o = I_p^2 R_L \text{ watts}$$

The plate dissipation may then be obtained by solving Equation (4-1);

$$P_a = E_{bb} I_b - I_b^2 (R_L + R_K) - I_p^2 R_L \qquad (4\text{-}2)$$

It is instructive to obtain P_a by another method. If e_b and i_b have their usual meanings, then the average power at the plate is

given fundamentally by

$$P_a = \frac{1}{T} \int_0^T e_b \, i_b \, dt \quad . \qquad (4\text{-}3)$$

where

$$e_b = E_{bb} - i_b \, (R_L + R_K)$$

and

$$i_b = I_b + i_p \, ,$$

then, for $i_p = \sqrt{2} \; I_p \sin \omega t$, the integral of Equation (4-3) results in

Equation (4-2), which shows that the maximum plate dissipation occurs when the power output is zero. If power is not delivered to the load or biasing resistors, it must be dissipated at the plate. The tube should therefore be operated with its plate dissipation at quiescence equal to the rated value.

Equivalent expressions for the power output may be obtained graphically from the plate diagram. Three typical triode plate characteristics and a load line are shown in Figure 4-2. Assuming linear operation, bias at $e_c = E_c$ volts (negative) and a grid swing of $e_{g_{max.}} = E_c$ volts, then by definition,

$$E_p = \frac{e_{b_{max.}} - e_{b_{min.}}}{2\sqrt{2}} \qquad (4\text{-}4)$$

and

$$I_p = \frac{i_{b_{max.}} - i_{b_{min.}}}{2\sqrt{2}} \qquad (4\text{-}5)$$

so that

$$P_p = \frac{(e_{b_{max.}} - e_{b_{min.}})(i_{b_{max.}} - i_{b_{min.}})}{8} \qquad (4\text{-}6)$$

The function of the power amplifier is to convert d-c to a-c power, controlled by the alternating voltage and resulting electric [illegible] The [illegible] efficiency of the amplifier is measured by what is called its plate-circuit efficiency, defined as the useful plate output power divided by the plate input power. If η_p is the plate-circuit efficiency, then

$$\eta_p = \frac{P_p \; I_p}{E_{bb} \; I_b} \qquad (4\text{-}7)$$

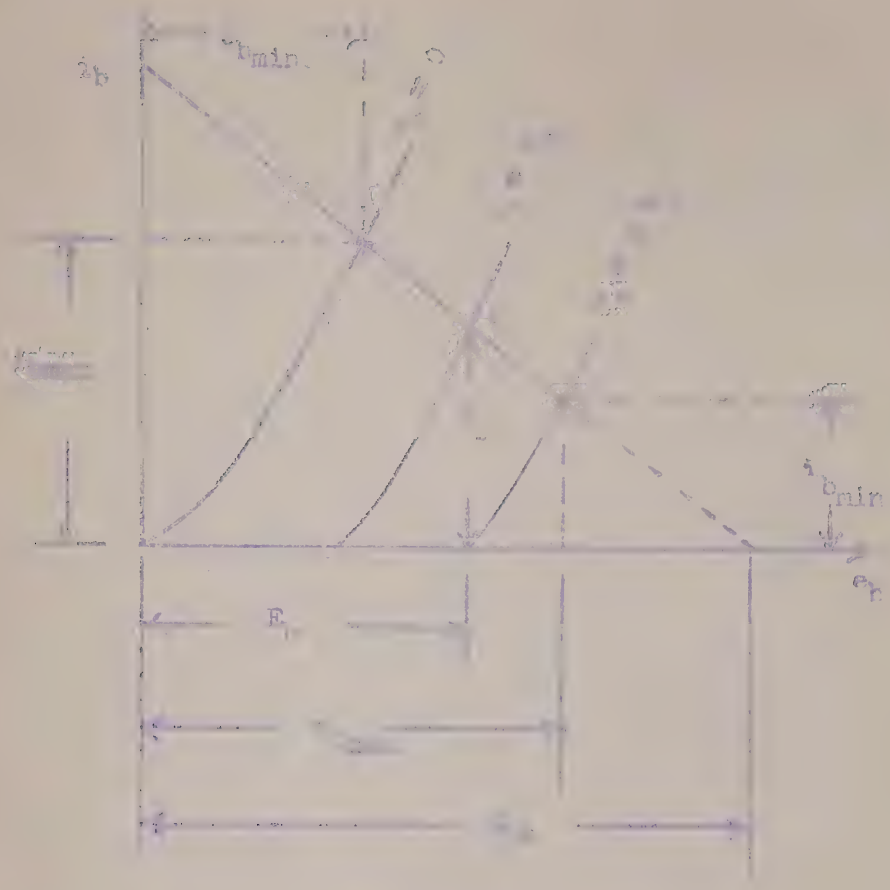


Figure 4-2

Load line and plate diagram for a power amplifier.

For the circuit of Figure 4-2, and from Equation (4-2),

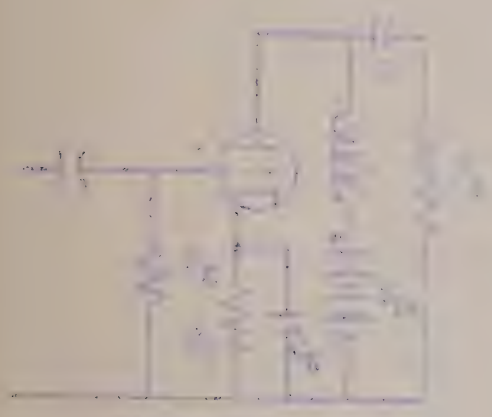
$$\eta_p = \frac{E_{bb}\, I_b - I_b^2 (R_L + R_K) - P_a}{E_{bb}\, I_b}$$

Evidently, the plate circuit efficiency could be considerably increased by using parallel feed to eliminate the loss $I_b^2 R_L$

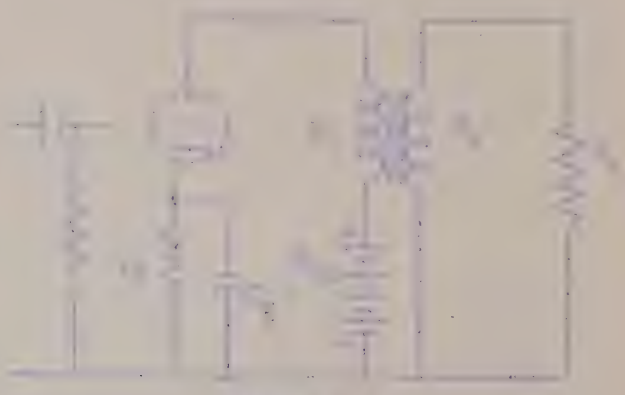
4-2 Load Coupling Circuits.

Series feed as in Figure 4-1 is wasteful of both power and battery voltage. Parallel feed, using a choke and condenser as shown in Figure 4-3, or transformer coupling, as shown in Figure 4-4, have the

advantage of greater plate circuit efficiency and the additional advantage that no direct current component flows in the load resistance. Because of the large value of the quiescent or average component of plate current in power amplifiers, the output transformer primary inductance may be reduced as a result of this operation. For this reason, a choke and a condenser may be desirable in coupling the tube to the output transformer. In such cases the impedance of these components need be shown in the equivalent a-c circuit.



Parallel-fed power amplifier.



Power amplifier with output transformer.

The location of the operating point on the plate diagrams for the circuits of Figure 4-3 and Figure 4-4 require the construction of what may be called the static load line. This is a load line drawn in the usual way from the point on the voltage axis corresponding to E_{bb} and with a slope determined by the d-c resistance of the choke or of the transformer primary plus the resistance of the biasing resistor. These resistances will be small compared with the effective load resistance for a power stage. If these resistances were negligible the static load line would be vertical. The operating point is then determined by the intersection of the static load line and the plate characteristic corresponding to the grid bias as shown in Figure 4-5. The dynamic load line is then drawn through the operating point with a slope corresponding to the a-c resistance of R_L, Figure 4-3, or to $\left(\frac{N_1}{N_2}\right)^2 R_L$ in Figure 4-4. The required plate supply voltage is approximately the operating voltage E_b. It is assumed that the biasing resistors are adequately by-passed, but this is difficult where the value of R_K is in the range 150 to 750 ohms.

Static and dynamic load lines for parallel-feed.

4-3. Optimum Value of Triode Load Resistance.

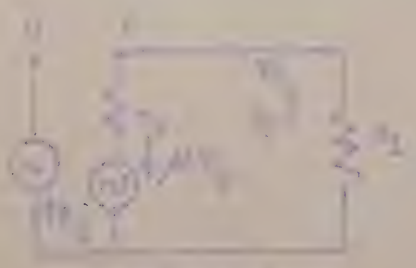
For triodes, definite values of load resistance correspond to [illegible] are usually considered. In the first, a small excitation voltage of fixed effective value, E_g, is specified; E_g is numerically less than the grid bias so that operation may be considered as linear, Class A_1. The equivalent circuit of Figure 4-3, for example, could then be used, and is shown in Figure 4-6. Then

$$P_o = I_p^2 R_L$$

$$= \left(\frac{\mu E_g}{r_p + R_L} \right)^2 R_L \,. \qquad (4\text{-}8)$$

Since the excitation is small, μ and r_p may be assumed constant, so that P_o and R_L are the variables. By differentiating P_o with respect to R_L it is easily verified that the maximum value of power output will be realized, at fixed excitation, for $R_L = r_p$

$r_p = R_L$



The conditions of the second case are more frequently encountered in the use of single tube power amplifiers. The only restriction upon the grid voltage is [illegible] can be tolerated. This means that the grid should not swing positive. It is then required to determine R [illegible] give a fixed value of the minimum plate voltage, $E_{p_{min}}$. To avoid distortion, the minimum value of the plate current must never be less than some arbitrary constant, I_1, Figure 4-7, determined by the curvature of the [illegible] harmonic distortion, which, by common consent, may be fixed at 5%. Summarizing, the conditions for maximum undistorted power output are:

(1) [illegible]

(2) $i_{b_{min}} = I_1$

(3) [illegible]

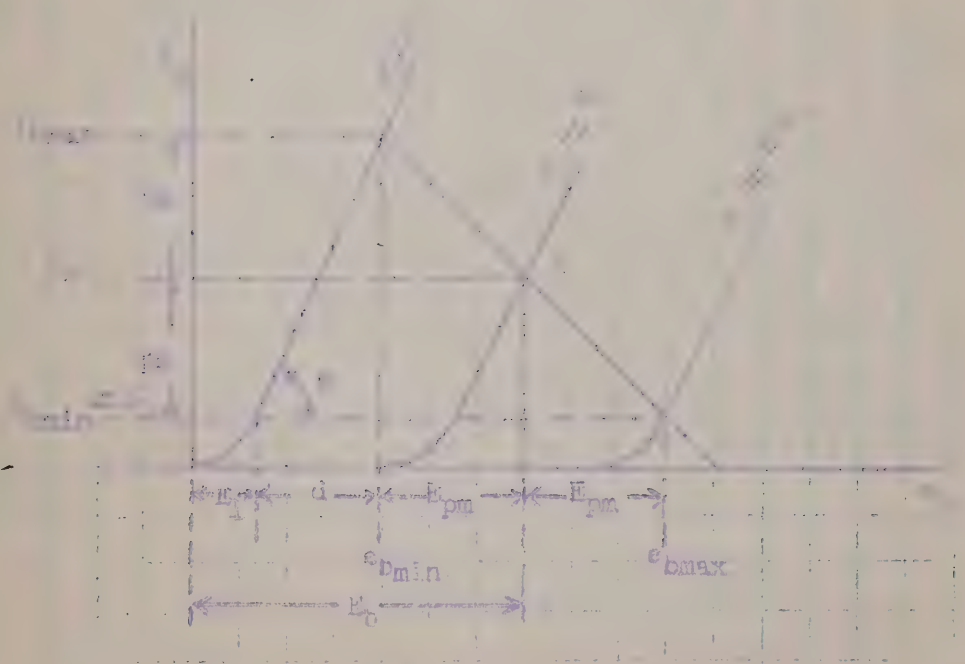


Figure 4-7

Now for an arbitrary selected value of bias voltage, E_c, the operating point Q is determined. The maximum and minimum values of plate current and of plate voltage, and hence also the value of the power output depend upon the resistance of the effective plate load, R_L, which determines

the slope of the dynamic load line. The magnitude of $i_{b_{min}} = I_1$ may be selected arbitrarily. If then an optimum load resistance is determined by analysis, the resulting harmonic distortion may be checked, and if a de-

load resistance involved is optimum in the sense that it provides maximum power output for the assumed value of I_1, and for the fixed values of E_b, E_c, and E_{gm}. The problem of analysis, therefore, is to determine the best value of R_L to use with the given tube for the specified conditions.

The analysis is quite straight-forward and proceeds as follows. The line $i_b = I_1$ intersects characteristic $e_g = 0$ at $e_b = E_1$. Let $\sqrt{2}\ E_p = E_{pm}$, and $\sqrt{2}\ I_p = I_{pm}$. Assuming that the characteristics are linear and evenly spaced, the values of E_{pm} and I_{pm} are as shown in Figure 4-7. The distance d may be expressed as

$$d = 2\ I_{pm} \cot \theta = 2\ I_{pm}\ r_p$$

Then, using Equation (4-6)

$$P_o = \frac{E_{pm}\ I_{pm}}{2} \qquad (4\text{-}9)$$

$$E_{pm} = I_{pm} R_L = E_b - E_1 \qquad (4\text{-}10)$$

so that, substituting in Equation (4-9),

$$P_o = \frac{1}{2}\ (E_b\ I_{pm} - 2\ I_{pm}^2\ r_p - E_1\ I_{pm}). \qquad (4\text{-}11)$$

In Equation (4-11) E_b, r_p, and E_1 are constants and P_o depends upon I_{pm}. The optimum value of I_{pm} may be determined by equating the derivative of P_o with respect to I_{pm}. Thus,

$$\frac{d\ P_o}{d\ I_{pm}} = \frac{1}{2}\left[E_b - 4\ I_{pm}\ r_p - E_1\right] = 0$$

$$I_{pm} = \frac{E_b - E_1}{4\ r_p} \qquad (4\text{-}12)$$

From Equation (4-10), $E_b - E_1 = E_{pm} + 2\ I_{pm}\ r_p$.

Since numerically $E_{pm} = I_{pm}\ R_L$,

$$E_b - E_1 = I_{pm}\ (R_L + 2\ r_p)$$

Substituting in Equation (4-12) and simplifying, the relation obtained is:

$$R_L = 2 r_p \qquad (4\text{-}13)$$

according to the conditions assumed. If the load resistance is made equal to twice the tube plate resistance. It is assumed, however, that the rated plate dissipation will not be exceeded at zero excitation, so that it must be determined in practice if the operating point corresponding to the assumed value of E_b can be used without exceeding allowable plate dissipation. If the maximum plate dissipation is specified, and no restrictions are placed on plate voltage, Nottingham[1] has shown that the optimum plate load for undistorted power output is in the range fifteen to twenty times the plate resistance.

To determine the proper value of grid bias, Equation (4-12) may be written as

$$\frac{E_{pm}}{R_L} = \frac{E_b - E_1}{4 r_p}$$

For $R_L = 2 r_p$,

$$I_{pm} = \frac{E_b - E_1}{4 r_p} \qquad (4\text{-}14)$$

$$I_{pm} = \frac{A E_c}{r_p} \qquad (4\text{-}15)$$

Since $E_{pm} = I_{pm} R_L$, (neglecting phase) Equations (4-14) and (4-15) may be solved for E_c. The result is

$$E_c = -\frac{1}{A} \frac{E_b - E_1}{4} \qquad (4\text{-}16)$$

Experience with power triodes shows that E_1 is approximately 0.1 E_b, so

$$E_c = -0.675 \frac{E_b}{\mu}$$

$$E_c = -0.7 \frac{E_b}{\mu} \qquad (4\text{-}16a)$$

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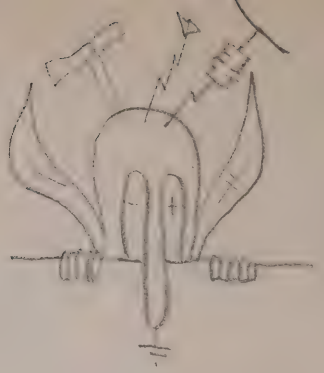
[1] Nottingham, W. B., "Optimum Load Condition for Maximum Power in Class A Amplifiers," Proc. I.R.E., 23, 642, 1935.



The optimum or maximum power output corresponding to $R_L = 2\,r_p$

$$= \frac{\mu E_b}{}$$

$$P_{o\,max} = \frac{1}{} \left(\right) E_b$$

$$= \frac{E_b}{} \qquad (4\text{-}17)$$

Using the approximate relation of Equation (4-16a) for E

$$P_{o\,max} = 0.054 \frac{E_b^2}{r_p} \text{ watts} \qquad (4\text{-}17a)$$

is the maximum power output which may be expected from a tube of resistance r_p operating at a plate voltage of E_b. Actually because of the approximations necessary in its derivation, Equation (4-17a) should be considered as a useful approximation of the maximum power output.

4-6. Theoretical Efficiency

If triode plate characteristics were perfectly straight lines, then I_1 in Figure 4-7 would be zero, and I_{pm} would be equal to I_b. This would be the ideal situation, and although the conditions are theoretical, the plate-circuit efficiency determined by assuming straight line characteristics is of value because it provides information as to the maximum possible theoretical efficiency of a triode power amplifier. Referred to Figure 4-7, the efficiency is (for $I_1 = 0$, straight line characteristics)

$$\eta_P = \frac{E_{pm}\, I_{pm}}{2\, E_{bb}\, I_b} \qquad (4\text{-}18)$$

according to Equation (4-7). For series feed, the d-c loss in the load resistor must be considered, so that $E_{bb} = E_b + I_b R_L$ Since $I_b = I_{pm}$

$$\eta_P = \frac{E_{pm}}{2\,(E_b + E_{pm})} \qquad (4\text{-}19)$$

If the plate characteristics are all straight lines, then that for which $e_c = 0$ would be expected to pass through the origin so that $E_{c_1} = 0$. Equation (4-10) may then be written as

$$i_b = \frac{E_{bb}}{r_p} + \frac{e_c}{r_p}$$

so that the efficiency may be expressed as

$$\eta_p = \frac{E_{pm}}{2\left(\frac{E_{bb}}{r_p} + 2 I_b\right) r_p}$$

which may be simplified by dividing numerator and denominator by E_{pm}. Finally for the series fed amplifier, the theoretical plate-circuit efficiency becomes

$$\eta_p = \frac{1}{4\left(1 + \frac{r_p}{R_L}\right)} \qquad (4\text{-}20)$$

The maximum possible plate-circuit efficiency of the series fed amplifier is, according to Equation (4-20), 0.25, or 25%. For the optimum power output with fixed E_{bb}, $r_p = R_L$, and the upper limit of the plate-circuit efficiency is 12.5%. Actual plate-circuit efficiencies are usually less than 10%.

A considerable improvement in plate-circuit efficiency may be realized by using parallel feed, thereby practically eliminating the d-c loss due to the quiescent or average value of plate current. For parallel feed, $E_{bb} = E_{pm}$, and Equation (4-18) becomes, with $I_b = I_{pm}$,

$$\eta_p = \frac{E_{pm}}{2 I_b r_p}$$

Again using Equation (4-10), with $E_{c_1} = 0$,

$$\eta_p = \frac{E_{pm}}{2\left(E_{pm} + 2 I_b r_p\right)}$$

$$= \frac{1}{2\left(1 + \frac{2 r_p}{R_L}\right)} \qquad (4\text{-}21)$$

is, from Equation (4-21), 12%. For the maximum undistorted power output, $R_L = 2 r_p$, the value of η_p drops to 2.5%. It should be noted that the maximum value of theoretical plate-circuit efficiency for [illegible] plate-circuit efficiency [illegible]

$$P_L = \frac{(\quad)}{(\quad)}\left(P_1\right)$$

$$\frac{\partial P_L}{\partial R_L} = \frac{1}{\quad}\left[\quad\right] = 0$$

$$-(\quad)^2(r_p + R_L)^2 R_L + (\quad)(r_p + R_L)^3 = 0$$

$$-2(r_p + R_L)R_L + (r_p + R_L)^2 = 0$$

$$r_p - R_L = 0$$

$$\boxed{r_p = R_L}$$

$E_L^N = \frac{\Delta P}{\quad}$ = maximum allowable power loss

Optimum Operation of Power Pentodes and Beam Tubes

for pentodes than for triodes while the grid-plate transconductance, g_m, is of the same order of magnitude. The shape of the static characteristics of pentodes is such that a much greater range of alternating plate [illegible] circuit efficiency are greater for pentodes than for triodes. Pentodes, however, introduce greater harmonic distortion than triodes, and since the minimum distortion is comparatively high, it is harmonic distortion [illegible] optimum operation does not occur for $R_L = 2\,r_p$ for either beam or pentode power amplifiers. In fact, R_L is usually a small fraction of r_p for best operation.

It was shown in Chapter II, Section 2-11, that the second harmonic amplitude, E_2, in pentode amplifiers can be reduced to zero by a proper choice of load resistance. This value of load resistance is a first [illegible] output. Since minimum distortion is the determining factor, the optimum load resistance is approximately that for which $E_2 = 0$. The best value of R_L is most easily determined experimentally. If determined by computation with the assumption of a value of $i_{b_{max.}}$ near the knee of the $e_c = 0$ pentode [illegible] trial value of E_b since $P_a \cong E_b\left(\frac{i_{bmax.}}{2}\right)$. A value of grid bias E_c is then chosen, and a trial R_L is determined by the operating point and the value of $i_{bmax.}$ on the $e_c = 0$ characteristic. If the value of I_b at the operating point differs appreciably from $\frac{i_{bmax.}}{2}$, the process is repeated. For each trial R_L, the harmonic distortion and the value of P_o are computed, and curves plotted to show the best combination of values. Unfortunately, maximum P_o does not coincide with minimum harmonic distortion.

The variation of P_o with R_L is easily understood if the power [illegible]

$$P_o = \frac{(e_{b_{max.}} - e_{b_{min.}})(i_{b_{max.}} - i_{b_{min.}})}{8}$$

[illegible] a load line, as shown in Figure 4-8. The slope of the load line corresponds [illegible]

an operating point of $E_b = 250$ volts, $E_c = -15$ volts, and a grid swing of 15 volts, is approximately

$$B_2 = \frac{132 + 19 - 2(80)}{4} = -13$$

$$= -3.25 \text{ ma}$$

$$B_1 = 1/3\left[i_{b\max.} + i_b(1/2) - i_b(-1/4) - i_{b\min.}\right]$$

$$= 1/3\left[132 + 112 - 43 - 15\right] = \frac{166}{3} = 55.3 \text{ ma}$$

The percent second harmonic distortion is $\frac{B_2}{B_1} \times 100$ or 5.8% Since

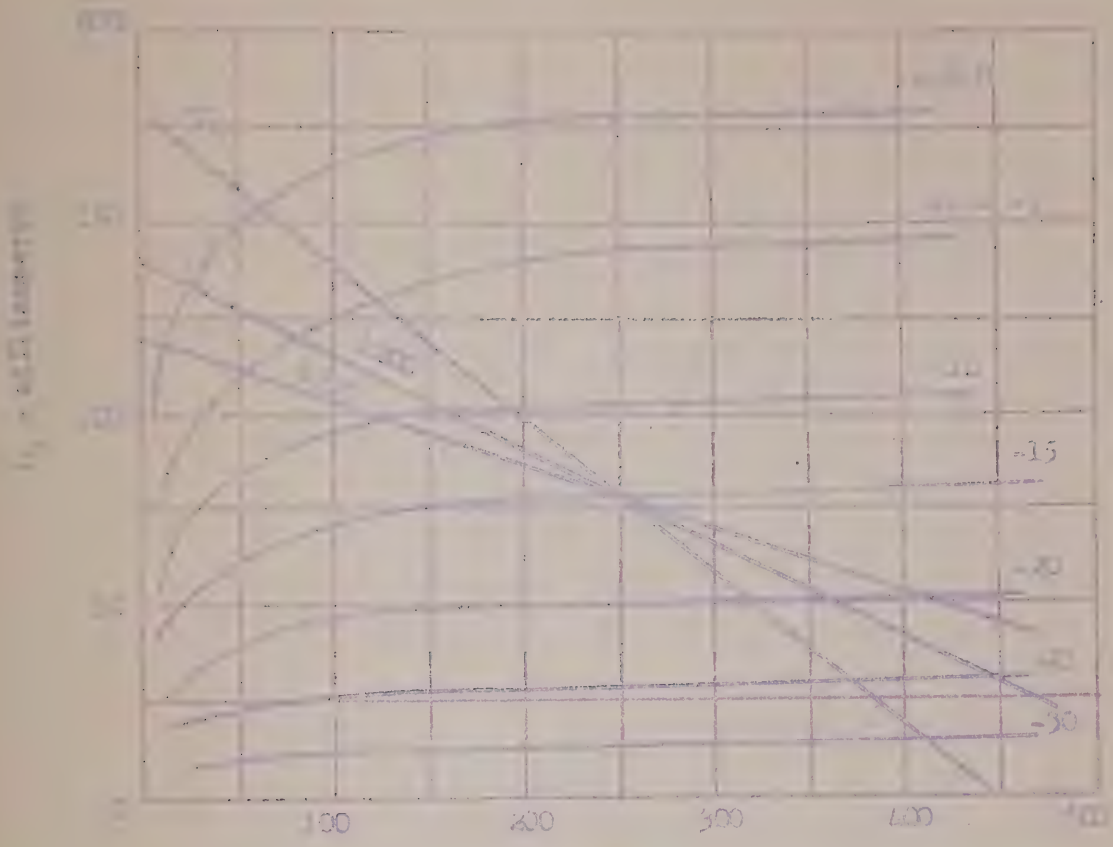
$$\frac{e_{b\max.} - e_{b\min.}}{2} = \frac{510 - 34}{2} = \frac{476}{2} = 238 \text{ volts}$$

$$P_o = \frac{238\ (55.3 \times 10^{-3})}{2} = 6.1 \text{ watts.}$$

If now R_L is increased to 6,250 ohms, $i_{b\min.}$ remains constant, but $i_{b\max.}$ decreases by about 15 ma. The plate swing increases, however, so that the power output is increased to approximately 8 watts, but the percent second harmonic distortion now exceeds 11% If R_L is decreased to 2500 ohms, P_o is reduced to 6 watts, but the second harmonic distortion is about 2.5%. Evidently, $R_L = 5000$ ohms is reasonably close to the optimum

In connection with Figure 4-8, it is important to notice that the value of $i_{b\min.}$ becomes quite small as R_L increases. It will be remembered from Chapter II that the screen grid current is high for very low plate voltages, and that, because of secondary emission, the plate current may actually be negative for very low values of plate voltage. It is important to remember that for pentodes or tetrodes the plate voltage must never be suddenly removed or greatly reduced unless the screen voltage is first removed because the flow of electrons to the screen may destroy the tube.

Effectively the same considerations as to optimum operation apply to beam power amplifier tubes as to pentodes. The beam power tubes have more nearly linear characteristics over a wider region of the plate diagram than do pentodes, and so are capable of delivering larger power output with less distortion than pentodes of equivalent size. Both pentodes and beam power tubes are frequently operated in push-pull to reduce harmonic distortion



Typical power pentode static characteristics

4-6 Class AB Power Amplifiers

Because of distortion, Class AB audio frequency amplifiers are always operated push-pull

Figure 4-9.

Composite Triode Characteristics for push-pull operation in Class AB. $E_c = -40^V$; $E_b = 275^V$; $(N_1/N_2)^2 R_L = 100\ \Omega$.

$i_{b_{max}} = -i_{b_{min}}$; $i_{b(1/2)} = -i_{b(-1/2)}$, and since $I_b = 0$, the harmonic amplitude equations of Section 2-10 become

$$B_1 = 2/3\,(i_{b_{max}} + i_{b(1/2)})$$

Application of these relations to Figure 4-9 shows that the third harmonic distortion is very small.

Composite characteristics for positive grid swings have been drawn for two 6L6 beam power amplifier tubes in push-pull, Class AB operation, and are shown in Figure 4-10. Although not straight lines, the composite characteristics are more evenly spaced than the individual characteristics, and the distortion is small. The composite characteristic for $e_g = 20$ volts differs very little from the individual $E_{c_1} = 0$ characteristic. Although the composite quiescent plate current, I_b, is zero, the individual quiescent plate currents are not zero. It was shown that the even harmonics are present in the midtap-to-cathode return, a - K, of Figure 3-19. In case a biasing resistor is used in Class AB operation, a by-pass condenser must be used, if [illegible] necessary in Class A_1 push-pull operation. In the biasing resistor of the Class AB operated push-pull amplifier, the biasing current is $I_{b\,av} = I_{b_1} + B_0$ at maximum excitation, where B_0 is the d-c component resulting from distortion. The grid bias, therefore, increases at high values of signal voltage, so that the distortion also increases with the excitation, requiring that the bias resistor be selected approximately as though the operation were to be limiting Class A.*

Because of the greater grid bias of Class AB operation, the operating plate current is smaller, and the operating plate voltage may be increased without exceeding the limiting plate dissipation. The result is a greater power output at higher efficiency. The power supplied to the tube increases with the signal voltage, E_g, for fixed E_{bb}, and since the power to the load also increases, the plate dissipation (which is the difference between the input to the tube and the output) may either increase or decrease. In Class AB operation, P_p generally increases with E_g. It will be remembered that the reverse is true for Class A operation.

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* By limiting Class A operation it is meant that $e_g = 0$ for the most negative value of instantaneous grid voltage.

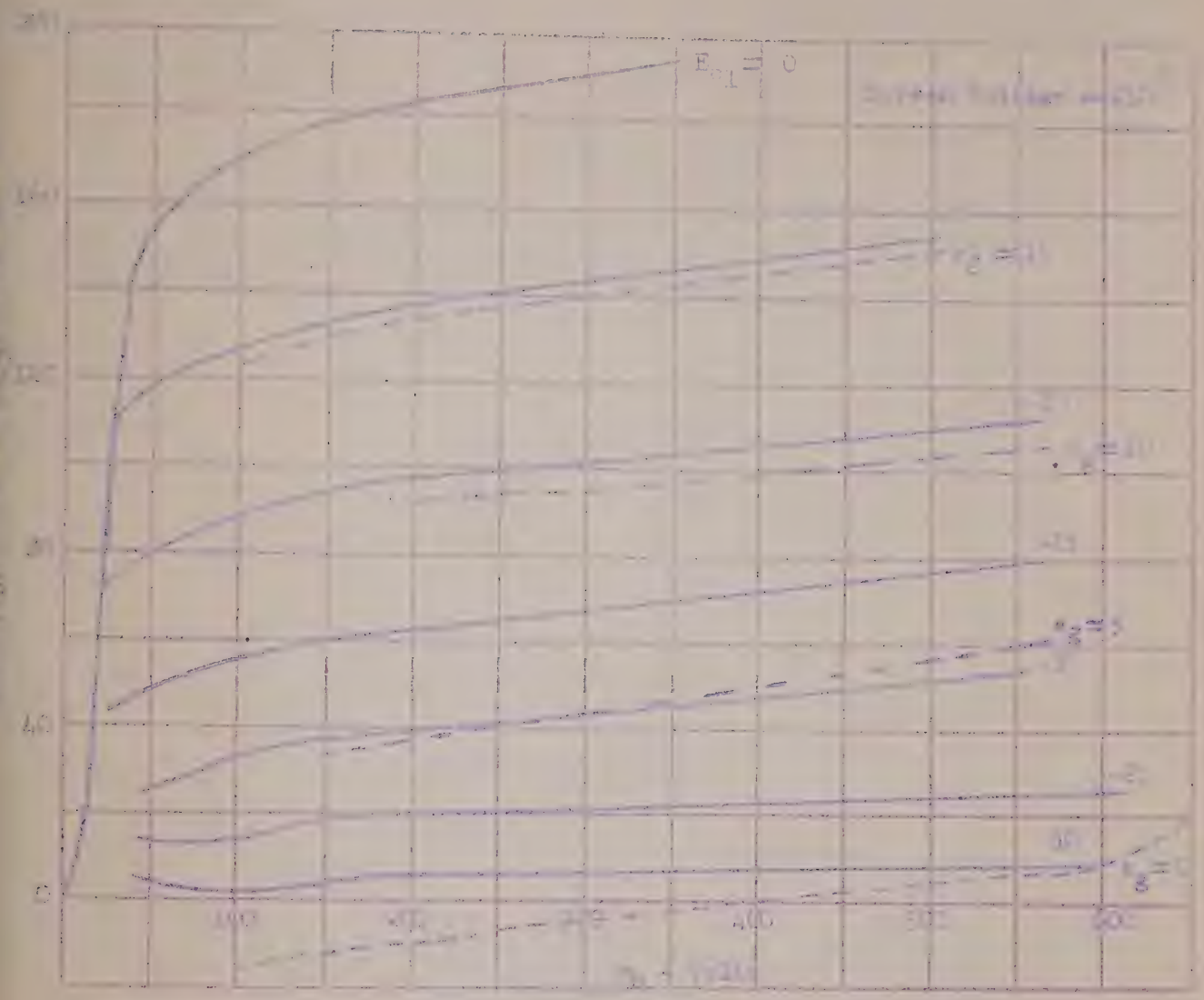


Figure 4-10.

Individual and composite static characteristics for two type 6L6 beam tetrodes in push-pull operation. $E_{c_1} = -20^{v}$, $E_b = 375^{v}$

4-7. Optimum Class A_1 and AB_1 Push-Pull Operation

For push-pull operation in either Class A_1 or AB_1 for triodes where the composite plate characteristics are equally spaced straight lines, Figure 4-11 applies. Since the distortion is justifiably neglected, and the

output transformer is assumed to be ideal, the power output is given by [illegible] and the grid swing may be varied from zero to the magnitude of the grid bias. Then, from Figure 4-11,

$$d = I_{pm} \cot \theta = I_{pm} R_{cp}$$

where R_{cp} is the reciprocal of the slope of the composite characteristics, and is usually obtained from the slope of the $e_g = 0$ characteristic. The maximum plate swing from quiescence is then

[illegible] (4-23)

so that $P_o = 1/2 (E_b - E_1 - I_{pm} R_{cp}) I_{pm}$. (4-24)

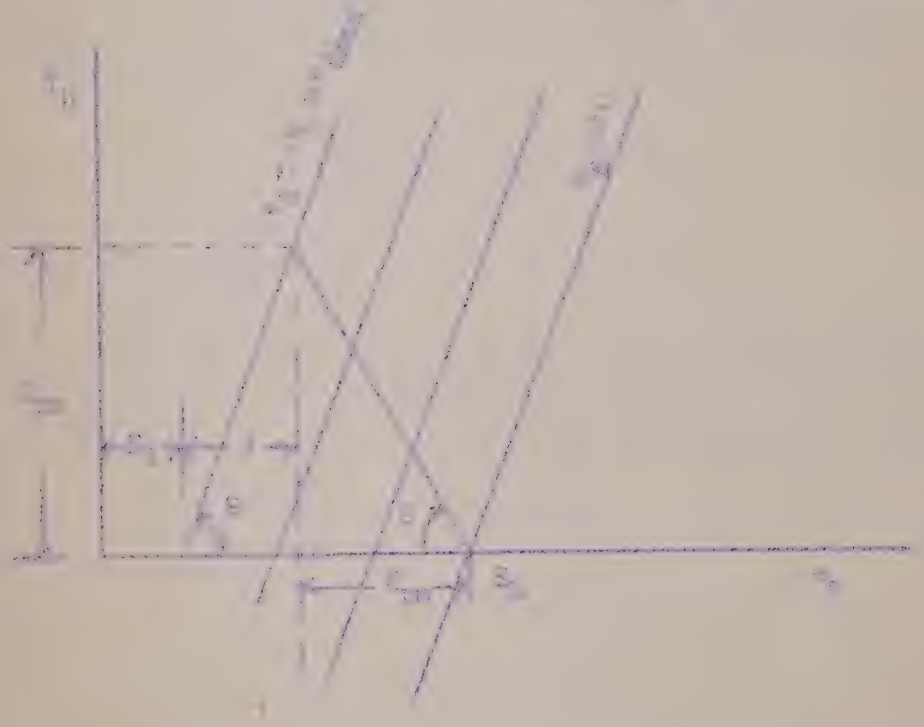


Figure 4-11

Composite static plate characteristics and load line for a push-pull triode amplifier.

It is evident from Equation (4-24) and Figure 4-11 that P_o depends upon I_{pm} and therefore upon the slope of the load line. For maximum P_o,

$$\frac{d P_o}{d I_{pm}} = [\text{illegible}] = 0$$

If the effective load resistance is R_L'', then $E_{pm} = I_{pm} R_L'$ and from Equation (4-23)

Substituting Equation (4-26) in Equation (4-25), the result is

$$R_L = R_{cp} \quad (4\text{-}27)$$

Therefore, for either Class A_1 or AB_1 push-pull operation of triodes the optimum operating load resistor is that for which the slope of the composite load line is equal numerically to the slope of the composite static characteristics, or

$$\left(\frac{N_1}{N_2}\right)^2$$

where R_L is the actual secondary load resistance.

In push-pull amplifiers using power pentodes or beam tetrodes, the optimum load resistance is determined by construction in almost the same way as for the single tube power amplifier. The effective resistance should be such that the composite load line intersects the composite characteristic for zero grid voltage in the vicinity of the knee of the curve.

4-1 A type 10 power amplifier triode is used in a circuit like that of Figure 4-1. The tube and circuit constants are: $r_p = 3000$ ohms, $\mu = 8$, $R_L = 10{,}000$ ohms, $E_{bb} = 630$ volts, $E_c = -30$ volts

(a) Locate the operating point (E_b, I_b) assuming fixed bias.

(b) For a grid swing of $E_{gm} = 30$ volts, determine the percent second harmonic distortion and the output voltage and power.

(c) Compute the plate dissipation and the plate-circuit efficiency

(d) If parallel feed is used as in Figure 4-3, find the new value of E_{bb} so that the same operating point and load line may be used as in part (a) and (b). Assume an ideal choke and condenser.

(e) Compute the power output, plate dissipation, and plate-circuit efficiency for the shunt feed system.

4-2 The power stage of a voltage amplifier uses a 2A3 triode operating at $E_b = 250$ volts, $E_c = -45$ volts. If $E_{gm} = 45$ volts, compute the second harmonic distortion, the power output, and the average or direct plate current for the following values of load resistance:

(a) $R_L = 400$ ohms

(b) $R_L = 800$ ohms

(c) $R_L = 1600$ ohms

(d) $R_L = 3200$ ohms

4-3 Given: T_1, a type 605, $\mu = 20$, $r_p = 10{,}000$; $R_{L1} = 50{,}000$; $R_1 + R_2 = 100{,}000$; $R_2 = 5000$ Reactances of all condensers negligible

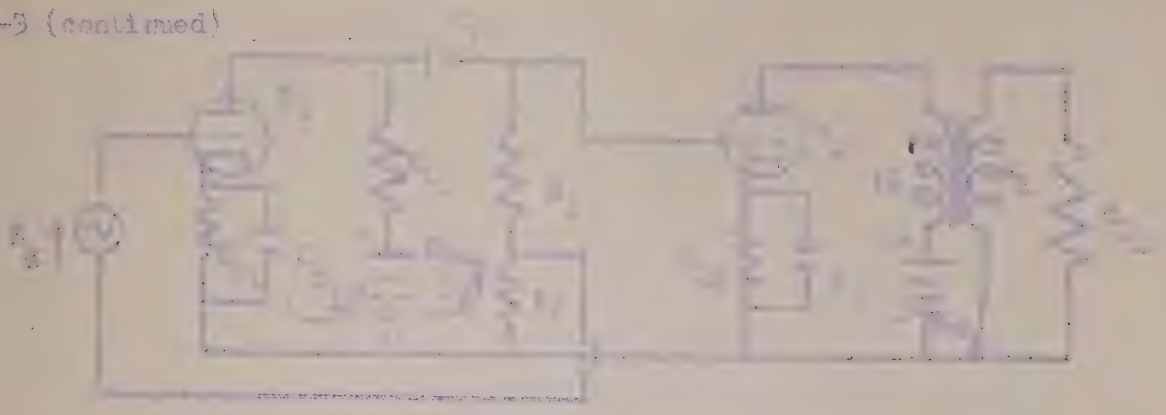
[illegible] For the transformer, ωL_1 and ωL_2 are much greater than the winding resistances.

(a) If T_1 operates at $E_c = -8$ volts, $I_b = 4$ ma., and T_2 operates at $E_c = -50$ volts, $I_b = 34$ ma., $E_b = 250$ volts, find: R_{c1}, R_{c2}; E_{bb_1}, E_{bb_2}

(b) Draw the equivalent a-c circuit.

(c) For $E_g = 5$ volts, find the input voltage to the second stage and the gain of the first stage. Do not neglect the feedback in finding the gain.

4-3 (continued)



4-4 In the circuit of Problem 4-3, compute the output power of the final stage, the plate circuit efficiency of each tube, and the plate dissipation of each tube. Assume that distributed capacitance in the transformer may be neglected.

4-5 A power amplifier stage consists of a 25A6 power pentode with an effective load resistance of 4000 ohms operating with $E_b = 135$ volts, $E_{c1} =$ [illegible] $I_b =$ [illegible] $E_{c2} =$ [illegible]

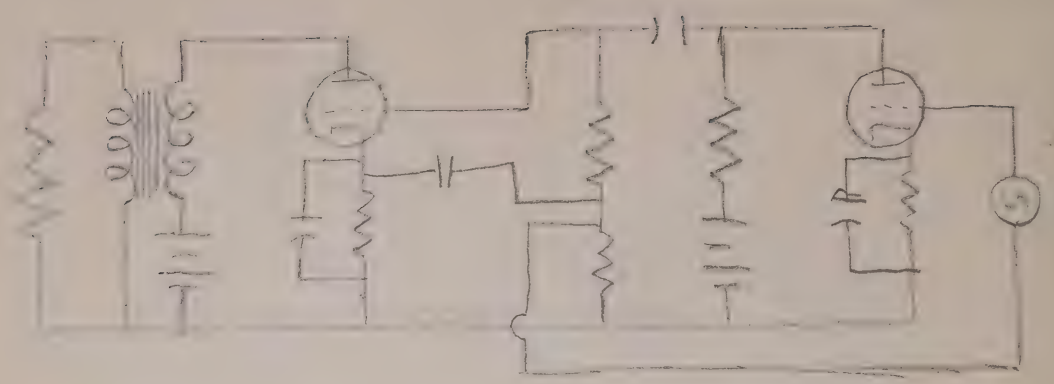
Determine the output power, plate dissipation, and plate-circuit efficiency if the tube is transformer coupled to the load resistance through an ideal transformer, and E_{gm} (sinusoidal) is 20 volts.

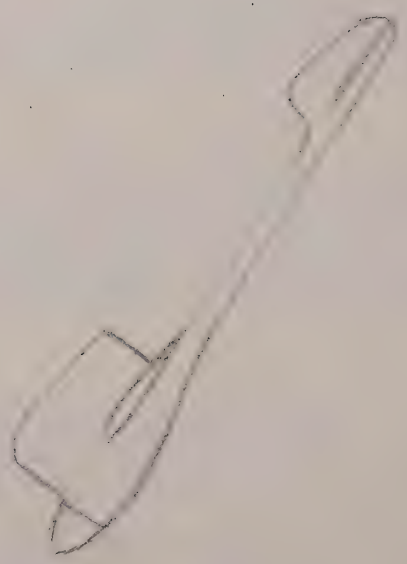
4-6 Determine the best value of load resistance to be used with the amplifier of Problem 4-5 by computing power output, dc components and total harmonic distortions up to the 3rd, and plate-circuit efficiencies for the following effective values of load resistance: 2000, 4000, 6000, 8000, 10,000, and 12,000 ohms. Tabulate and plot the results.

4-7 Determine the proper value of load resistance to be used with a power amplifier stage in which the tube is a 6L6 operating with a fixed bias of $E_{c1} = -15$ volts, $E_b = 250$ volts, $E_{c2} = 250$ volts.

The load is coupled to the tube through an ideal transformer of turns ratio 2:1 step-down (tube to load). It is desired that the load resistance be chosen so that the second harmonic amplitude B_2 will be zero. For this value of load resistance and with $E_{gm} = 15$ volts compute:

(a) The fundamental and third harmonic amplitudes;
(b) The d-c component of plate current;
(c) The fundamental and total power outputs;
(d) The plate dissipation and plate-circuit efficiency.





AB_2. The plate supply voltage is 400 volts, $E_{c_1} = -25$ volts, $E_{c_2} = 250$ volts, and the peak value of the sinusoidal input voltage, grid-to-cathode is 35 volts. The transformer is assumed to be ideal and has a turns ratio, tube-to-load of 2:1.

(a) Prepare a set of composite plate characteristics.

(b) [illegible] values of effective plate-to-plate load resistance as follows: 5000 ohms, 10,000 ohms, and 20,000 ohms.

[illegible] harmonic distortion, the d-c supply current, and the power output.

4-9. Two 2A3 triodes are connected in push-pull and operated Class AB_1, at $E_b = 300$ volts, $E_{c_1} = -60$ volts. What should be the value of load resistance for maximum undistorted power output if the ideal output transformer has a step-down turns ratio of 3:1?

[illegible] a 250 ohm load if the maximum input signal voltage is 0.2 volt. Draw the complete circuit and compute all circuit constants. Discuss [illegible]

Gas-filled Tubes as Circuit Elements

The electron tubes described in the circuit applications of preceding chapters have been high-vacuum tubes. The vacuum in a high-vacuum tube is desirably as high as possible, although it is probable that many high-vacuum tubes of the receiving type operate quite satisfactorily at pressures of the order of 10^{-3} millimeters of mercury. The residual gas pressure for high vacuum tubes should be in the range 10^{-6} to 10^{-7} millimeters of mercury or better.

It is the objective of the present chapter to present the gas-filled electron tube as a circuit element. The extremely interesting and highly important internal physics of the gas-filled tube will be ignored, as was done in the case of the high-vacuum tube, until the behavior of the tube has been described in terms of simple current and voltage measurements made at the available external tube terminals. Then, after having achieved at least a degree of familiarity with the circuit behavior of both vacuum and gas-filled tubes, the student should find that a more extended analysis into the internal physical behavior of electron tubes in general will serve to answer a number of important questions which will have occurred to him, and will enable him to use and to select tubes for specific applications with considerable assurance.

5-1. Effect of Gas Upon the Diode Characteristic.

Essentially the addition of certain gases to the envelope of a high-vacuum electron tube converts the tube from a variable resistance to a switch. Relatively few gases are satisfactory for the purpose, and the [illegible] gases helium, argon, neon, krypton, and zenon. Hydrogen has been used for some applications. Many gases cannot be used because they "poison" the oxide coated cathodes used in most tubes, reducing the available emission current. Gas pressures used in gas-filled tubes are roughly in the range .003 to 0.2 millimeters of mercury.

The fact that low pressure gas added to the envelope of a high-vacuum diode gives to the tube the properties of a switch may be very briefly explained. The diode characteristic shown in Chapter I, Figure 1-2, is typical of the high-vacuum diode, and of the triode for a constant grid voltage. If the anode voltage, Figure 1-2, is increased sufficiently to draw to the anode all of the available emission current from the cathode, the diode characteristic curve would flatten horizontally at this voltage and would remain flat at higher voltages since additional current is not available. This behavior is illustrated by Figure 5-1, where the current is shown as emission limited at a voltage e_b'. The cathode of the tube operates at a fixed heater voltage, and therefore just as much current is available at anode voltages, less than e_b' as at voltages greater than e_b'. It will be shown quantitatively in a later chapter that the anode current is a definite function of the anode voltage for values of

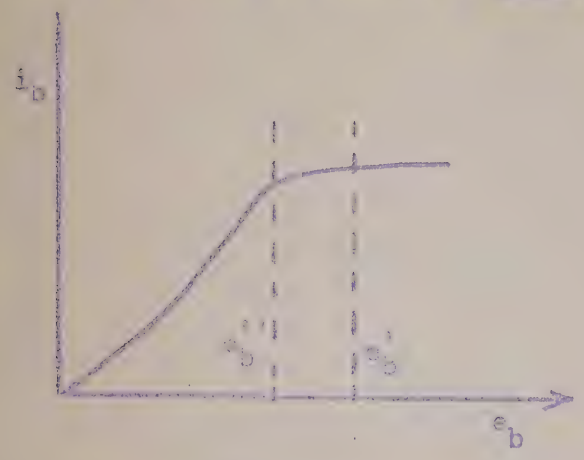


Figure 5-1.
Emission limitation of anode current for high-vacuum diode.

the anode because the electrons already on their way to the anode exert large repulsive forces on electrons very near the cathode, forcing many of them to return to the cathode. This effect is described by saying that the current is limited by space charge. Thus, in Figure 5-1, the anode current is limited by space charge for voltages less than e''_b, and by the ability of the cathode to emit electrons at its operating temperature for voltages greater than e'_b. The region between e'_b and e''_b is a transition region between the two conditions.

If now mercury vapor is added to the high vacuum tube, enough mercury atoms are ionized by electron collision to provide the positive ions which neutralize the electron space charge. Such neutralization is possible because the positive ions move very slowly compared with the electrons, and move toward the negative cathode. The restraining effect of electron space charge upon electron flow from the cathode is removed at very low voltages, of the order of 10 to 20 volts for mercury, and the full emission current is made available at 10 or 20 volts instead of 400 volts, for example. As soon as the anode voltage is increased enough to provide the necessary positive ions, the full cathode emission current suddenly becomes available. Thus the diode behaves as a switch, open for anode voltages less than the critical value, closed for anode voltages equal to or greater than the critical value.

5-2. Volt-ampere Characteristics of the Gas-filled Diode.

Two or three general classifications of gas-filled diodes are in general use, depending upon the mechanism of electron release at the cathode. For present use, the tubes will be classified as cold cathode or as hot cathode tubes. The characteristic of a typical cold cathode gas-filled diode is shown in Figure 5-2. The tube begins to conduct at around 120 volts, but after the tube "fires" the operating range of plate current extends from 5 to 30 or 35 milliamperes at an anode voltage of 105 volts. The flow of current in the tube is accompanied by a luminous discharge of color depending upon the nature of the gas used. The particular value of current after discharge is initiated depends not at all upon the tube but upon the external circuit. It is therefore necessary to include sufficient circuit impedance in series with the tube to limit the current to values within its rating.

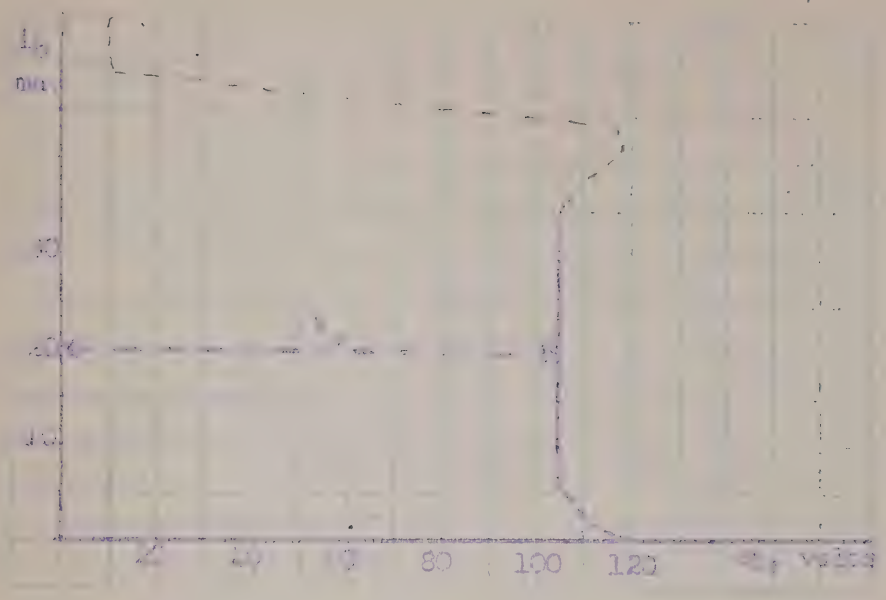


Figure 5-2.
Plate current-plate voltage characteristic of the VR 105 cold cathode tube.

The constant voltage drop V_o across the gas tube during conduction is 105 volts for the cold cathode tube of Figure 5-2. The particular value of V_o for a given tube depends upon the tube design and the nature of the gas used. The thermionic or hot cathode gas tube differs from the cold cathode tube particularly as to the magnitude of V_o and the respective current ratings. A typical mercury vapor diode characteristic for a hot cathode tube is shown in Figure 5-3. The tube drop V_o is 15 volts over the entire operating range of plate current.

Figure 5-3.
Characteristic of a hot cathode mercury vapor diode.

Cold cathode tubes are also referred to as glow discharge

Cold cathode glow discharge tubes are quite rugged and are being designed for switching operations in communication applications to
tube is not so durable because of its heated cathode. The cathode coating used for hot cathode tubes is nearly always a barium or strontium or mixed barium and strontium oxide coating. It is found experimentally that oxide coatings can withstand ionic bombardment from mercury ions without damage if V_b remains below about 22 volts. If sufficient electron
voltage, the tube drop quickly falls to 10 or 15 volts when anode voltage is applied. However, if the cathode has not been given sufficient time to reach the rated emission temperature, the available current may not
of the safe value of 22 volts, so that the cathode may be destroyed. Therefore, the manufacturer always specifies a preliminary heating time for hot cathode gas tubes to ensure adequate emission before the application of plate voltage.

Rating of Gas-Filled Diodes

Gas-filled diodes are designed primarily for rectifier service. The circuit of a half-wave rectifier is shown in Figure 5-4, in which R_L is the load resistor, and the inductance L and capacitor C are used to filter the d-c output across the load. The dot near the cathode indicates that the tube is
designed to hold the circuit open against the inverse peak voltage of the transformer secondary when the anode is negative. Such tubes are given the following ratings by the manufacturers:

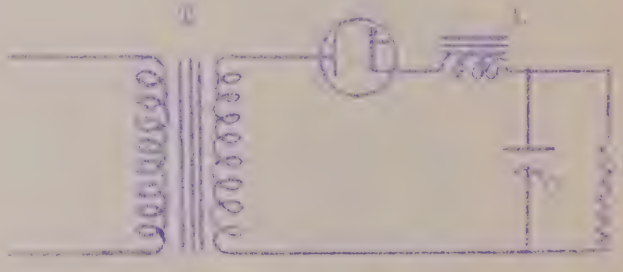


Figure 5-4

(1) Filament or heater current and voltage and filament heating time before the application of anode voltage.

(2) Tube voltage drop, or the voltage across the tube, anode-
mercury-vapor tube it is usually between 10 and 20 volts, and may be assumed to be 15 volts in consideration of circuit behavior.

(3) The maximum instantaneous anode current is specified, and is usually about four or more times the maximum average anode current. A direct current equal to the maximum average anode current may be continuously delivered by the tube without injury provided that the maximum values of the peaks of the current wave during the conducting period are not in excess of the maximum instantaneous anode current rating. The maximum time of averaging the anode current is specified by the manufacturer. For example, the 323A Western Electric mercury vapor and argon-filled tube is rated 1.5 amperes maximum average anode current with time of averaging 5 seconds or less. The maximum instantaneous anode current is 6.0 amperes. If 2.5 amperes flow for three seconds and no current flows for two seconds of the five, the average current will be $(3 \times 2.5)/5 = 1.5$ amperes over the five seconds; if five amperes flow for 1.5 seconds of each five seconds, $I_{AV} = (5 \times 1.5)/5 = 1.5$ amperes. However, if 6 amperes flow for 5 seconds, and [illegible] this cycle of operations is repeated, the average current is $(5 \times 6)/20 = 1.5$ amperes over the 20-second period of the cycle, but the tube rating is greatly exceeded since during the maximum time of averaging the anode current (5 seconds) the average anode current is $(5 \times 6)/5 = 6$ amperes. Fundamentally, the rating of tubes like other electrical apparatus, depends upon the ability of the tube to rid itself of the heat generated at anode or grid. In the example quoted, the approximate rated tube voltage drop is 15 volts. If the tube operates on the 5 amperes -1.5 seconds, zero amperes -3.5 seconds cycle, the number of joules or watt-seconds of energy dissipation at the anode in 20 seconds would be $5(15)(1.5)(4) = 450$ watt-seconds. For the 6 amperes -5 seconds, zero amperes -15 seconds cycle, the plate energy dissipation in 20 seconds is $6(15)(5)\cdot1 = 450$ joules. The total energy involved is the same in each case, but the [illegible] at which heat energy may be radiated, conducted, or convected away from the anode of this tube is $(1.5)(15) = 22.5$ joules per second, according to its rating. For the 5 - 1.5, 0 - 3.5 cycle, the average rate is not exceeded over the 5, or over the 20-second interval, but in the 6-5, 0-15 cycle, the average dissipation rate is $6 \cdot 15 = 90$ joules per second over the first five seconds. As the anode may be expected to get rid of only 22.5 joules per second during this interval, heat accumulates and the probable result would be the cracking of the glass envelope around the base, and the loss [illegible]

(4) The maximum inverse peak voltage is the maximum instantaneous [illegible] and without breakdown when the anode is negative. For the 323A previously mentioned, this rating is 500 volts.

(5) Deionization time is the time required for recombination of ions with electrons after the anode voltage becomes negative. For the Western Electric 323A, the nominal deionization time is 1000 microseconds. [illegible]

$= 220 \sqrt{2} \sin (\pi + 0.377)$

$= -220 \sqrt{2} \sin 0.377 = -315$ volts.

This voltage might be sufficient to initiate a glow discharge in the reverse direction if sufficient ions remained in the tube. It is obvious that the deionization time will limit the frequency at which the tube will operate successfully as a rectifier.

Grid-Controlled Gas-Filled Tubes

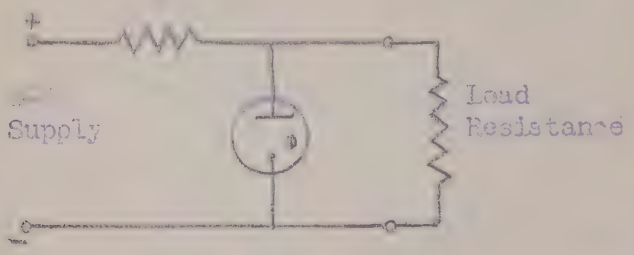
Two types of gas-filled triodes are in common use. The grid-glow tube operates with a cold cathode and a glow discharge, the thyratron has a hot cathode and an arc discharge. In both types, the anode current is limited to the very short interval of the positive value of anode voltage at which ignition will be initiated between cathode and anode. After ignition or breakdown occurs, the grid becomes surrounded by a sheath of positive ions so that its field does not extend beyond the sheath, and the grid loses control over the discharge. It merely controls the voltage at which current begins to flow, and so establishes the point in time when the tube fires. The same statement applies somewhat less emphatically to grid-glow tubes. For both grid-glow tubes and thyratrons an additional rating is necessary -- the [illegible] voltage, which is the [illegible] for which the grid can prevent the ignition [illegible].

The construction of a Westinghouse WL-631 thyratron is illustrated [illegible]. The cathode shown in (a) is of the inward radiating type, open only at the top, and heated indirectly by a heater element in the inner cylinder. The outside of the inner cylinder, the anode, and the inside of the outer cylinder are coated with a highly emissive oxide layer. The assembly of anode, grid and cathode gives an arrangement such that the grid almost completely shields the cathode, and is provided with a baffle between anode and cathode. For decreased [illegible] thyratron grids frequently are made with regularly spaced perforations in the grid to allow ion recombination as rapidly as possible after arc extinction.

5-5. Glow Tubes and Applications

Construction and ratings of glow tubes are so very different for tubes are used as light sources, voltage regulators, rectifiers, relaxa- is a glow tube used as a light source, as are the glow tubes of the strobotac

A very important application of glow tubes is that of voltage regulation. The d-c supply voltage of Figure 5-6 may be slightly vari- tic of the tube is shown in Figure 5-2. Since the anode voltage remains practically constant for a wide range of current, the voltage across the crease of the supply voltage, the increase in current will flow through the tube, the voltage across the load resistor remaining fixed.



Glow tube used as a voltage regulator.

The electrode arrangement of the glow tube of Figure 5-2, the tubes may be greatly reduced by providing a projection extending from the cathode almost to the anode. As current increases, progressively larger areas of the cathode (outer cylinder) become covered with glow.

Figure 5-7.

$$(E - e_c)\,dt = cR\,de_c$$

$$\int \frac{dt}{cR} = \int \frac{de_c}{E - e_c}$$

$$q = Ce_c$$

$$i = \frac{dq}{dt} = C\frac{de_c}{dt}$$

$$E = iR + e_c$$

$$E = cR\frac{de_c}{dt} + e_c$$

$$\frac{t}{RC} = \ln(E - e_c) + K$$

$t = 0$, $e_c = 0$

$$K = -\ln E$$

$$\frac{t}{RC} = -\ln(E - e_c) + \ln E$$

$$= \ln\left(\frac{E}{E - e_c}\right)$$

$$t = RC\ln\left[\frac{E}{E - e_c}\right]$$

$$e^{\frac{t}{RC}} = \frac{E}{E - e_c}$$

$$e_c = E - Ee^{-\frac{t}{RC}}$$

$$e_c = E(1 - e^{-\frac{t}{RC}})$$

A glow tube connected in parallel with a circuit element serves to protect that element from voltages higher than its voltage rating.

In the circuit of Figure 5-8, a glow tube is shown connected as a relaxation oscillator. When the switch S is first closed, the capacitor C begins to charge through resistance R, and the voltage e_c across tube and capacitor rises until the ignition potential of the tube is reached. At this voltage the tube breaks down, the capacitor discharges through the tube, and the voltage e_c begins to rise again. If the voltage across the capacitor is plotted against time, Figure 5-9, it will be seen that across the capacitor is plotted against time. Since the voltage e_c is a transient voltage at the terminals of a charging or discharging capacitor, it is not difficult to obtain an approximate formula for the frequency of oscillation. The charging current may be expressed as

$$i = \frac{dq}{dt} = C \frac{de_c}{dt}$$

If E = the d-c supply voltage, then

$$E = RC \frac{de_c}{dt} + e_c \quad (5\text{-}1)$$

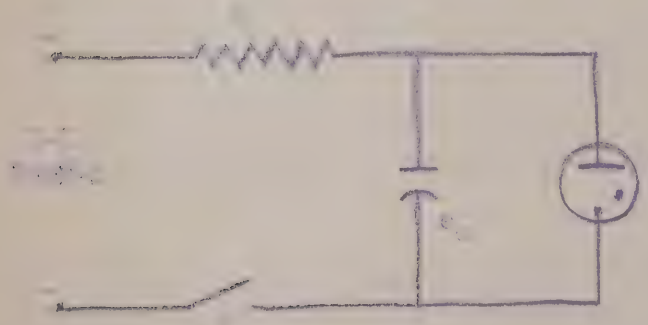


Figure 5-8.
Relaxation Oscillator

The solution of (5-1), with zero initial condenser voltage, is

$$e_c = E\left(1 - \epsilon^{-t/RC}\right) \quad (5\text{-}2)$$

If the discharge time is neglected, the frequency of the oscillation obtained from equation (5-2) and from Figure 5-9 is

$$I = I_1 - I_2$$

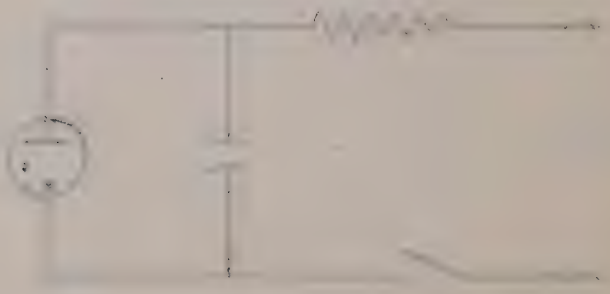
$$I_1 = I_{01} \exp\left(\frac{E_F - E_1}{kT}\right)$$

$$I_2 = I_{02} \exp\left(\frac{E_F - E_2}{kT}\right)$$

$$I = I_{01} \exp\left(\frac{E_F - E_1}{kT}\right) - I_{02} \exp\left(\frac{E_F - E_2}{kT}\right)$$

$$I = I_{01} \exp\left(\frac{E - E_{0K}}{E - E_1}\right)$$

$$f = \frac{1}{RC \ln\left(\frac{E - E_2}{E - E_1}\right)}$$



$$RC \log \frac{\quad - E_e}{\quad}$$

It is important that the charging rate of the R-C circuit of Figure 5-8 be slow enough relative to the discharge rate of the condenser through the tube that the voltage e_c will quickly fall below the value required to maintain the glow. Otherwise, the voltage of the condenser may reach a steady value intermediate between E_e and E_i, while the current through R is conducted continuously through the tube. In such a case the condenser is said to "block" the oscillation. Blocking is avoided by choosing R large enough that the available continuous current through the tube when conducting is not large enough to maintain the discharge.

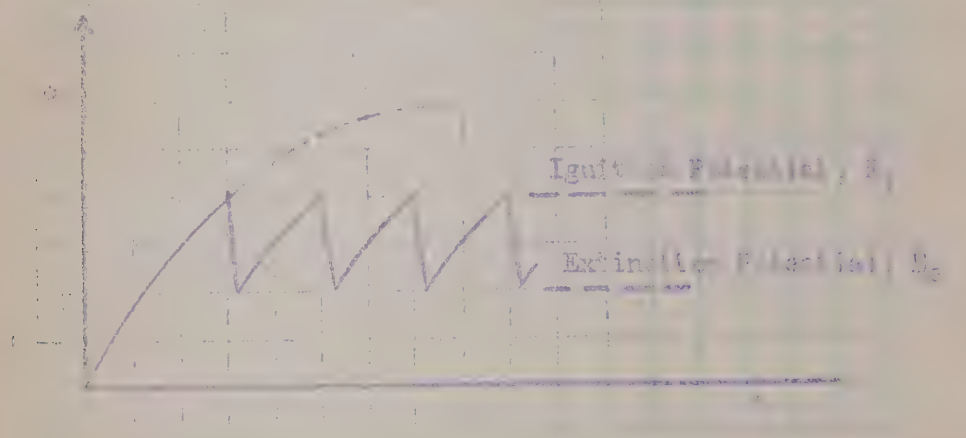


Figure 5-9.
Saw-tooth Wave Form of Relaxation Oscillator.

equation (5-3), by adjusting the product RC. The oscillation frequency can high enough to be in the upper audio range where deionization time imposes an upper limit on the frequency.

The wave shape of the relaxation oscillator is controllable by adjusting the voltage of the supply circuit. If a linear rise of potential before breakdown is desired, a high potential is used so that breakdown occurs relatively low down on the e_c vs. t curve.*

5-6. Arc Tubes and Applications

In tubes designed to carry relatively high currents -- several amperes -- at low tube voltages, arcs rather than glows are required. Arc discharges are obtainable by providing, as previously mentioned, a source of electrons at the cathode. Two types of electron source are in common use. These are the thermionic, oxide-coated cathode, and the mercury pool cathode. Thermionic cathodes are used for rectifier and special control tubes carrying currents ranging from a few milliamperes to several amperes. The mercury pool tube is a very rugged device capable of supplying hundreds of amperes but requiring special arc-initiating devices, all of which are designed to initiate or maintain what is called a cathode spot upon the surface of the mercury. Although an adequate and universally accepted explanation of the mechanism of electron emission from a cathode spot is still to be found, it is known that such a spot is capable of delivering almost unlimited currents. In the large metal-tank, mercury-arc power rectifiers used in supplying large amounts of d-c power in electric traction, elevator, and other similar applications, a special auxiliary anode is required to operate continuously to maintain the cathode spot. A number of anodes, one or more for each phase of a 3-phase system, are commonly operated from the same pool. It is the function of the auxiliary anode or "keep-alive" circuit to maintain the cathode spot at all times. The operating characteristics of this type of rectifier will be described further in Chapter VII.

The efficiency of a tube depends upon its energy dissipation. With their almost completely enclosed and inward-radiating cathodes and their low tube drop, the mercury-vapor arc tubes are the most efficient of the gas-discharge tubes.

Applications of the arc tubes are extremely numerous, and only a few fundamental applications will be selected for discussion. These include rectifiers, control applications, and inverters.

5-7. Tungar Rectifiers

The tungar rectifier is a two-electrode, gas-filled tube using a thermionic cathode and differing from other rectifiers in one important feature -- the use of a relatively high-pressure gas, usually argon, or [illegible] The gas pressure is usually between one and five millimeters of mercury. The cathode is a very heavy filament of thoriated or oxide-coated tungsten, operated at high temperature to provide the large currents -- 6 to 8 amperes -- required in battery charging, which is the principal application of these tubes. It is the function of the high pressure gas in the tube to prevent the evaporation of the coating of the filament, as well as to supply the ions required for removing the limitation due to space charge.

5-4. Thyratron Characteristics and D-C Control

The grid of the thyratron has only "trigger" control over anode current. The adjustment of grid bias serves to select a particular value of anode voltage at which the tube fires. When direct anode voltages are used, the grid may be biased to hold the anode circuit open, and a separate circuit may be employed to control the bias such that the tube fires as the result of some event actuating the grid-control circuit. Before giving circuit diagrams and specific examples, it is necessary to examine the grid-control characteristics of typical thyratrons. In Figure 5-10 are the grid-control curves for a typical negative-grid thyratron, the

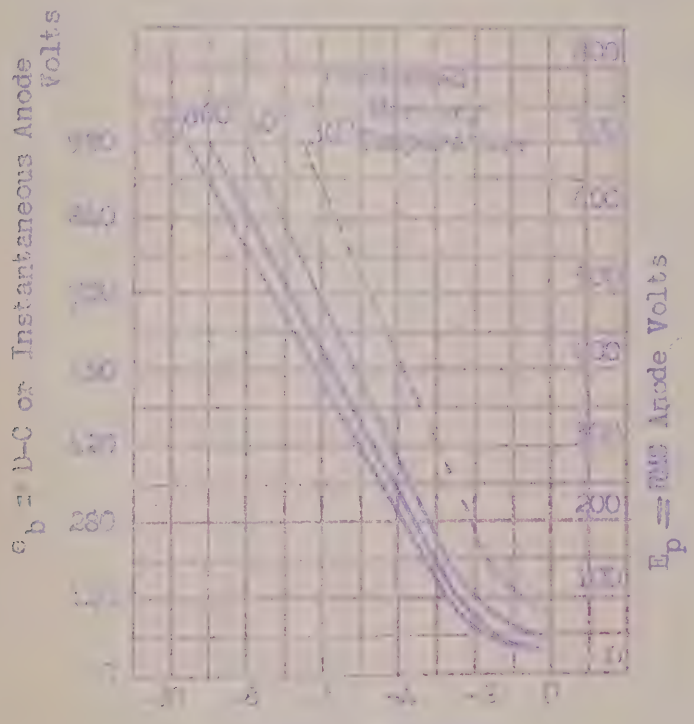


Figure 5-10.

Average control characteristics of a WL-631

Westinghouse WL-631. Similar characteristics for a positive-grid thyratron, the General Electric FG-33 are shown in Figure 5-11. In the latter tube, shielding of the anode by the grid is complete enough that positive grid voltage is required to fire the tube. A tube in which the type of grid-control characteristic, positive or negative, may be selected is illustrated in Figure 5-12. This is a four-electrode gas tube known as a shield-grid thyratron. The sketch was drawn from a Westinghouse WL-632 tube. A few control characteristics for this tube are shown in Figure 5-13.

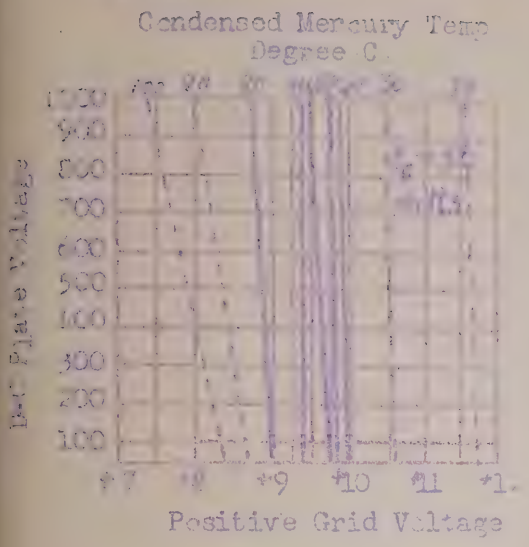


Figure 5-11.

Grid control characteristics of the Type FG-33 Thyratron.

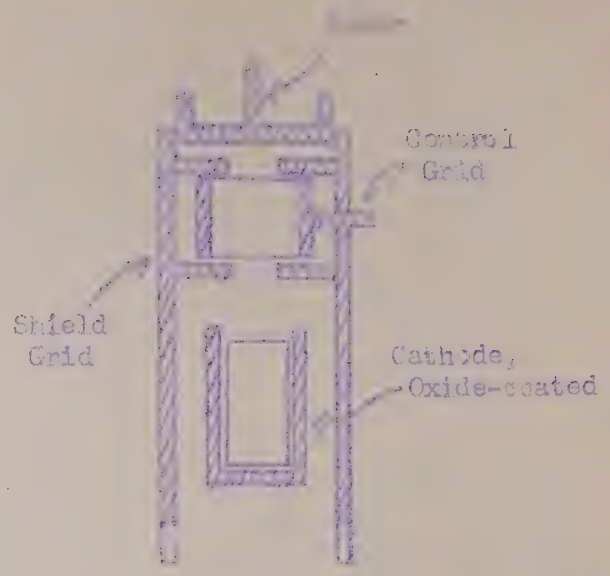


Figure 5-12

Electrode arrangement of a shield-grid thyratron.

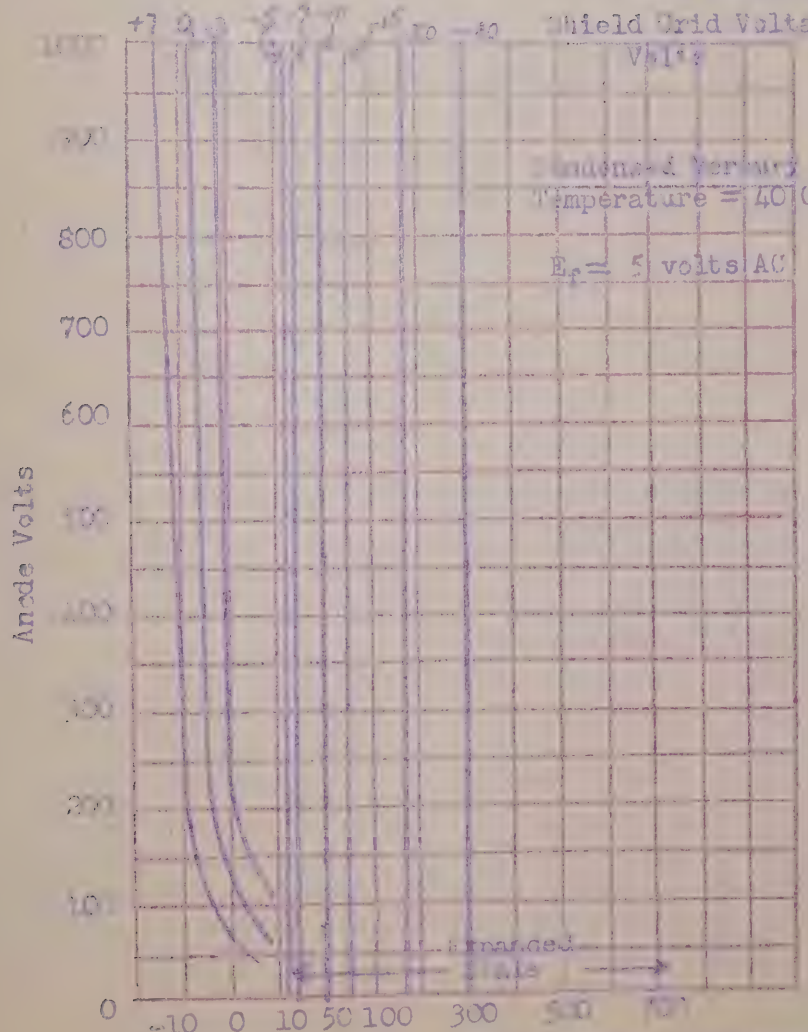


Figure 5-13.

Average control characteristics of a WL-632.

When direct voltages are used on thyratron anodes, the grid loses control after triggering the tube, and if the anode current is to be reduced to zero, either the anode circuit must be opened by a switch, or by electrical means. Means of d-c control vary in the methods used to interrupt the anode current, but most of these methods employ the voltage of a condenser to apply a momentary positive voltage at the cathode with respect to the anode, preventing electrons from leaving the cathode long enough for deionization to take place, and for the grid to regain control. One such circuit is that of Figure 5-14. When switch S_1 closes, the grid voltage becomes zero and the tube fires. The current through R_1 is given by

$$i_1 = \frac{E - V_o}{R_1} \qquad (5\text{-}4)$$

where V_o is the tube drop when conducting. The current through R_2 can be obtained by solving the circuit equation

$$E = R_2 i_2 + e_c + V_o \qquad (5\text{-}5)$$

since $i_2 = C \frac{de_c}{dt}$, where e_c is the instantaneous potential across the condenser terminals. The resulting value of i_2 is

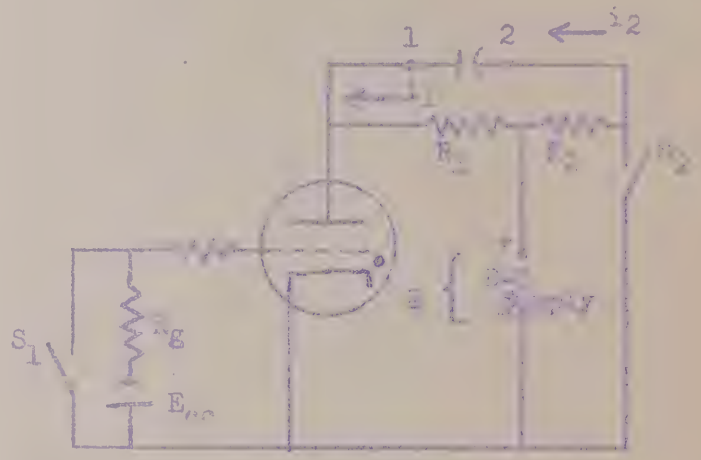


Figure 5-14.

Parallel control of thyratron anode current.

$$i_2 = \frac{(E - V_o)}{R_1} \varepsilon^{-\frac{t}{R_2 C}} \qquad (5\text{-}6)$$

The charging current i_2 flows through the tube in the forward direction from anode to cathode, and the ultimate voltage across the condenser will be just that across R_1, or $E - V_o$. If switch S_2 closes after the condenser charging current has become approximately zero, the full voltage of the condenser, $(E - V_o)$, is impressed across the tube, but since terminal 2 is positive, terminal 1 negative, the anode becomes negative with respect to the cathode; the cathode emission current is held at the positive cathode, deionization occurs, and the anode current is interrupted. Meanwhile, if S_1 has been opened, the grid regains control, provided that the anode voltage does not reach the reignition value before sufficient deionization has occurred. With S_2 closed and the tube not conducting, the condenser begins to charge in the opposite direction through R_1 and ultimately would reach the supply potential E, with terminal 1 positive. A graph of condenser voltage versus time for the sequence of

events just described is shown in Figure 5-15. The original direction of condenser voltage, with the potential of terminal 2 greater than that of terminal 1 by $(E - V_o)$ volts, is shown as a positive voltage, and the reverse, with terminal 1 at a higher potential than 2, is shown as negative on the graph. At time t_1, switch S_1 closes, the tube fires and the condenser begins to charge. At t_2 switch S_2 closes, S_1 opens, and the condenser begins to discharge. At t_3 the condenser discharge is complete, and the condenser begins to charge in the opposite direction. If, at t_4, switch S_1 closes (S_2 opens), the tube fires and the condenser again discharges. At t_5 the entire sequence of events begins to repeat.

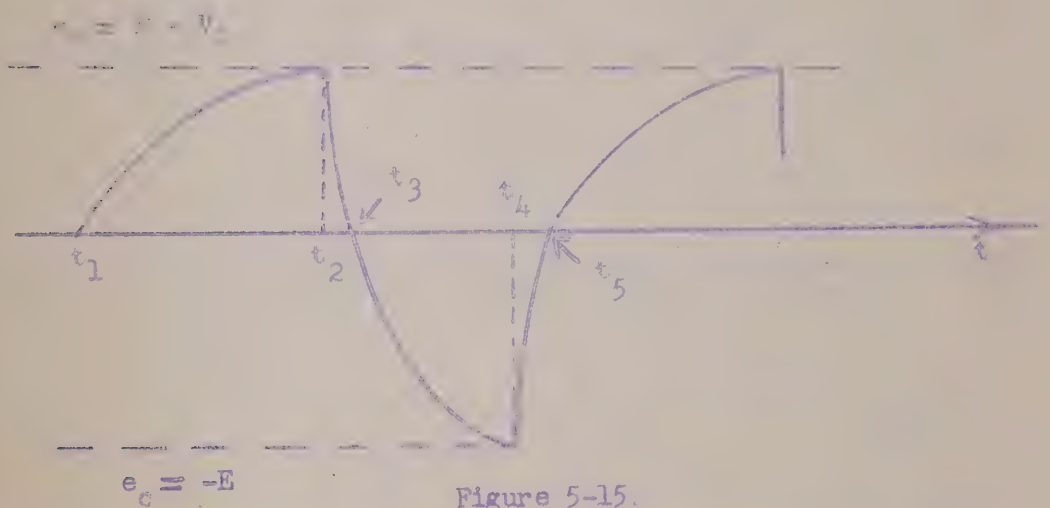


Figure 5-15.

Condenser voltage-time curve for the circuit of Figure 5-14.

In applications of the basic circuit of Figure 5-14, switch S_1 may be controlled, as a relay, by a photo-tube or high-vacuum tube circuit or both. If S_2 is replaced by another thyratron, the circuit is that of Figure 5-16. The graph of Figure 5-15 applies to the circuit of Figure 5-16 except that the lower extreme value of voltage is

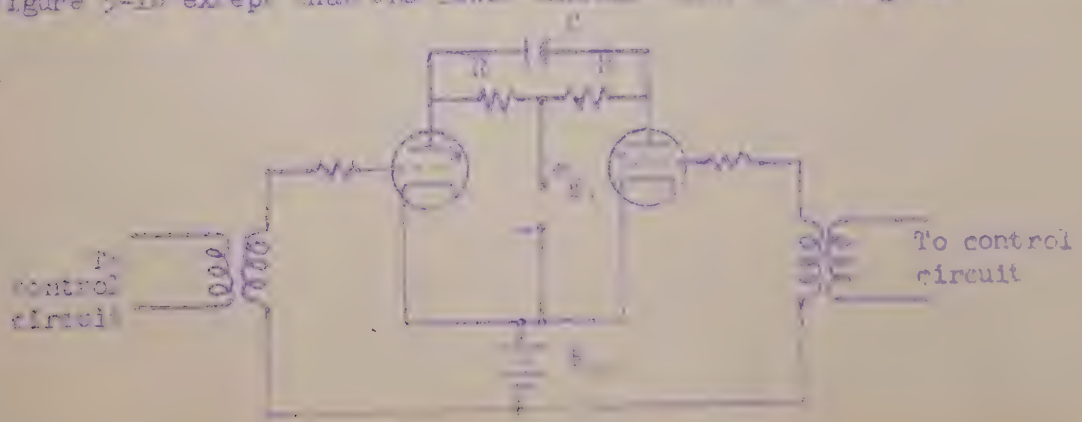


Figure 5-16

Basic counter or inverter circuit.

$e_g = -(E - V_o)$ instead of $-E$, since the thyratron acts as an imperfect switch requiring a voltage of V_o volts to keep it closed. The circuit of Figure 5-21 is [illegible] be described later, and has been used for high speed counting

Another method of d-c control is that illustrated by the circuit of Figure 5-17, in which the condenser C is in series with the inductive load and the supply voltage. The grid current is the charging current $C \frac{de_c}{dt}$ of the condenser. Before firing, the tube itself acts as a condenser of small capacitance, plate-to-cathode, in series with C. Since the internal [illegible] will be small, the greater voltage q/C_{pk}, will exist across the tube. When the tube fires as a result of a positive grid impulse from the control circuit, the following equation is [illegible] the plate circuit

$$E = RC \frac{de_c}{dt} + LC \frac{d^2e_c}{dt^2} + e_c + V_o \qquad (5\text{-}7)$$

Since current [illegible] circuit will be open when the condenser becomes fully charged.

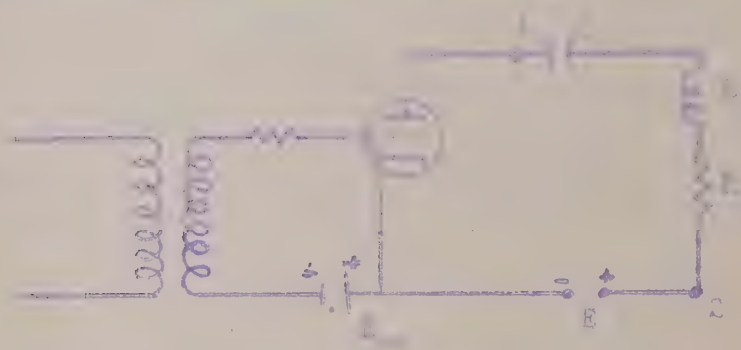


Figure 5-17.
Basic series control circuit.

The solution of (5-7) for the condition $\frac{R^2}{4L^2} < \frac{1}{LC}$ is

$$e_c = [illegible] - (E - V_o)$$

where

$$\alpha = \frac{R}{2L} \qquad \beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

If $C \gg C_{pk}$, then $\frac{q_o}{C} \ll \frac{q_o}{C_{pk}}$; where q_o = the initial condenser charge before the tube fires. Neglecting the initial value of e_c before the tube fires, the following boundary conditions may be used:

(1) $e_c = 0$ at $t = 0$

(2) $i = 0$ at $t = 0$

From these,

$$i = -(I_1 - I_2)$$

$$e_c = -\frac{i}{C}(R - r_d)$$

and

$$e_c = (E - V_o)\left[1 - e^{-at}\left(\frac{b \cos bt + a \sin bt}{b}\right)\right] \qquad (5\text{-}8)$$

The solution in (5-8) does not take into account the fact that the voltage across the tube will change suddenly from V_o to a much larger value when the current i becomes too small for deionization to occur. However, the equation does show that the condenser voltage oscillates about the value $(E - V_o)$, and would ultimately approach that value if the tube continued to conduct and the tube drop remained constant at V_o. By differentiation of equation (5-8) the maximum value of e_c may be shown to approximate $2(E - V_o)$ for R small. The graph of Figure 5-18 shows the theoretical variation of e_c with time after the tube fires. For R large, the circuit is not oscillatory, and the maximum value of e_c is the limiting value, $(E - V_o)$.

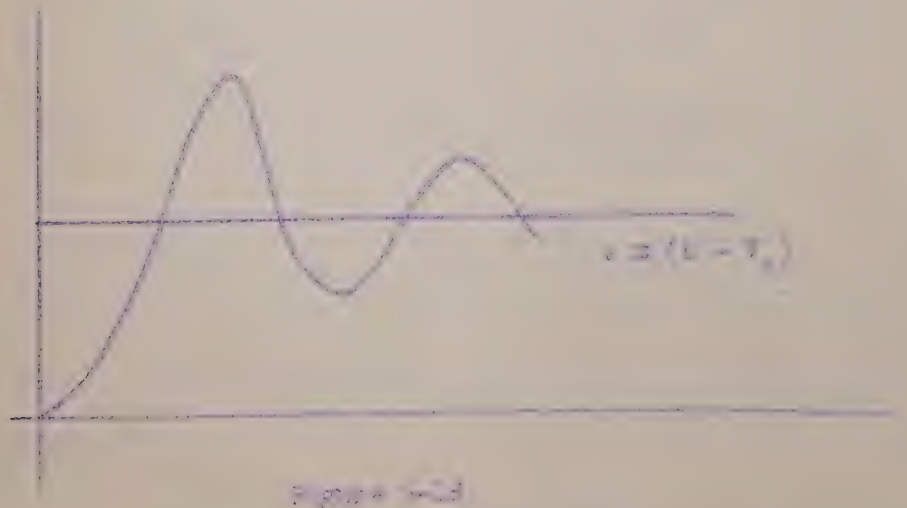
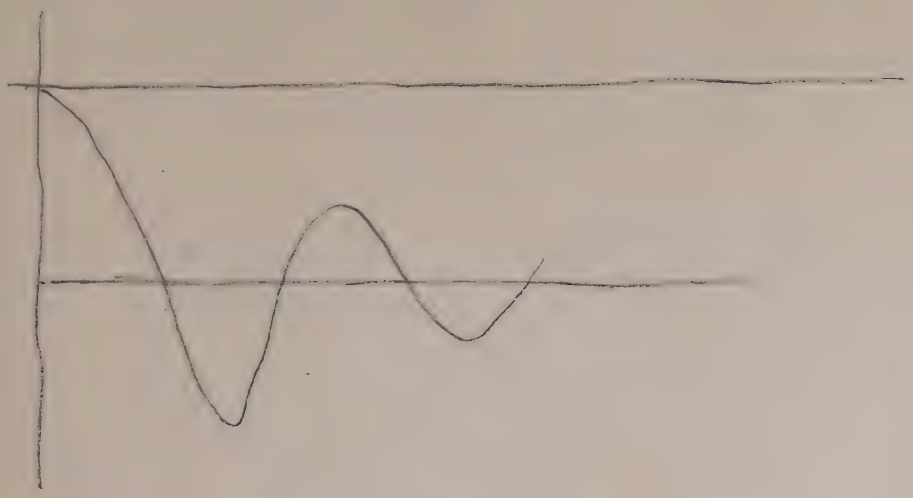


Figure 5-18

Sketch of equation (5-8) for small resistance R.



( ) [( )] (2-8)

In order for the tube to fire again, the condenser must be discharged. This may be accomplished by the use of another thyratron If the second thyratron is connected with its cathode to terminal 1 and its anode to terminal 2, Figure 5-17, the condenser would discharge through tube No. 2 and load impedance, but if tube No. 1 were still conducting, the d-c supply would be short-circuited. In order to avoid this, the circuit of Figure 5-19 may be used.

In Figure 5-19, if tube No. 1 is conducting when tube No. 2 fires, but if at that time the condenser is nearly charged, then the condenser discharge current flows in the direction i_{32} through the inductance L_1. The induced voltage in L_1, opposing this current is such as to make terminal 1, and therefore the anode of tube No. 1 negative, so that the arc is extinguished in tube No. 1. If L is the primary of a transformer, the circuit becomes that of a series inverter in which an alternating-current output is available at the secondary of the transformer. The grid control circuits may be controlled either from a separate a-c source or from the proper selection of constants to provide an oscillating, self-excited circuit. In the former case, the frequency of the a-c output is controlled by the frequency of the grid supply; in the self-excited circuit, it is controlled by the circuit constants.

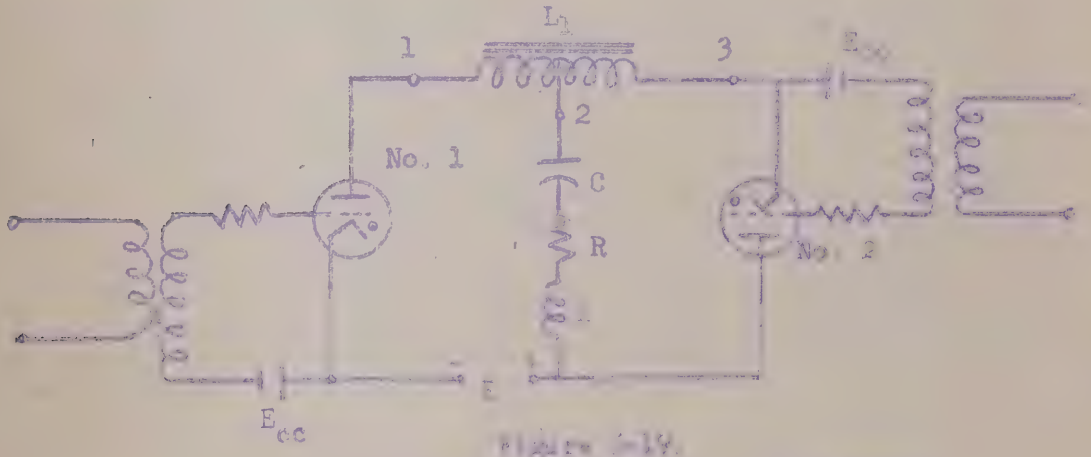
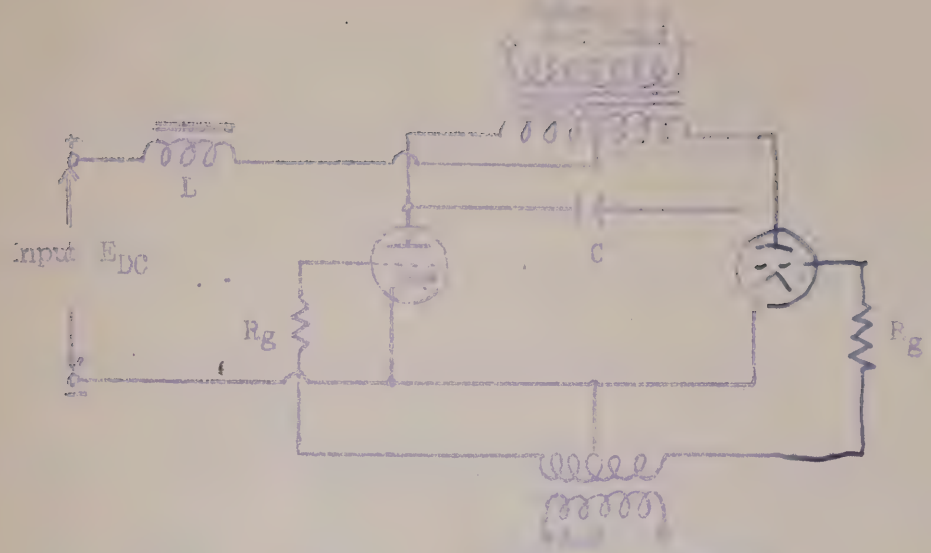


Figure 5-19.

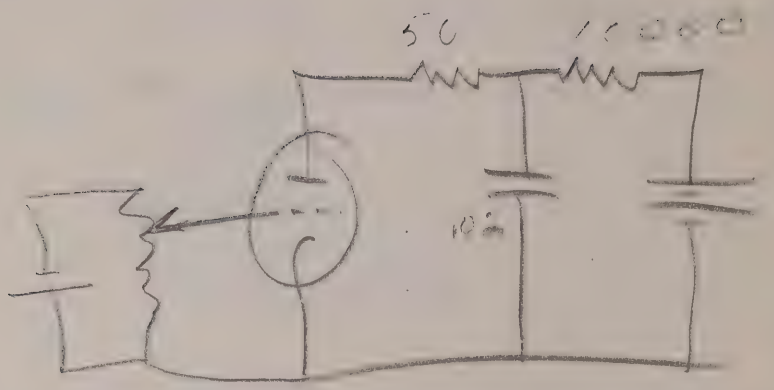
Series Control Circuit or Series Inverter using two Tubes.

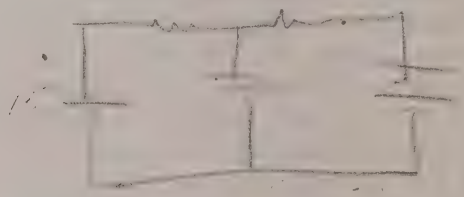
The circuit of Figure 5-20 is that of a parallel inverter. An inverter is a circuit providing alternating-current power from a direct-current source. The behavior of the circuit has been briefly explained in the discussion of Figure 5-16. Inverters have been developed which, technically, would make possible long-distance d-c power transmission, and this may eventually become an important electronic application in the power field. Power would be generated as alternating current, perhaps at some remote waterfall, such as Victoria Falls in Africa. The voltage



Parallel Inverter.

would then be raised by high-voltage step-up transformers to 300 kv, or more. This would be rectified, and then transmitted by means of a d-c transmission line. Because of the capacitance and inductance of long transmission lines, it is impossible to transmit even commercial power frequencies a distance of 300 miles, approximately the distance of Victoria Falls from the nearest industrial center. But with d-c transmission, power could be transmitted at high voltage much longer distances. It is used at the receiving end, however, the high voltage must be reduced. The first step in this process is efficient inversion to a.c., and then transformation downward to the 600, 440, 2[illegible], or 11 volts now used for distribution and utilization. As has been stated, such a process is entirely feasible, technically. It has not as yet become justified, economically.





$15 + 30 (\quad \times 166 + \rho_c$

$15 + 30(i_{b2} + i_{c2}) = 300 - (\quad) i_{b1}$

$i_{b1} = .1875$ amps.

$e_g = 3.75$ volts

Assume that tube cannot conduct plate current below .3 amps.

C (dia - pc by $\frac{374.25}{374.25 - 344.75}$

...-1. An RCA VR-105 tube is used as a voltage regulator in the circuit of Figure 5-5. If the load current varies from a minimum of 40 ... (Note the operating range, Figure 5-6.) D.C supply voltage = 125 volts

...-2. Using the value of R determined in Problem 5-1, how much supply voltage fluctuation is possible without changing the output voltage?

...-3. Derive completely equation (5-3).

...-4. A thyratron is used in the relaxation circuit of Figure 5-4. The tube is a WL-631, Figure 5-10, operating at a condensed mercury temperature of 40° C. A 50-ohm resistor is used in the anode circuit of the thyratron, R of Figure 5-4, is 10,000 ohms, C = 0.02 mfd. ...

(a) Grid bias voltage required.
(b) Frequency of the saw-tooth wave.
(c) The time required, approximately, for the condenser to discharge when the tube fires.

... drop, is 15 volts, E = 250 volts d-c.

(a) When tube No. 1 fires and steady conditions have been reached, what are the currents through the resistors and the voltages, relative to cathode, of the anodes?

(b) Tube No. 2 fires. Draw an equivalent circuit and find how long a time is required for the condenser voltage to pass through zero.

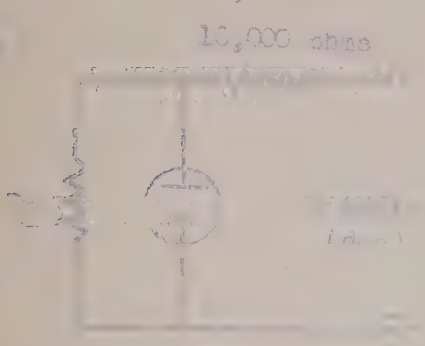
(c) Sketch the voltage across the condenser, assuming each tube fires when condenser voltage reaches 215 volts. Approximate the frequency of the condenser voltage wave.

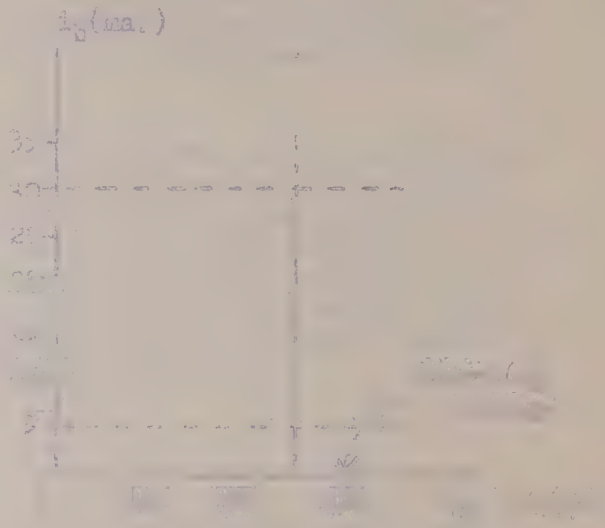
5-6. If the charging current of the condenser in Figure 5-4 is constant and equal to I, show that (neglecting condenser discharge time) the frequency of oscillation of the resulting saw-tooth wave is

$$f = \frac{I}{C\,(E_1 - E_2)}$$

and is therefore independent of the magnitude of the ...

volts, compute the oscillation frequency if $E_1 = 110$ volts, $E_c = 10$ volts, $C = 2$ microfarads.





The characteristic for the glow discharge tube T is given by the $i_b - e_b$ graph.

(a) If $R_1 = 10{,}000$ and $E = 300$ volts, find the voltage across R_1, the current in each resistor, and the tube current, if the tube is conducting.

(b) What are the maximum and minimum values of E such that the voltage across R_1 remains constant and the tube current is above 30 ma. but below 5 ma ?

Chapter VI

Single-phase Rectifiers and Power Supplies

Electric power is generated as alternating current. When direct current power is required, conversion from alternating to direct current is accomplished by use of a commutator on the shaft of the generator, or by the use of rectifiers. By definition,[1] "a rectifier is a device which converts alternating current into unidirectional current by virtue of a characteristic permitting appreciable flow of current in only one direction".

Diodes of copper and of selenium have rectifying properties and are widely used for low voltage applications. A vacuum tube conducts only when the anode is positive with respect to its cathode, and, as shown for the diode of Chapter 4, may be used as a rectifier. The discussion of this chapter will be confined to thermionic high vacuum tube rectifiers and to gas-filled thermionic and mercury-pool cathode tube rectifiers in single-phase circuits. Such equipment may be used to supply d-c power in many applications, particularly to vacuum tube plate circuits.

6-1. The Half-wave Rectifier Circuit.

The tube of Figure 6-1 will conduct current only during that portion of the transformer secondary a-c cycle during which the tube anode is positive. The positive half cycle is shown in Figure 6-1(c). If the tube is a high vacuum diode, the anode current will begin to flow when the anode becomes positive, as indicated in Figure 6-1(e). If the tube is a gas-filled diode, current will not flow until the anode-to-cathode voltage has reached its breakdown value. This occurs when $e = E_i$, $\omega t = \alpha_i$, as shown in Figure 6-1(b) and (e). The idealized tube characteristics are shown in Figures 6-1(d) and (f). It is assumed in the drawings that the breakdown voltage is equal to the tube drop.

The current through the load resistance R of Figure 6-1(a) is unidirectional but pulsating. From the point of view of Fourier analysis of the wave form, the current i has a d-c or average value and many a-c harmonic components. An important measure of rectifier circuit performance is the ripple voltage, which is defined as the alternating component of the unidirectional voltage. The percent ripple is defined as the ratio of the effective or root-mean-square value of the ripple voltage to the average or d-c value of the total voltage, expressed in percent. Other criteria of rectifier circuits used as power supplies are voltage regulation, power output, and efficiency. The definitions of these quantities are the same as for any other electrical equipment to which they may apply.

6-2 The Half-wave Rectifier, High-vacuum Tube Circuit.

An ideal rectifier would have the properties of a perfect switch, closing when the voltage is in the forward or conducting direction, and

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[1] American Standard Definitions of Electrical Terms, p. 79, published by AIEE

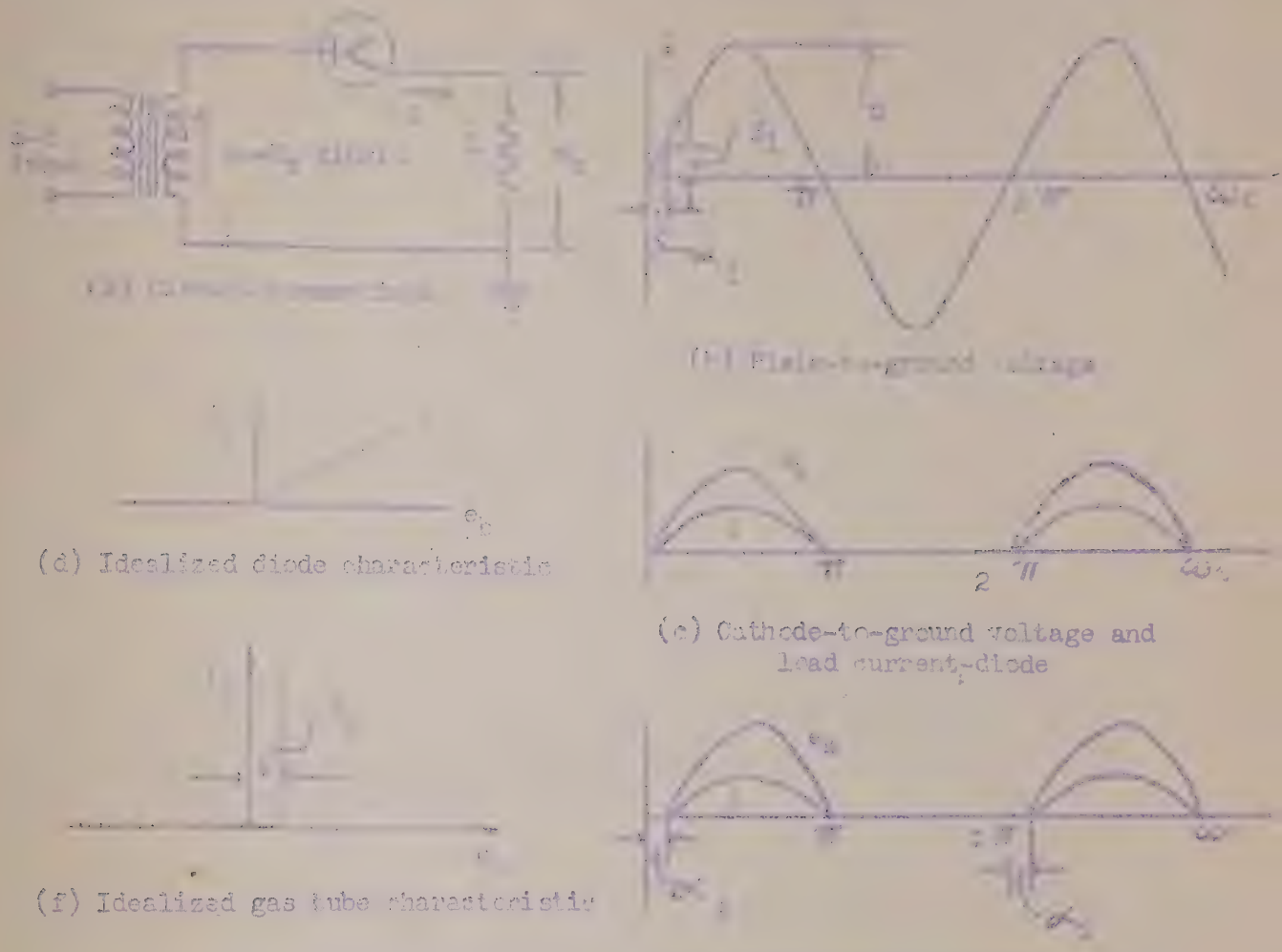

Figure 6-1.

Half-wave Rectifier Circuit and Wavefor s.

spening when the voltage reverses. The idealized tube characteristic of

acteristics. An expression for the current i, approximate because R_b is not a constant, may then be written as follows:

$$0 < \omega t < \pi \quad , \quad i = \frac{e}{R + R_b} = \frac{E_m}{R + R_b} \sin \omega t \qquad (6\text{-}1)$$

...ximum value of the output voltage e_k is then $\frac{E_m}{R}$ R . The average ...r direct current is defined by the relation

$$I_{dc} = \frac{1}{2\pi} \int_{0}^{2\pi} i \, d(\omega t) = \frac{1}{2\pi} \int_{0}^{\pi} I_m \sin \omega t \, d(\omega t) \qquad (6-2)$$

$$I_m = \frac{E_m}{(R + R_b)}$$

...om (6-2)

$$I_{dc} = \frac{1}{\pi} I_m = \frac{E_m}{\pi(R + R_b)}$$

$$E_{dc} = I_{dc} R = \frac{E_m R}{\pi(R + R_b)} = \frac{1}{\pi} \left[\frac{E_m}{1 + \frac{R_b}{R}} \right] \qquad (6-4)$$

If the origin for the load current is chosen as in Figure 6-2, the Fourier analysis of the wave may be carried out in the usual way[2] by evaluating only the cosine coefficients. Because of the choice of origin no sine terms will be obtained. Then

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} i \cos n(\omega t) \, d(\omega t) \qquad (6-5)$$

Equation (6-5) applied to Figure 6-2 yields the expression

$$a_n = \frac{I_m}{\pi} \left(\frac{2}{n^2 - 1} \right) \cos n \pi/2, \quad n \neq 1. \qquad (6-6)$$

For $n = 1$, Equation (6-5) shows that

$$a_1 = \frac{I_m}{2}$$

The Fourier series for the current i is then as follows

$$i = \frac{I_m}{\pi} + \frac{I_m}{2} \cos \omega t + \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t + \frac{2}{35} \cos 6\omega t$$

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[2] E. E. Y., Alternating Current Circuits, Chapter 7

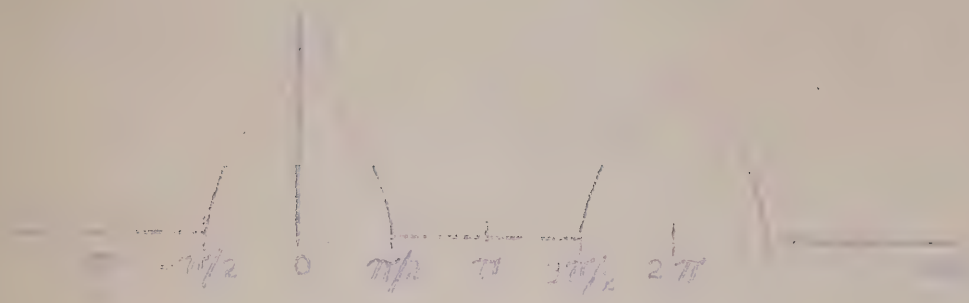


Figure 6-2.
Lead current wave from

$-\pi < \omega t < -\pi/2$, or $\pi/2 < \omega t < \pi$, $i = 0$

The current is shown, by Equation (6-7), to contain the fundamental and all the even harmonics of the supply frequency. If the effective value of the voltage e_k is required, it may be obtained from the current series by [illegible] the fundamental relation [illegible] the effective value, E, of the sum of any [illegible] is given by the square root of the sum of the squares of the average and of the effective values of the harmonic components. Thus if E_0, E_1, E_2, E_4, E_6, etc. are the effective values of the respective harmonic components in the wave $e_k = iR$, then

$$I = \sqrt{E_{dc}^2 + E_1^2 + E_2^2 + E_4^2 + \ldots + E_n^2} \qquad (6\text{-}8)$$

In the same way, the effective value of the [illegible] of the voltage wave is expressed as

$$E_{ac} = \sqrt{E_1^2 + E_2^2 + E_4^2 + \ldots + E_n^2} \qquad (6\text{-}9)$$

From (6-8) and (6-9)

$$I_{ac} = \sqrt{I^2 - I_0^2} \qquad (6\text{-}10)$$

The ripple factor, by definition, is expressed as the ratio

$$r = \frac{I_{ac}}{I_0} = \frac{\sqrt{I^2 - I_0^2}}{I_0} = \sqrt{\frac{I^2}{I_0^2} - 1} \qquad (6\text{-}11)$$

[illegible]

I represents the effective current of the non-sinusoidal wave. For the half-wave vacuum tube rectifier, the effective voltage is obtained by definition, from the relation

$$E = RI = R\sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} i^2 \, d(\omega t)}$$

The current i is expressed as in Figure 6-2. The result is:

$$E = R\left[\frac{I_m}{2}\right] \qquad (6\text{-}12)$$

From (6-4), and (6-12) $E_{dc} = \frac{I_m}{\pi} R$, and

$$\frac{E}{E_{dc}} = \pi/2$$

Then

and the percent ripple is 121%

Examination of Equation (6-4) shows that the circuit load voltage may be expected to vary considerably with variation of R unless R is always much larger than R_b. Equation (6-4) may also be rearranged as follows:

$$I_{dc}(R + R_b) = \frac{E_m}{\pi} \qquad (6\text{-}4a)$$

$$I_{dc} R = E_{dc} = \frac{E_m}{\pi} - I_{dc} R_b . \qquad (6\text{-}14)$$

Equation (6-14) shows that the no-load d-c voltage is equal to $\frac{E_m}{\pi}$, but that the voltage decreases linearly with current at a rate dependent upon the magnitude of R_b. An expression for the voltage regulation in terms of the load current may be obtained from (6-14).

The d-c power output of the rectifier is expressed as the product of the d-c components of current and voltage, or, from (6-3) and (6-4),

$$P_o = E_{dc} I_{dc} = \frac{E_m^2 R}{\pi^2 (R + R_b)^2} \qquad (6\text{-}15)$$

The power supplied by the secondary of the transformer delivering effective current I may be expressed as

ore, from (6-12)

$$I_{dc} = \frac{E_m}{\pi R} = \frac{i_m}{\pi} = \frac{E_m}{\pi (R + R_p)}$$

$$i_m = \frac{E_m}{\frac{1}{2}(R + R_p)}$$

$$\eta = \frac{P_{dc}}{P_{in}} = \frac{4R}{\pi^2 (R + R_p)} = \frac{0.406}{\left(1 + \frac{R_p}{R}\right)} \qquad (6$$

he theoretical maximum efficiency of the half-wave vacuum rectifier is evidently 40.6%. It may easily be shown from Equation (6-15) that the maximum power output occurs for $R = R_p$. For this value of load resistance the efficiency of rectification is only 20.3%. The dependance of the efficiency of rectification upon the load current is conveniently sult is:

$$\eta = 0.406 \left[1 - \frac{\pi}{E_m} I_{dc} R_p\right]. \qquad (6-17)$$

Example problem: The plate resistance of a certain rectifier istic to be a straight line passing through the origin and one point on the characteristic. If $R_p = 500$ ohms, and the tube is connected as a half- rent and voltage (d-c), the power output, and the rectifier efficiency if the transformer secondary voltage is 350 volts (rms)

Solution: $E_m = 350\sqrt{2} = 495$ volts; from Figure (6-1a),

$$I_{dc} = \frac{E_a}{R + R_p} = \frac{\quad}{5500}$$

$$I_{d.c.} = \frac{I_{dc}}{\quad} = 28.5 \text{ ma}$$

$$E_{d.c.} = R\, I_{d.c.} = 143.0 \text{ volts}$$

$$P_{d.c.} = E_{d.c.}\, I_{d.c.} = 4.1 \text{ watts}$$

effective current in the transformer secondary is

$$I = \frac{I_m}{\quad} =$$

$$\eta = \frac{0.406}{500} = \frac{0.406}{1.1} = 0.368$$

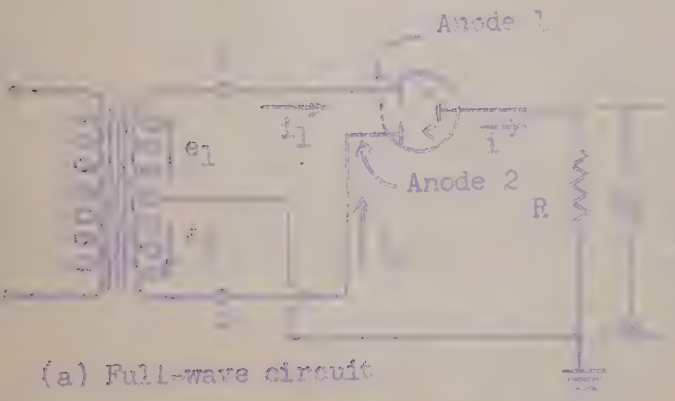
The Full-wave Vacuum Tube Rectifier Circuit.

The negative half cycle of the applied voltage wave of Figure 6-1 may also be utilized if applied to the anode of another tube, or to a second anode in the same tube, connected as shown in Figure 6-3a to the center-tapped seondary of a transformer. When anode 1 is negative, anode 2 is positive and current continues to flow through R in the direction [illegible] winding at any instant is 20 volts, with, for example, anode 2 positive, then anode 2 is 10 volts higher in potential than the ground or mid-tap connection, and anode 1 is 10 volts lower than ground. At every instant, the voltages of anodes 1 and 2 with respect to ground are equal but opposite This may be expressed mathematically by the relations

$$e_1 = E_m \sin \omega t$$

The secondary voltage rise may be expressed as

$$e_{21} = -e_2 + e_1 = 2 E_m \sin \omega t$$



(a) Full-wave circuit

) current, voltage and current.

Figure 6-3
Full-wave Rectifier circuit

The voltage wave forms of Equation (6-18) are shown in Figure 6-3b and the load current and voltage, cathode-to-ground, in Figure 6-3c. As shown in Figure 6-3b, anode 1 conducts during the half cycle in which it is positive, that is below zero and that of anode 2 becomes positive. The currents i_1 and i_2 of anodes 1 and 2 are identical in form, but i_2 is displaced 180 degrees along the time axis from i_1. If an origin is chosen as shown by the dotted line, the Fourier series for i_1 is (from 6-7)

$$i_1(\omega t) = \frac{I_m}{\pi}\left[1 + \frac{\pi}{2}\cos\omega t + \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t + \frac{2}{35}\cos 6\omega t - \ldots\right] \quad (6-\ )$$

and $i_2 = i_1(\omega t + \pi) = \frac{I_m}{\pi}\left[1 - \frac{\pi}{2}\cos\omega t + \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t \right.$

$$\left. - \frac{2}{35}\ldots\right]$$

The current i through the load may be obtained as the sum of $i_1 + i_2$ or

$$i = \frac{2}{\pi} I_m\left[1 + \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t + \frac{2}{35}\cos 6\omega t + \ldots\right]. \quad (6-19)$$

Since the load current does not contain the fundamental, smoothing or filtering [illegible] for the half-wave circuit. The d-c components of the tube currents flow through the secondary in opposite directions, and the [illegible] of the core is avoided, and the necessary a-c magnetizing component is smaller than that required in the half-wave circuit. The necessary number of secondary turns may be expressed as $N_2(i_1 - i_2)$, where N_2 is the total secondary turns. The primary ampere turns for a primary load current of i_o would be $i_o N_1$. Then, $i_o N_1 = N_2(I_m \cos\omega t)$, obtained by subtracting i_2 [illegible] in circuit [illegible] difference with reactance and [illegible] is much less than in the case of the half-wave circuit. In Figure 6-3 with the origin as in (c) the load current and voltage, a-c and d-c components may be expressed as follows:

$$-\pi/2 < \omega t < \pi/2, \quad i = \frac{E_T}{R_b + R}\cos\omega t, \quad e_L = E_m\cos\omega t,$$

$$\pi/2 < \omega t < 3\pi/2, \quad i = \frac{[illegible]}{R_b + R}$$

The peak value of the rectified current wave is

$$I_m = \frac{E_m}{R_b + R}. \quad (6-21)$$

and the effective (r-m-s) value of the rectified current is

$$I = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} i^2 \, d\omega t} = \frac{I_m}{\sqrt{2}} \qquad (6\text{-}22)$$

The effective (r-m-s) value of the rectified voltage wave is

$$E = IR = \frac{I_m R}{\sqrt{2}} \qquad (6\text{-}23)$$

and the average or direct-load current is

$$I_{dc} = \frac{2}{\pi} I_m \ . \qquad (6\text{-}24)$$

The direct load current is

$$I_{dc} = \frac{2}{\pi} I_m = \frac{2}{\pi} \frac{E_m}{R + R_b} = \frac{2}{\pi} \frac{E_m}{R\left(1 + \frac{R_b}{R}\right)} \qquad (6\text{-}25)$$

The ripple factor is

$$r = \frac{I_{ac}}{I_{dc}} = \frac{\sqrt{I^2 - I_{dc}^2}}{I_{dc}} \ ; \quad \text{since } \frac{E}{I_{dc}} = \frac{\frac{R I_m}{\sqrt{2}}}{\frac{2}{\pi} I_m} = \frac{\pi R}{2\sqrt{2}}$$

$$r = \sqrt{\frac{\pi^2}{4} - 1} = 0.48 \ . \qquad (6\text{-}26)$$

The power output, power input to the rectifier and the efficiency are obtained as for the half-wave case:

$$P_o = E_{dc} I_{dc} = \frac{4}{\pi^2} I_m^2 R \ . \qquad (6\text{-}27)$$

$$P_{in} = I^2 (R + R_b) = \frac{I_m^2}{2} (R + R_b) \ ; \qquad (6\text{-}28)$$

$$\eta = \frac{P_o}{P_{in}} = \frac{8}{\pi^2} \left(\frac{1}{1 + \frac{R_b}{R}} \right)$$

$$= \frac{0.81}{1 + \frac{R_b}{R}} = 0.812 \left(1 - \frac{\pi}{2} \frac{R_b I_{dc}}{E_m}\right) \qquad (6\text{-}29)$$

The theoretical maximum efficiency of the full-wave rectifier is then just twice that of the half-wave rectifier, and the ripple factor has been reduced from 1.21 to 48%

The Full-wave Rectifier Circuit with Gas-filled Tubes

The conduction period during the positive half cycle of a gas-filled diode rectifier is somewhat less than 180 degrees because the tube fails below this value. The ignition voltage, E_i, is equal, practically, to the operating tube drop, V_o. The wave forms of transformer secondary voltage e_1 and e_2, and of output voltage and current are shown in Figure 6-4 for a gas-filled tube in a circuit identical with that of Figure 6-3a. The conduction period extends over an interval of 2θ radians in each anode's positive half cycle. For anode 1, i_1 becomes zero when $e_1 = V_o$, or

$$\cos \theta = \frac{V_o}{E_m} . \qquad (6\text{-}30)$$

For mercury vapor tubes, V_o is usually between 10 and 20 volts. If $E_m = 500$ volts, $V_o = 15$ volts, $\theta \cong 88.3^\circ$. In this case, little error would result if tube drop were neglected.

During the conduction period for anode 1, the expression for current i may be obtained by applying Kirchhoff's voltage law.

$$i = \frac{E_m \cos \omega t - V_o}{R} , \quad -\theta < \omega t < \theta \qquad (6\text{-}31)$$

The load voltage e_k is

$$e_k = i R = E_m \cos \omega t - V_o . \qquad (6\text{-}32)$$

The curves for [illegible] this ratio V_o/E_m has been made large enough that curves for e_k and e_1 are easily distinguishable. The average output voltage, E_{dc}, as obtained from (6-32) is as follows:

$$E_{dc} = \frac{1}{\pi} \int_{-\theta}^{\theta} (E_m \cos \omega t - V_o) \, d(\omega t)$$

$$= \frac{1}{\pi} \left[\qquad \right] = \frac{1}{\pi} \left[\qquad \right] \qquad (6\text{-}33)$$

$$\frac{E_{dc}}{E_m} = \frac{1}{\pi} \left[\qquad \right] \qquad (6\text{-}34)$$

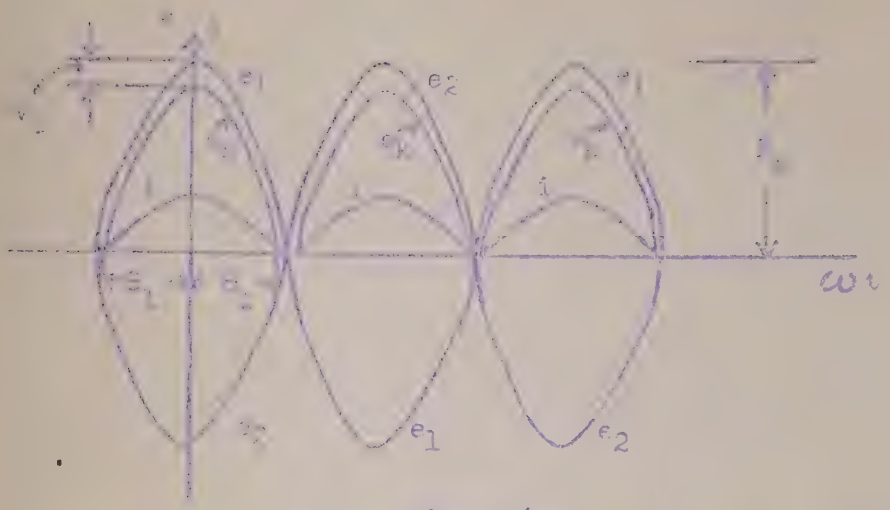


Figure 6-4.
Wave forms of input and output voltage and output current of a full-wave rectifier using gas-filled tubes.

The d-c power output is

$$P_{dc} = E_{dc}\, I_{dc} = \frac{E_m^2}{\pi^2 R}\left[\sin\theta - \frac{V_o}{E_m}\,\theta\right]^2 \qquad (6\text{-}35)$$

The average power input to the rectifier is given by

$$P_{in} = \frac{1}{\pi}\int_{-\theta}^{\pi/2} e_1\, i\, d(\omega t) = \frac{2}{\pi}\int_{-\theta}^{\theta} E_m \cos\omega t \left(\frac{E_m \cos\omega t - V_o}{R}\right) d(\omega t)$$

$$= \frac{E_m^2}{\pi R}\left[\theta - \frac{V_o}{E_m}\sin\theta\right]. \qquad (6\text{-}36)$$

Finally, the rectifier efficiency is expressed as

$$\eta = \frac{P_{dc}}{P_{in}} = \frac{4}{\pi}\left[\frac{\left(\sin\theta - \frac{V_o}{E_m}\,\theta\right)^2}{\left(\theta - V_o/E_m \sin\theta\right)}\right] \qquad (6\text{-}37)$$

If V_o is small, the expression for efficiency becomes

e maximum theoretical efficiency would be obtained if $V_g = 0$. From ation (6-38) for $V_g = 0$, $\eta = 0.812$, which is the same as the maximum oretical efficiency of the high vacuum tube full wave circuit. How- ver, the efficiency is independent of the load, which is a rather im- portant gas-tube rectifier property not possessed by the high vacuum tube circuit.

If tube drop is neglected, the ripple factor for the full-wave rectifier using vapor tubes is the same as for the high vacuum tube full wave circuit, but is greater with decreasing values of conduction angle 2θ.

6-5. Filters

Circuit elements which store energy, e.g. capacitors and in- ductors, may be used to reduce the ripple in the rectifier output. Com- binations of capacitances and inductances used to reduce ripple in a rectifier output are known as filters. The action of the filter is to store energy when the rectifier voltage is high and to discharge energy to the load when the rectifier voltage drops. The behavior of single filter elements will be considered first, and combinations of capacitive and inductive elements will then be discussed.

The Capacitance Filter.

A full-wave circuit with capacitance filter and gas-filled tubes is shown in Fig. 6-5(a). It will be shown in the discussion to follow that such a filter should not be used with vapor type tubes, but may be used with high vacuum tubes. In Figure 6-5, the positive half cycles of voltage are shown for each tube. The current, i_c, through the con- denser is determined by the time rate of change of the voltage across the condenser. If e_c is this voltage,

$$i_c = C \frac{de_c}{dt} \qquad (6-39)$$

The voltage across the resistor is always the same as that across the con- denser, since they are in parallel. The current i_R in the resistor is

$$i_R = \frac{e_c}{R} \qquad (6-40)$$

Figure 6-5.

Tube current, load current, condenser current and load voltage for a full-wave rectifier with condenser filter.

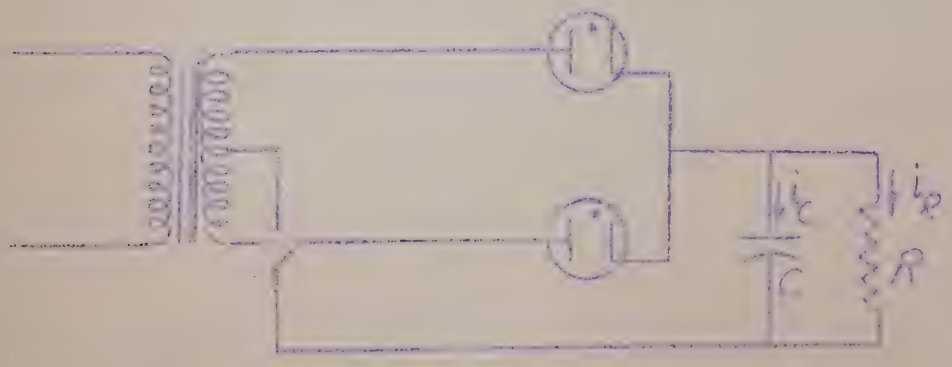


Fig. 6-5(a)

and the tube current, i, is the sum

$$i = i_c + i_R = C \frac{de_c}{dt} + \frac{e_c}{R} \quad (6\text{-}41)$$

The current is limited by the restriction that it can never become negative since the tubes conduct only in the positive direction. If at any instant i becomes zero, then the load current i_R will be determined entirely by the discharge of the condenser, for the tubes will deionize and the circuit from transformer to load will be open.

In the analysis of the circuit of Figure 6-5, t = 0 will be chosen at the instant when the applied voltage e_1 has reached its positive maximum, or

$$e_1 = E_m \cos \omega t \quad (6\text{-}42)$$

If the tube drop is V_o volts when conducting,

$$e_c = e_1 - V_o$$

$$= E_m \cos \omega t - V_o, \quad (6\text{-}43)$$

and at $t = 0$, $\frac{de_c}{dt} = 0$, and the capacitor current is zero. Tube 1 will be conducting, and the tube current will be

$$i = i_c + i_R = C \frac{d}{dt}(E_m \cos - V_o) + \frac{E_m \cos \omega t - V_o}{R}$$

$$= \omega C E_m \sin \omega t + \frac{E_m \cos \omega t - V_o}{R} \quad (6\text{-}44)$$

Equation (6-44) will hold as long as the current is positive, that is, until $i = 0$. If $\omega t_1 = \theta_1$ when $i = 0$, then, neglecting tube drop,

$$\frac{E_m}{R} \cos \theta_1 = \omega C E_m \sin \theta_1$$

$$\text{or} \qquad \tan \theta_1 = \frac{1}{\omega CR} \quad (6\text{-}45)$$

The angle θ_1 may be referred to as the "cut-out" angle. After i becomes zero, equation (6-41) may be solved for e_c. Thus,

$$= \left(\frac{de_c}{dt} + \frac{e_c}{R} \right) \quad (6\text{-}46)$$

and

$$e_c = A \epsilon^{-t'/RC} \qquad (6\text{-}46b)$$

where t' is measured from that time at which $t = 0$. Then, referred to the original time axis,

$$t' = t - t_1$$

or

$$\omega t' = \omega t - \theta_1 \, . \qquad (6\text{-}47)$$

Equation (6-46) now becomes

$$= \epsilon^{-\frac{(\omega t - \theta_1)}{\omega RC}} \qquad (6\text{-}48)$$

and since $e_c = E_m \cos \theta_1$ (neglecting tube drop) when $\omega t = \theta_1$, then

$$= E_m \cos \theta_1$$

and

$$= \epsilon^{-\frac{(\omega t - \theta_1)}{\omega RC}} \qquad (6\text{-}48b)$$

Condition (6-48b) will hold for values of t between t_1 and the value $t_2 = t_1$ at which time tube 2 is fired by the rising voltage, e_2, equals the exponentially decreasing voltage of equation (6-48b). Then, when tube 2 fires,

$$E_m \cos \theta_1 \epsilon^{-\frac{(\omega t - \theta_1)}{\omega RC}} = E_m \cos \theta_2$$

or

$$\epsilon^{-t_1/\omega CR} \cos \theta_1 \epsilon^{-\theta_2/\omega CR} = -\cos \theta_2 \, . \qquad (6\text{-}49)$$

Equation (6-49) is most conveniently solved for θ_2 by plotting the curves

$$y_1 = k_1 \epsilon^{-(\theta_2/\omega CR)} \quad \text{where } k_1 = \epsilon^{\theta_1/\omega CR} \cos \theta_1$$

and

$$y_2 = -\cos \theta_2$$

with t_2 as abscissa. The angle θ_2 may be called the "out-in" angle

The curves of Figure 6-5 show the effect of the condenser upon the output voltage. During the interval $\theta_1 < \omega t < \theta_2$, the load voltage is prevented from dropping to zero by the condenser. Filtering would be perfect if the load current were zero, since then the condenser could not discharge and e_c would remain at the peak value E_m. Effective filtering also depends upon the magnitude of the capacitance of the condenser. As [illegible] approaches zero, -- as C increases. Also, the larger the value of C, the [illegible] Equation (6-46a). However, it must be remembered that a condenser does not pass direct current. The average value of the current i_c of Figure 6-5 must therefore be zero, requiring that the cross-hatched areas above and below the ωt axis be equal. Then as C increases, it is evident from Figure 6-5 that the interval $(\theta_2 - \theta_1)$ increases since θ_1 approaches zero and θ_2 increases toward π; at the same time, interval ϕ decreases. Since [illegible] more and more peaked. The surge of current i at cut-in may greatly exceed the rating of a gas-filled rectifier tube even though the average current is [illegible] For this reason, condenser-input filters are not used with vapor tubes, but may be used with high vacuum [illegible]

The output voltage wave form of the rectifier with condenser filter may be approximated by drawing straight line segments as shown in Figure 6-6. The average of such a triangular-topped wave is

$$E_{dc} = E_m - \frac{E_r}{2} \qquad (6\text{-}50)$$

where E_r is the maximum change in voltage during discharge of the condenser.

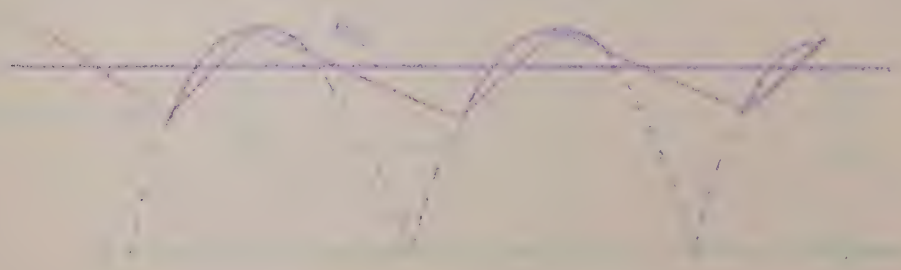


Figure 6-6.
Use of straight line segments to approximate output voltage of Figure 6-5

[illegible] condenser has been assumed to be discharging at a constant rate, [illegible] current delivered by the condenser to resistance R during this

segment over the interval θ_2 may be obtained as follows:

Since $e_c = \frac{q}{C}$

$$\frac{de_c}{} = \frac{1}{} \frac{dq}{} = \frac{I_{dc}}{}$$

$$\frac{de_c}{} = \frac{I_{dc}}{}$$

during the interval θ_2. From Figure 6-6, over the interval θ_2, $e_c = e_R$ and

$$\frac{}{d(\omega t)} \quad \frac{}{d(\omega t)} \quad \frac{}{\theta_2}$$

so that

$$E_r = \frac{I_{dc}\,\theta_2}{\omega C} \qquad (6\text{-}52)$$

It is easily shown that the effective (rms) voltage of the ripple portion of the wave (Figure 6-6) is the same for all three segments and depends only upon the peak value of the voltage. If the time axis were made to coincide with E_{dc} in Figure 6-6, the root-mean-square value of the ripple voltage obtained in the usual way is

$$\frac{}{} \qquad (6\text{-}53)$$

The ripple fraction for the rectifier is then, by definition,

$$r = \frac{E_{ac}}{E_{dc}} = \frac{E_r}{2\sqrt{3}\ E_{dc}}$$

$$\frac{I_{dc}\,\theta_2}{(2\sqrt{3})\,\omega C\ R\ I_{dc}}$$

$$= \frac{\theta_2}{2\sqrt{3}\ \pi f\ C\ R} \qquad (6\text{-}54)$$

The value of θ_2, in radians, corresponding to the ripple fraction may be found from Figure 6-5(b). The ripple fraction decreases with increasing θ_2. Inspection of the curve in Figure 6-5(b) shows, however, that an increasing θ_2 is accompanied by a decrease

value of E_r since the point of intersection of the line segment with the rising cosine curve will move upward along the curve. A commonly used method of approximation of the ripple fraction in this case is to assume that the condenser discharges for one-half cycle. This corresponds to $\theta_2 = \pi$, and results in a less exact but more convenient expression for r.

From the foregoing analysis, it should be evident that the condenser is to effect [illegible], that is, for R large and ω small. Also, filter action improves with increasing C, and the output voltage is high; but the [illegible] are peaked and [illegible] should not be used. The voltage regulation is also very poor.

6-7. The Voltage Doubler

A practical application of the principles discussed in the preceding section is found in the circuit of the voltage doubler, Figure 6-7. The circuit is that of a full-wave rectifier, with two filter condensers in series shunting the load resistance. The voltage e_R across the load is at any instant equal to the sum of the two condenser voltages, $e_{c1} + e_{c2}$. Condenser C_1 charges when tube 1 is conducting, a-c supply terminal a positive. Condenser C_2 charges during the next half cycle, when tube 2 conducts and terminal b is positive. When steady state conditions are reached, the condensers charge and discharge as shown by the graphs of Figure 6-7. The anode

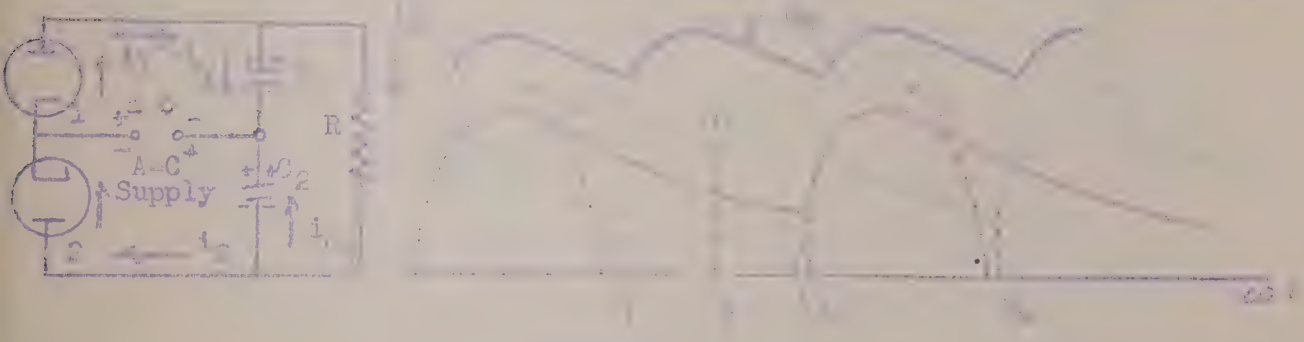


Figure 6-7.
Voltage doubler circuit and condenser voltage wave forms.

of tube 1 becomes positive with respect to its cathode at time t_1; tube 1 conducts until time t_2, delivering current to the load and charging C_1. At t_2, the tube current i_1 becomes zero and would reverse except that the tube will not pass inverse current. The circuit to the a-c supply is then open, and C_1 supplies current to R, aided by C_2 in series, until tube 1 again conducts at time t_4. Meanwhile the a-c supply is again connected to the load through [illegible] C_2 has

adding ordinates of the e_{c1} and e_{c2} curves. For light loads, the voltage e_R is approximately twice the peak voltages of the supply, neglecting tube drop. The circuit has been used as a power supply for loads requiring small current and has the advantage of requiring no transformer. The average load voltage E_d is twice the average voltage of either condenser, and the ripple magnitude is the same as that of e_{c1} or e_{c2}. The ripple frequency of e_R is twice the ripple frequency of e_{c1} or of e_{c2}. The ripple factor may be obtained by the same procedure as that used in section 6-6

6-8. The Inductance Filter

A single inductance used as a filter in a full-wave circuit is connected in series with the load as shown in Figure 6-8. Essentially, the filter action of the inductance is to oppose any change in the current through it, and to add its voltage to that of the rectifier. It is desirable that the iron of the inductance should not saturate with load current. If it may be assumed that there is no cut-out or cut-in of tube current, as was the case for the condenser filter, the open-circuited loop voltage is identical with the series for current, Equation (6-19), secondary voltage between anode and mid-tap. The tube drop or tube resistance will be neglected in the analysis to follow: transformer secondary leakage reactance and resistance, and also the resistance of the choke will be neglected in the preliminary analysis. The voltage applied at terminals a-b is given by

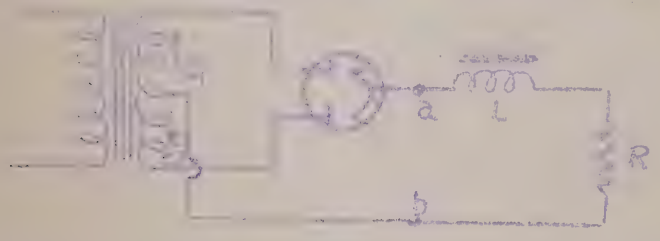


Figure 6-8

Full-wave current with inductance filter

$e_1 = E_m \sin \omega t \qquad e_2 = -E_m \sin \omega t$

$$e_{ab} = \frac{2E_m}{\pi}\left[1 + \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t + \frac{2}{35}\cos 6\omega t - \cdots\right] \tag{6-45}$$

applying the superposition theorem, e_{ab} may be thought of as represent-

frequencies $\frac{2\omega}{2\pi}$, $\frac{4\omega}{2\pi}$, $\frac{6\omega}{2\pi}$ etc. Currents in the load circuit

from the application of the several generators may be obtained one at a time, by superposition, by using one generator at a time. For example, the peak value of the second harmonic component of load current is

$$I_{2m} = \frac{\frac{4}{3\pi} E_m}{\sqrt{R^2 + (2\omega L)^2}} \quad (6\text{-}56)$$

The fourth harmonic amplitude is

$$I_{4m} = \frac{\frac{4}{15\pi} E_m}{\sqrt{R^2 + (4\omega L)^2}} \quad (6\text{-}57)$$

Now for effective filtering, $2\omega L$ is large compared with R. Then, approximately,

$$\frac{I_{4m}}{I_{2m}} = \frac{2}{5}\,\frac{R}{2\omega L}$$

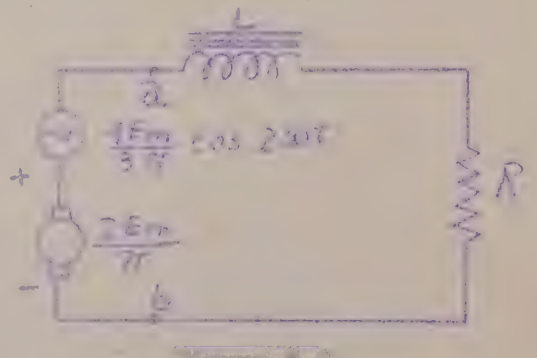
and the higher harmonics are progressively smaller fractions of I_{2m}. It is therefore justifiable, as a useful approximation, to replace the circuit to the left of terminals a-b in Figure 6-8 by an equivalent circuit consisting of a d-c generator and an a-c second harmonic generator in series, as shown in Figure 6-9. The currents in the load are then as follows:

$$I_{dc} = \frac{2E_m}{\pi R} \quad (6\text{-}58)$$

and

$$i_2 = \frac{\frac{4}{3\pi} E_m}{\sqrt{R^2 + (2\omega L)^2}} \cos(2\omega t - \theta), \quad (6\text{-}59)$$

where $\theta = \tan^{-1}\left(\frac{2\omega L}{R}\right)$



Approximate equivalent to the circuit of Figure 6-8.

The ripple is produced by the second harmonic of load current. The ripple fraction is, by definition,

$$\frac{I_{2m}/\sqrt{2}}{I_{dc}} = \frac{\frac{2\sqrt{2}}{3\pi} E_m}{\frac{2}{\pi} E_m \sqrt{1 + \frac{2R^2}{R^2}}} \quad (6\text{-}60)$$

Equation (6-60) shows that, for a given choke, the effectiveness of the filter is [illegible]

therefore increases with increasing load current. This contrasts with the capacitance filter which, as has been noted, was most effective for

If in Equation (6-60) $4\omega^2 L^2/R^2$ is large compared with unity,

that, with a given load resistance, the ripple fraction decreases with

ents and the choke for large currents, it may be surmised that the use of series choke and shunt condenser together will provide effective filter action over most of the load range of a power supply.

choke-condenser or L-section filter is shown in Figure 6-10. The resistance R_b is known as a "bleeder" resistance, and is permanently connected across the load terminals of the rectifier. The required value of R_b will be found in the analysis of the circuit. Experimentally, the load voltage as a function of load current follows a curve similar to that shown in Figure 6-11. A large voltage drop occurs for a small increase of current from the no-load value. After point A is passed, the voltage decrease with load is at a very much smaller rate. It is therefore desirable for improved voltage regulation to operate the rectifier only for values of current to the right of point A on the curve. A bleeder

rectifier will operate at point A. The reason for the large change of voltage to the left of point A is that the tube anodes are not conducting for their full half cycles, but cut-out and cut-in are

Typical filter circuit values require an inductance around 20

a 60-cycle power source, $2\omega L \cong 15{,}000$ ohms; $x_c = \frac{1}{2\omega C} \cong 300$ ohms. For

practical purposes, the reactance of the condenser should be small compared to the effective load and bleeder resistances. In the absence of

out and cut-in, the equivalent applied voltage at terminals a-b of

Figure 6-10 is the same

inductance filter. For R_b = the effective load resistance (actual load in parallel with R_b), and $x_c \ll R$,

$$\frac{R\,(+\,j\,x_c)}{\qquad} \cong \qquad j\,x_c$$

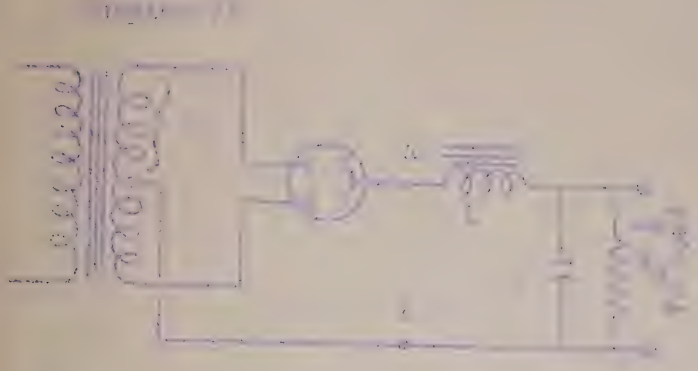


Figure 6-10
Full-wave rectifier with L-section filter and bleeder resistance

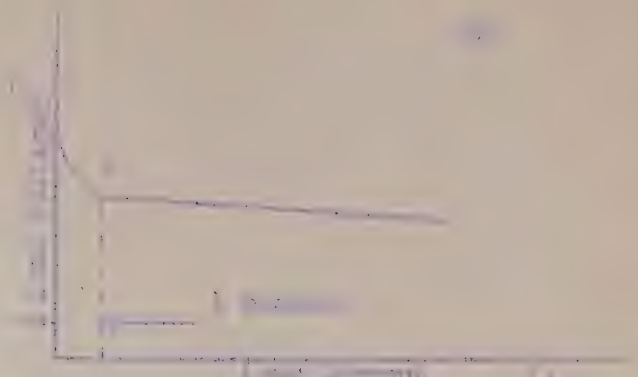


Figure 6-11
Load voltage curve for a full-wave rectifier.

$$I_{2m} = \frac{4 E_m / (3\pi)}{j\left(2\omega L - \frac{1}{2\omega C}\right)} \quad (6\text{-}62)$$

The peak ripple voltage across the condenser, and also across the load, is $I_{2m}(-j x_c) = E_{rm}$ which, in magnitude, is

$$|E_{rm}| = \frac{4 E_m / (3\pi)}{(4\omega^2 L C - 1)} \quad (6\text{-}63)$$

The ripple fraction is

$$r = \frac{\frac{1}{\sqrt{2}} |E_{rm}|}{E_{dc}} = \frac{\frac{2\sqrt{2}\, E_m}{3\pi(4\omega^2 L C - 1)}}{\frac{2 E_m}{\pi}}$$

$$= \frac{\sqrt{2}}{3(4\omega^2 L C - 1)} \quad (6\text{-}64)$$

A sketch of the ripple current is given in Figure 6-12, where the second harmonic ripple current is shown superimposed upon the direct component. If the load resistance increases, the value of I_{dc} decreases, according to Equation (6-64), a critical value of R is reached as

m Figure 6-12(b),

$$I_{2m} = I_{dc}$$

since $2\omega L >> \frac{1}{2\omega_1}$,

$$\frac{4 E_m}{3\pi(2\omega L)} = \frac{2 E_m}{\pi R_B} \qquad (6\text{-}65)$$

$$R_B = 1.31 L \qquad (6\text{-}66)$$

Figure 6-11 corresponds to a value of $R_B = 1000$ L. In view of the approximations used in the derivation of (6-66), the confirmation is [illegible] results if $R_B = 500$ L. The latter value of R_B is recommended, particularly for gas tubes, in order to decrease the ratio of the peak a-- to the direct

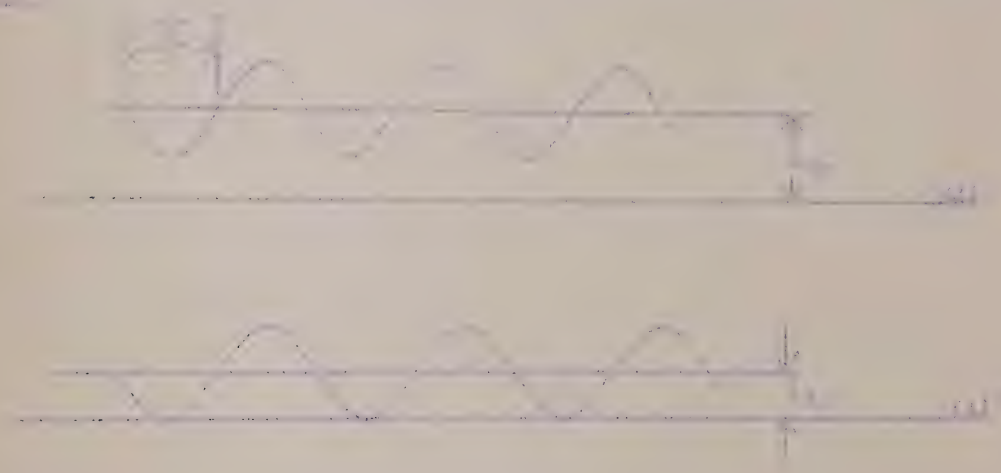


Figure 6-12.
Second harmonic ripple current.

Equation (6-66) has been interpreted as giving the critical value for a given L. It may equally well be interpreted as yielding the critical value of inductance L for a given load resistance, R_B. Using the recommended relation of Dellenbaugh and Quiney

$$L = \frac{R_B}{1.31}$$

full load and no load, even though L decreases, by proper design of an air gap in the iron core of the inductance. Such an inductance is called a swinging inductance. It is designed by proving a value of L which is available with load, always exceeds the critical value specified by equation (6-67).

If the ripple of a power supply, the permissible ripple factor may be specified, the necessary value of LC may be expressed as follows:

$$LC = \frac{1}{4\omega^2}\left[1 + \frac{0.471}{r}\right] \quad (6\text{-}68)$$

Usually, more than one filter section is used. If n identical sections are used in tandem, it may easily be shown that the value of r is given approximately by

$$r = \frac{\sqrt{2}}{3\left[4\omega^2 LC - 1\right]^n} \quad (6\text{-}69)$$

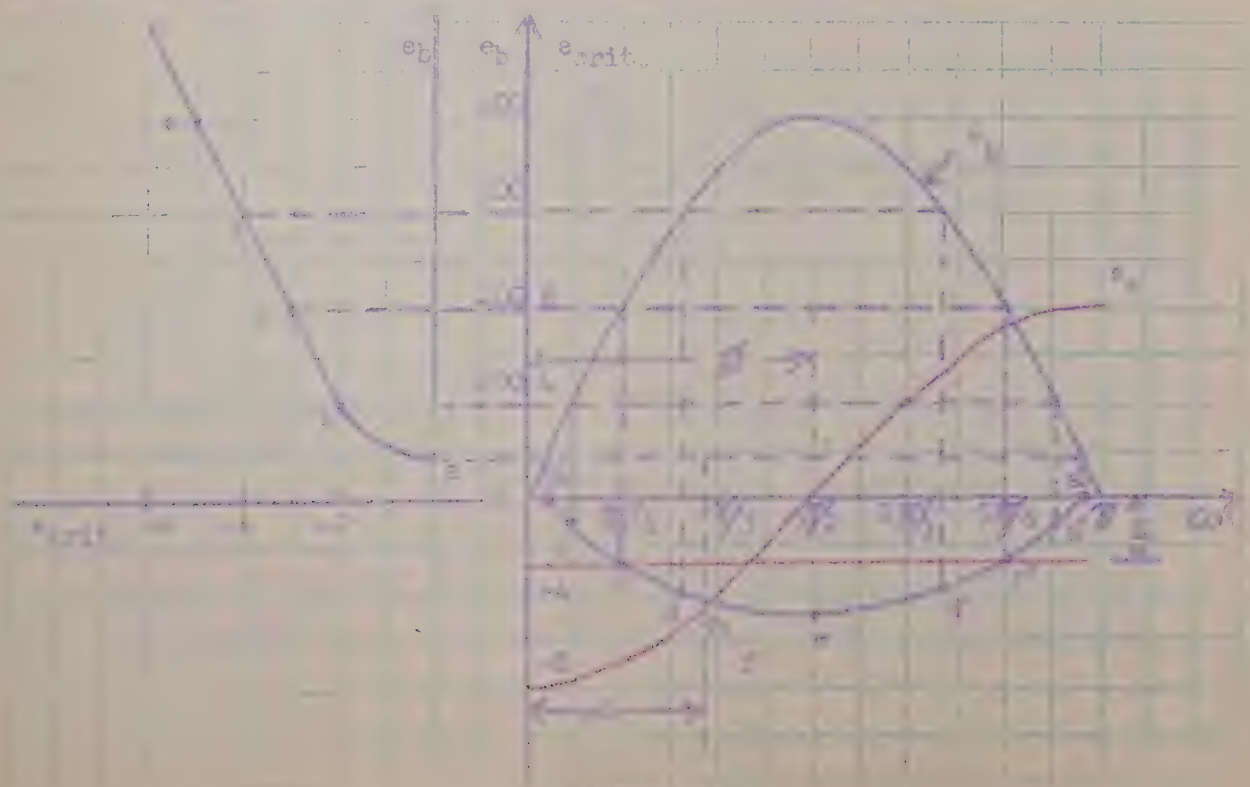
$$LC = \frac{1}{4\omega^2}\left[1 + \sqrt{1 + \left(\frac{0.471}{r}\right)}\right]$$

6-10. Grid-Controlled Rectifiers.

When gaseous rectifiers with grids are used in rectifier circuits, the function of the grid is to delay the initiation of tube conduction until a definite, calculable time in the a-c cycle of positive anode voltage After conduction begins, the field of the grid does not extend beyond its ion sheath and it exerts no further control upon the anode current. With an alternating voltage applied to the anode, however, the anode voltage falls below the value necessary to maintain the arc as the positive half cycle approaches completion, so that conduction ceases, and the grid regains control during the negative half cycle. There are many different circuits used in controlling the time of initiation of arc discharge in thyratrons, but the methods may be calssified as d-c, a-c, or combinations of the two depending upon the nature of the grid voltage. With these various grid-control circuits, it is possible to control the magnitude of the rectified direct load voltage or current continuously or with a fixed magnitude to specify whether the load current flows or does not flow. The latter method is referred to as on-off control. The general principles of such control methods will be analyzed briefly in the following paragraphs.

6-11 Critical Control Curves.

The grid-control characteristics of three typical thyratrons were shown in Figures 5-10, 5-11, and 5-13 of Chapter V. In rectifier applications, the anode voltage is a function of time, and it is desirable to be able to specify the critical firing control-grid voltage also as a function of time. Since the anode and critical firing voltages are related experimentally as shown, for example, by Figure 5-10, it is easy to obtain a critical firing voltage curve as a function of time. The process is illustrated in Figure 6-13, in which points from a control characteristic



have been transferred to a graph of the time varying positive half cycle of the anode voltage. Points a, b, c, d, and e on the critical control characteristic transfer to points a', a'', b', b'', c', c'', d', d'', and e during the positive half cycle. If the grid voltage is also a function of time, the tube begins to conduct at that instant in the positive half cycle at which the actual grid voltage crosses the critical firing curve from below. Two actual grid voltages are shown in red. The horizontal line at −3 volts corresponds to a fixed d-c grid bias. The intersection of this line with the critical firing curve shows that conduction would begin at a time corresponding to point f or at $\omega t = 77/6$ in the cycle. If an alternating voltage of peak value 2 volts and phase lag 90° (referred to the anode voltage) is applied to the grid, the firing point corresponds to the intersection at point f. Evidently, the ignition angle, α, may be varied from approximately zero to approximately π radians by varying the angle of phase lag, ϕ, of the alternating grid voltage behind the anode voltage, but α can be varied only between approximately zero and $77/2$ radians by varying only the amount of d-c bias grid voltage. If both an a-c and a d-c component of voltage are simultaneously applied to the grid, and if the a-c component lags the anode voltage in phase by $77/2$ radians, then considerable variation in α may be achieved by varying the d-c component alone. These three methods of varying α are known as d-c bias, phase shift, and bias-phase control. Circuits for accomplishing the required variations in α are described in later sections.

6-12. Load Current Analysis, Resistance Load.

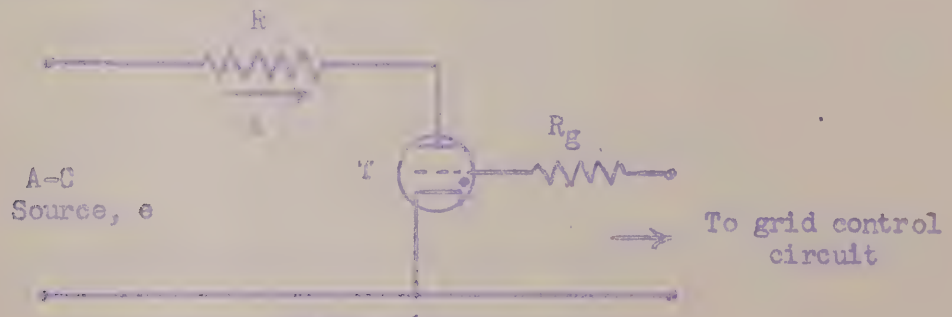


Figure 6-14.

Single-phase, grid-controlled rectifier.

The resistance R in the circuit of Figure 6-14 is the load resistance in which it is desired to control the magnitude of the rectified direct current by adjustments in the grid control circuit of the thyratron, T. If the grid control circuit is adjusted so that the ignition angle is α, Figure 6-15, then the tube conducts during the interval $\alpha < \omega t < (\pi - \lambda)$ during the positive half cycle. The angle λ depends upon the tube drop, since conduction ceases when the anode voltage falls below the value necessary to maintain the arc, and cannot begin until the applied anode voltage exceeds this value.

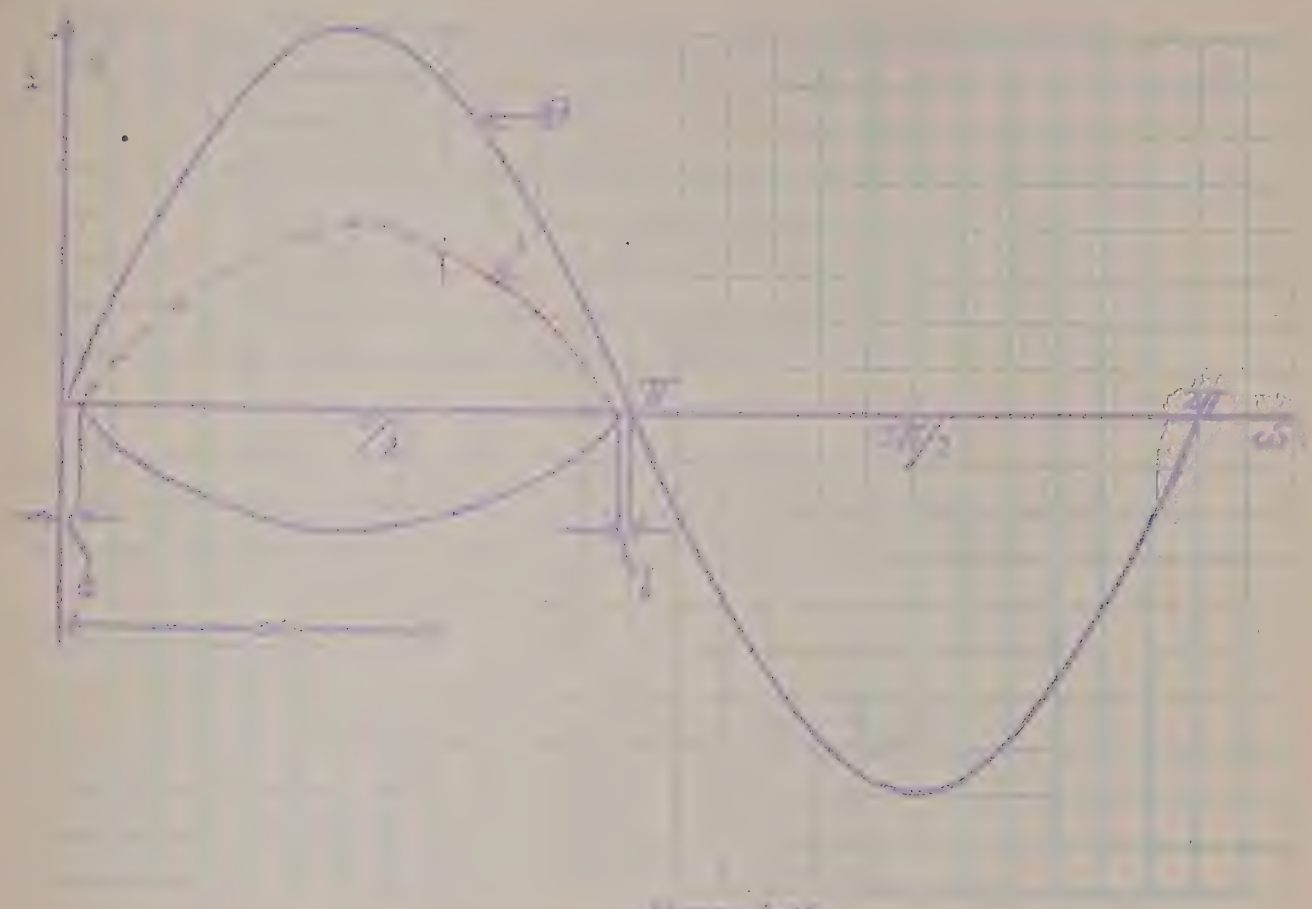


Figure 6-11

Current wave form, half-wave, grid-controlled rectifier.

If the applied voltage e is a sinusoid given by

$$e = E_m \sin \omega t,$$

then the instantaneous anode current during a complete a-c cycle is expressed as follows:

$$\left.\begin{aligned} i &= \frac{E_m \sin \omega t - V_o}{R} \quad \text{for } \alpha < \omega t < \pi - \delta, \\ i &= 0 \quad \text{for} \quad 0 < \omega t < \delta, \\ \text{and} \quad i &= 0 \quad \text{for} \quad \pi - \delta < \omega t < 2\pi \end{aligned}\right\} \quad (6\text{-}71)$$

The direct current in the load resistance R is then given by

$$I_{dc} = \frac{1}{2\pi} \int_{\alpha}^{\pi - \delta} \left(\frac{E_m \sin \omega t - V_o}{R} \right) d(\omega t) \qquad (6\text{-}72)$$

the applied a-c anode voltage, δ is a very small [illegible] and may be neglected. On page [illegible]

$$I_{dc} = \frac{1}{2\pi R}\left[E_m (1+\cos\alpha) - V_o (\pi-\alpha)\right] \quad , \qquad (6\text{-}73)$$

and $$E_{dc} = R\, I_{dc} \qquad (6\text{-}74)$$

Neglecting tube drop completely, the load direct current may be varied from a minimum of zero, for $\alpha = \pi$, to a maximum of

$$I_{dc} = \frac{1}{\pi}\,\frac{E_m}{R} \qquad \text{for } \alpha = 0.$$

[illegible] control of large values of current may be accomplished quite economically by controlling the ignition angle through the use of low voltage, low current control grid circuits.

6-13. Bias Control

A circuit for bias control involves any arrangement by means of which a variable direct voltage may be applied to the grid. One such arrangement is shown in Figure 6-16. Instead of the potentiometer, a voltage divider consisting of a fixed resistor in series with a phototube may be used, thus providing a control voltage dependent upon light intensity.

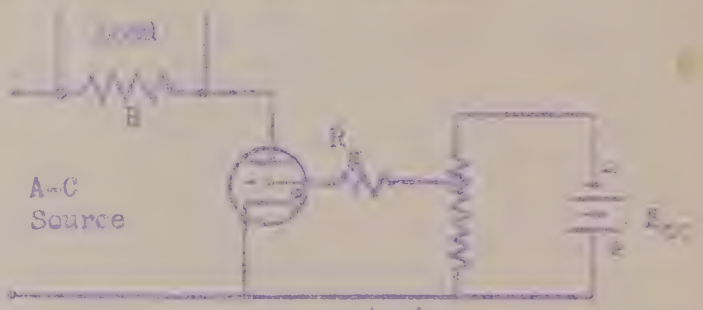


Figure 6-16.
Bias control circuit.

6-14. On-Off Control

An on-off circuit in which the main load current is interrupted or permitted to flow by switching in the grid circuit is shown by Figure 6-17. If the switch S is open, the load current is zero, since the tube is biased below the critical control voltage. If switch S is closed, the grid voltage is made equal to that of the anode and the tube conducts with a conduction period of $\delta < \omega t < (\pi - \delta)$ in the positive half cycle. Other positions of the switch and of R_1 are possible and should suggest themselves to the student.

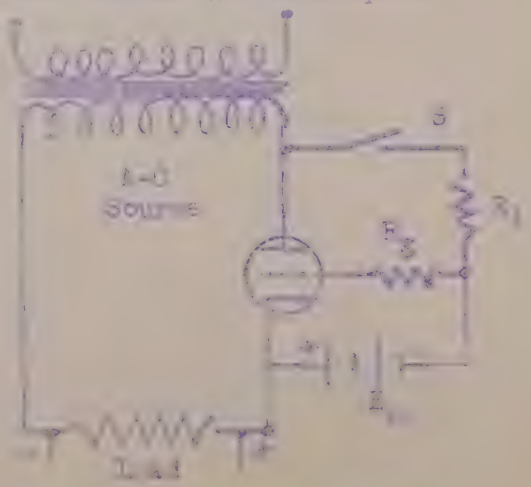


Figure 6-17. On-off control circuit

6-15. Phase-shift Control.

The control of average current in a thyratron circuit by shifting the phase of the alternating grid voltage with respect to the anode voltage is the method best adapted to most thyratron control applications, and is often used where automatic control or servomechanisms are employed. The control of the firing angle, θ, of Figure 6-13, may be accomplished by one of the general circuits of Figure 6-18(a). The vector diagram for the circuit is shown in Figure 6-18(b) where the impedance Z_1 is a pure resistance, and Z_2 is a pure inductive reactance. Thus, the current, I, lags the voltage by angle θ, where

$$\tan \theta = \frac{\omega L}{R}$$

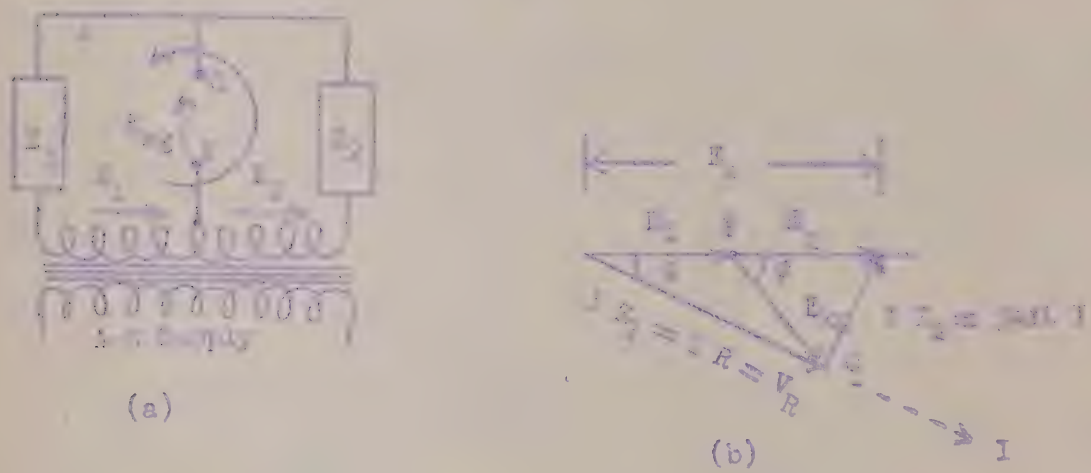


Figure 6-18.
Circuit and vector diagram for thyratron phase shift

From Kirchhoff's voltage law, $E_s = E_1 + E_2 = I\,(Z_1 + Z_2)$. Also,

$$E_1 = V_{kg} + I\,Z_1 \qquad (6\text{-}75)$$

where E_1 is the voltage rise across the indicated half of the supply transformer secondary and V_{kg} is the voltage drop between terminals K and G. If the voltage rise between these two terminals is specified, Equation (6-75) becomes

$$E_1 + E_{kg} = I\,Z_1 = I\,R \qquad (6\text{-}76)$$

and

$$E_2 = I\,Z_2 + E_{kg} = j\omega L\,I + E_{kg} \qquad (6\text{-}77)$$

diagram,

$$V_R = (E_1 + E_2) \cos \theta$$

$$= E_s \cos \theta \quad . \qquad (6\text{-}78)$$

is $E_s/2$ (see footnote) so that

$$\left| E_{xc} \right| = E_{s/2} \quad . \qquad (6\text{-}79)$$

For the physical case in question, the point C moves in a semi-circle, and the circuit will provide continuous control of the magnitude of the average anode current, provided the applied anode voltage is in phase with E_s.
Figure 6-19.

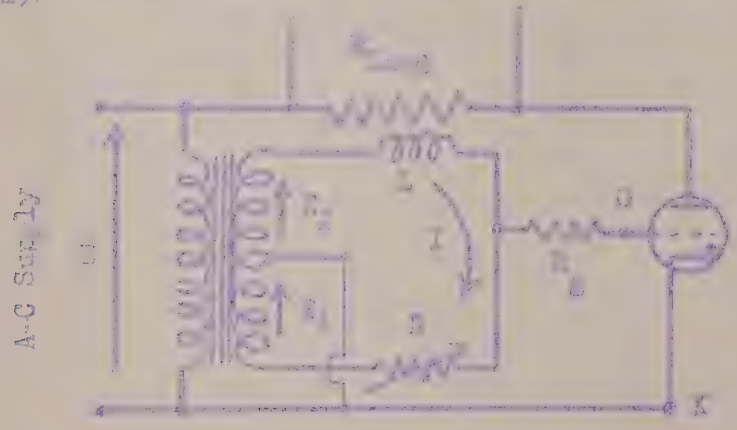


Figure 6-19.
Phase-shifting circuit for thyratron control.

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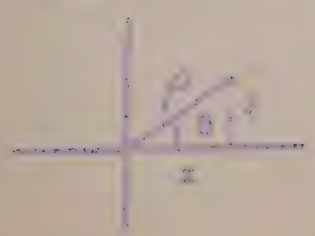
transformation to rectangular coordinates



$$\rho = \sqrt{x^2 + y^2} \quad , \quad \cos\theta = \frac{x}{\sqrt{x^2 + y^2}} \quad , \quad \sqrt{x^2 + y^2} = \frac{ax}{\sqrt{x^2 + y^2}}$$

or $x^2 + y^2 - a\,x + \frac{a^2}{4} = \frac{a^2}{4}$, and finally, $\left(x - \frac{a}{2}\right)^2 + y^2$

$= \left(\frac{a}{2}\right)^2$, the equation of a circle, center at (a/2, 0), radius a/2.

It is assumed in the circuit of Figure 6-19 that the phase shifting transformer secondary voltage $E_g = E_{g_m} \sin \omega t$ is in phase with the anode supply voltage, E. If so, an increase of R will cause a decrease in angle ϕ and an increase in angle ψ. Before the tube fires, the circuit K to G is assumed to be open; also grid limiting resistor R_g is large compared with the R-C impedance in the phase shifting circuit, and so may be neglected. The case is thus similar to that in Figure 6-13(a) where the terminals K to G are open. Since, then, an increasing R is accompanied by a decreasing phase lag ϕ, the average current will increase with increase of R.

The wave forms of the tube current and of the voltage across the tube are shown in Figure 6-20 for the circuit of Figure 6-19 with a pure

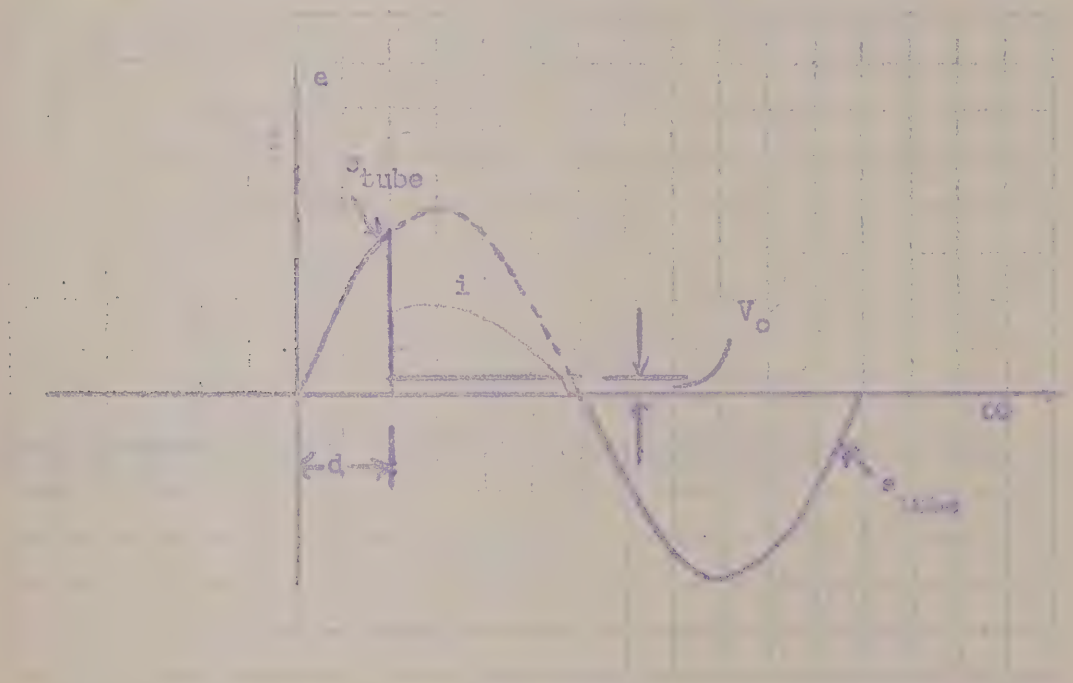


Figure 6-20.
Tube voltage and current wave forms, thyratron with resistance load.

until the tube fires. At $\omega t = \alpha$, the ignition angle, the tube voltage falls to a value equal to the conduction tube drop, V_o, and remains at this voltage until the current decreases to zero. This occurs approximately at the instant in the applied a-c cycle when the applied voltage [illegible] (the end of the positive half cycle). At this instant, the applied voltage is just sufficient to maintain the arc so that the voltage across the resistance is zero. The conduction current becomes zero and the tube voltage for the remainder of the cycle is equal to the applied voltage.

If R and L, Figure 6-15, are interchanged in position in the circuit, the result is a shift of 180° in the phase of voltage E_{kg}. The new position of E_{kg} is shown by the vector diagram of Figure 6-21. From the circuit and vector diagrams,

$$E_1 = V_{kg} + I\,(j\omega L) = -E_{kg} + I\,(j\omega L)$$

or

$$E_1 + E_{kg} = I\,j\omega L$$

and

$$E_2 = I\,R + V_{GK} = I\,R + E_{kg}.$$

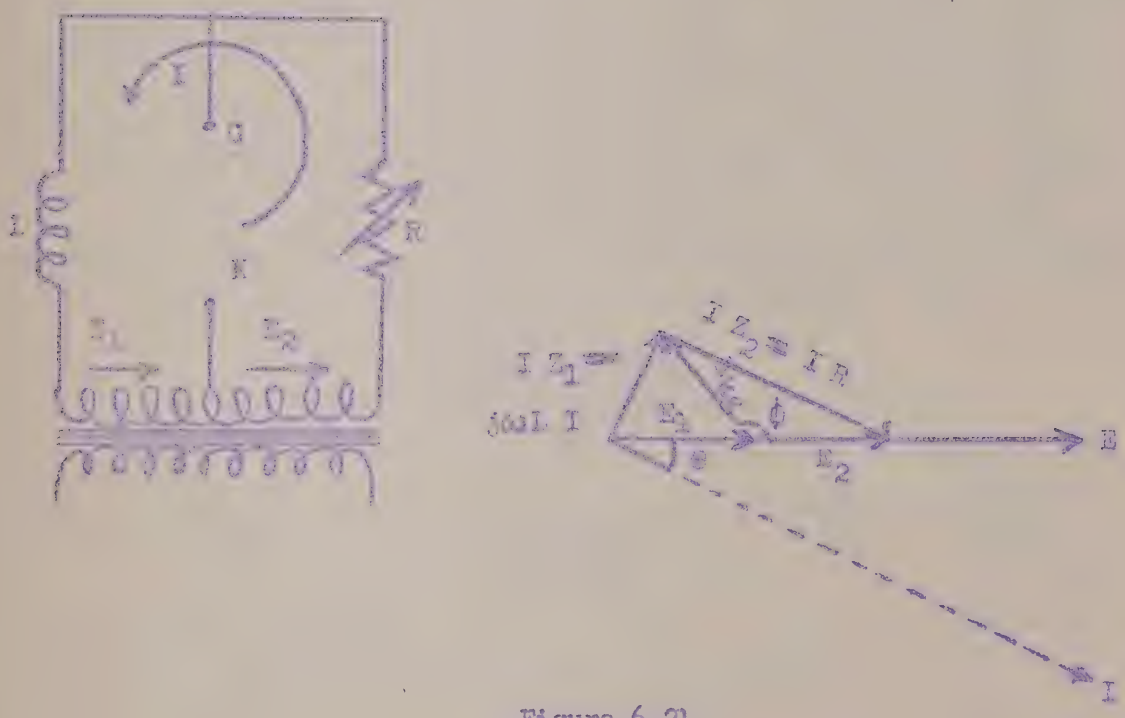


Figure 6-21.

Phase shift circuit with no control.

The grid voltage E_{kg} now leads the applied anode voltage, E, which is assumed to be in phase with $E_s = (E_1 + E_2)$. The tube will fire as early as possible in the anode positive half cycle, and a change in phase angle ϕ will have no effect upon the average current. In other words, there is no control. In case a phase shift control circuit is incorrectly connected and exerts no control over the average current, control may be obtained either (a) by interchanging the positions of R and L in the circuit, or (b) by reversing the phase of the phase shifting transformer secondary voltage.

Phase shift control may be provided by using an R-C instead of an R-L circuit. In this case the current will lead the voltage E_g, and control will be obtained if R and C are connected in the proper positions with respect to the terminals of the phase shifting transformer in exactly the same way as explained for the R-L circuit. It should be noted that ϕ must always be an angle of lag of the grid voltage E_{kg} behind the applied anode voltage E in order to obtain control.

6-16. The Use of Saturable Reactors in Phase Shift Control.

Saturable iron-core reactors have been used in phase-shift control circuits because of the wide range of reactance variation controllable by means of a few milliamperes of direct current. The direct current may be supplied from a small, receiving type, high vacuum triode. The grid voltage of the triode may then be used to control the direct voltage of the output circuit. It is possible by this means to control the armature and field currents of a d-c motor supplied from an a-c source, and thus to control the speed of the motor through the variations of the grid voltage of a small triode. A typical phase shift control circuit using a saturable reactor is shown in Figure 6-22. Two thyratrons are used in a full-wave rectifier circuit. The a-c windings of the saturable reactor are wound in such a way that their fluxes cancel in the d-c winding. The d-c winding

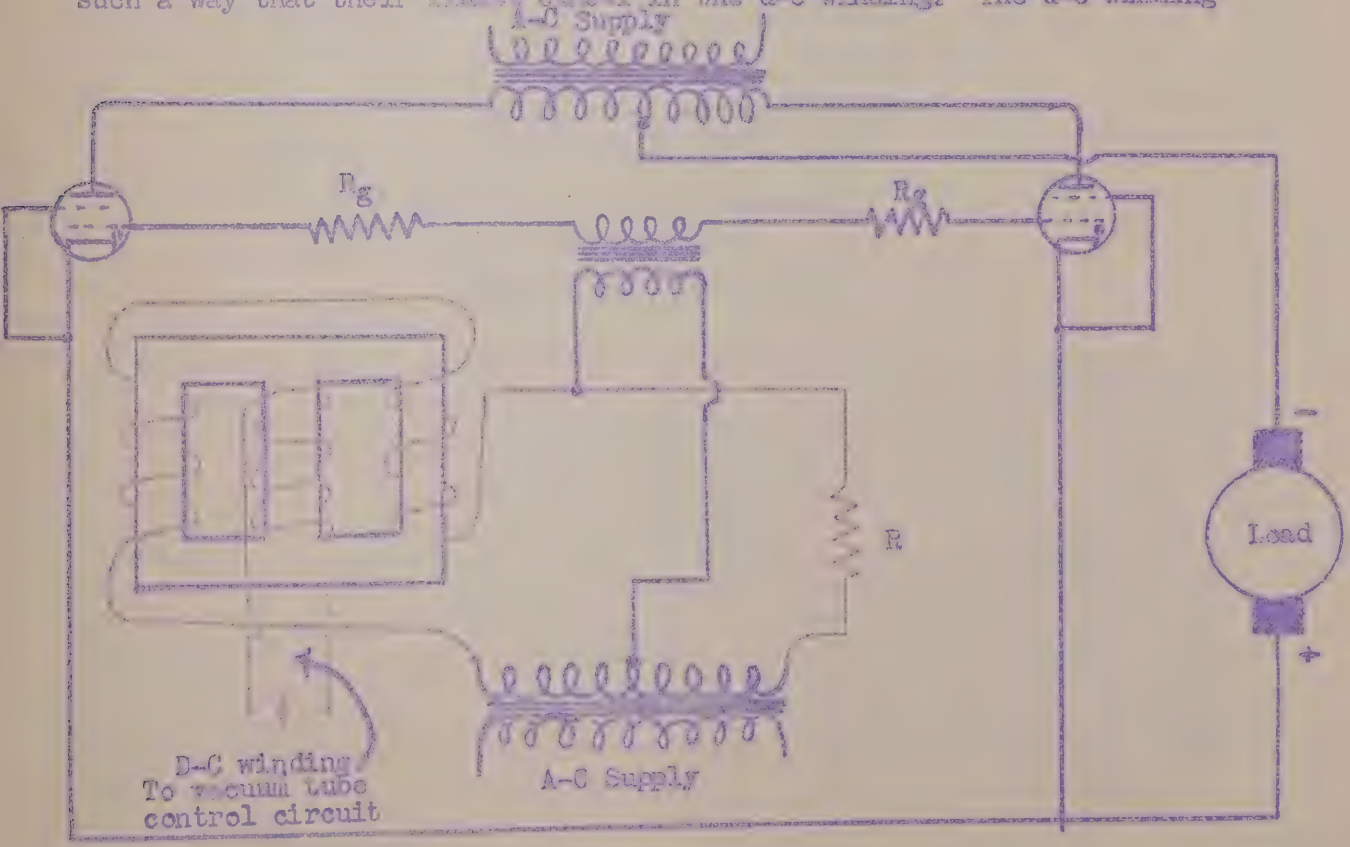


Figure 6-22.

Phase shift control circuit using saturable reactor.

has many more turns than the a-c winding. High permeability iron is used. In a typical [illegible] with a [illegible] winding, 2.5 milliamperes of current will saturate the reactor. A typical saturation curve is shown in [illegible] 6-23, in which the a-c flux is plotted against the control direct current. As the current i increases beyond the knee of the saturation curve, the inductance L decreases rapidly, which means that the angle ϕ of phase lag decreases, since

$$\phi = 2 \arctan \frac{\omega L}{R} .$$

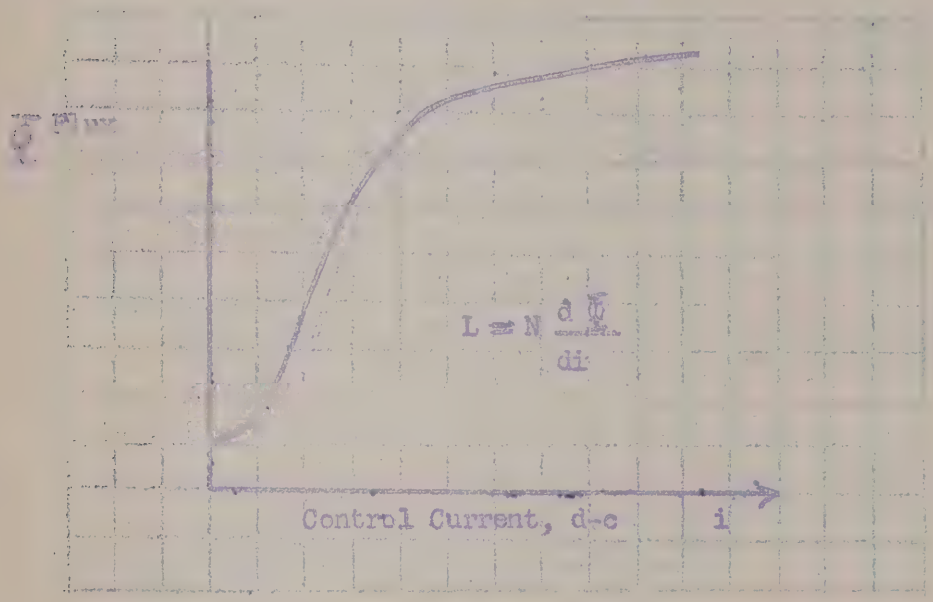


Figure 6-23.
Typical saturation curve

The tubes therefore conduct longer, and the load voltage is increased. This method of control has been applied to d-c motor speed control in applications where extremely accurate and extensive speed control is necessary. A circuit similar to Figure 6-22 is used for the field excitation, and another separate circuit for the armature supply. A standard reference voltage is provided from a small full wave rectifier circuit using cold cathode voltage regulator tubes. A portion of this fixed voltage is compared with a voltage proportional to armature speed. If the speed is not as specified, the difference or error voltage actuates the grid of a vacuum triode in which the plate current in turn controls the grid voltage of a second triode. The plate current of the second triode is the d-c control current of the saturable reactor. Thus control can be made auto[illegible], and the motor may be driven at any predetermined, fixed speed, even with variable load, where the necessary vacuum tube control circuits are utilized.*

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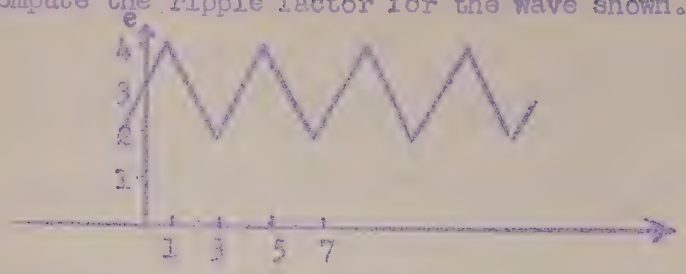
* Moyer, E. E., "Electronic Control of D-C motors", ELECTRONICS, Vol. 16, May, June, July, Sept., & Oct. 1943.

6-1. Compute the ripple factor for the wave shown.



6-2. A type 5Z4 diode full-wave rectifier is connected as shown in Figure 6-3(a). The plate characteristic (for one plate only) may be plotted from the following (e_b, i_b) values: (10 volts, 45 ma.); (17.5, 100); (22.5, 150); (27.5, 200); (40, 355).

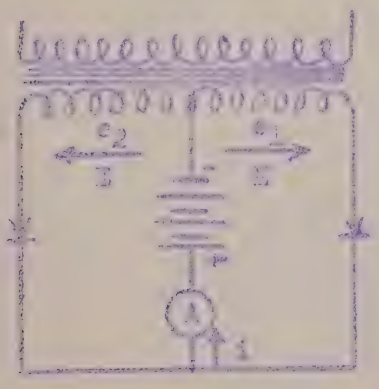
(a) Plot the diode plate characteristic.

(b) For R (Figure 6-3(a)) = 5000 ohms, obtain the composite characteristic.

(c) If $e_1 = 200 \cos \omega t$, where $\omega = 377$ rad/sec, obtain data for and sketch a complete cycle of the voltage between anode No. 1 and the cathode. What is the maximum inverse peak voltage?

6-3. Apply circuit analysis to the circuit of Figure 6-3(a) and compute I_m, I_{dc}, E_{dc}, P_o, P_{in}, and the efficiency for a load resistance of $R = 5000$ ohms. Use the tube of Problem 2.

6-4.



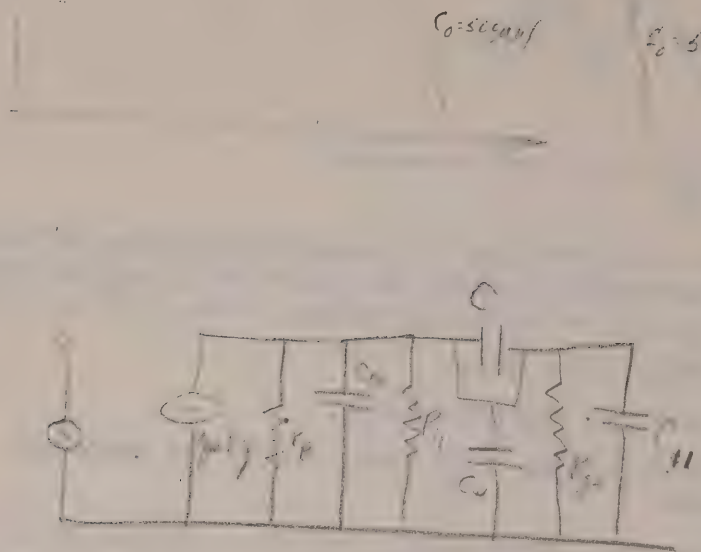
The rectifier circuit shown uses copper oxide rectifiers for battery charging. The voltage E, to center tap, is 8.48 volts, rms. Resistance and reactance of the transformer are negligible. The battery voltage is 6 volts, its internal resistance 0.1 ohm. The rectifier resistance is 0.5 ohm in the conduction direction.

(a) Sketch the voltage and current wave forms, and write the equations for the instantaneous voltages e_1, e_2, and the current i.

(b) If the ammeter is a direct-current meter, compute its reading.

(c) If the ammeter is an a-c meter reading rms current, compute its reading.

Effect of emitter resistance



$$f_{c1} = \frac{1}{2\pi C_0 R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{h_{fe}} + \frac{1}{R_{in}}$$

$$f_{c2} = \frac{1}{2\pi C_0 \left(R_{out} + \frac{R_L R_C}{h_{fe} + R_C}\right)}$$

Decrease R_{eq}, f_{c1} } ... f_H

Increase R_{in}

Increase C_0, R_{eq} } Decrease f_L

6-5. For Problem 4, compute

(a) The power delivered by the transformer.

(b) Power in rectifier and battery resistances.

(c) Total losses and power delivered to the battery.

6-6. A gas diode is used in a half-wave circuit to supply a 500-ohm resistance load from 220 volt a-c mains. The breakdown and maintaining voltages are constant at 10 volts. Calculate the readings of the following instruments:

(a) A d-c ammeter in series with the load.

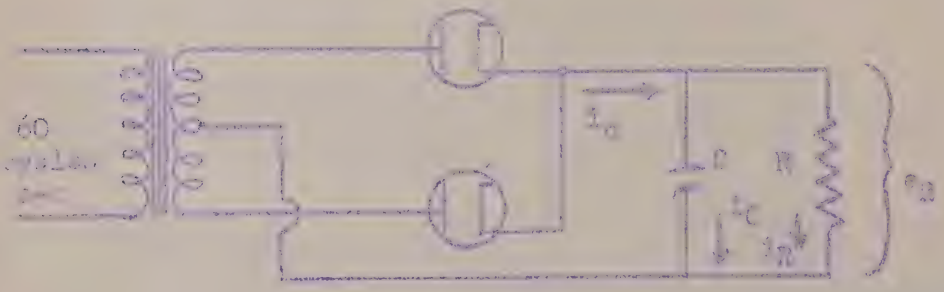
(b) An a-c ammeter in series with the load.

(c) A d-c voltmeter placed across the tube.

(d) An a-c voltmeter placed across the tube.

(e) A wattmeter connected with its current coil in series with the load and its voltage coil across the input.

6-7.

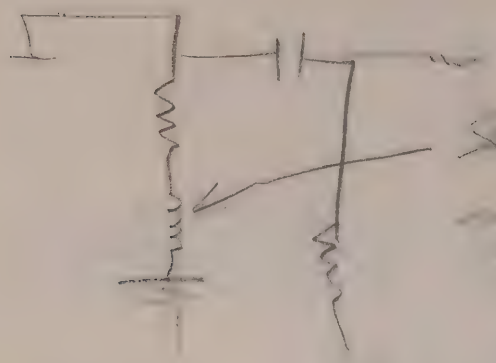


In the circuit shown, the transformer secondary voltage, plate-to-plate, is 500 volts rms (center tapped). For $C = 4$ microfarads, $R = 1000$ ohms, find the angles, as measured on the time axis, at which cut-out and cut-in occur; obtain equations for and plot i_C, i_R, i_o, and e_R for cut-in and cut-out periods. Neglect tube drop.

6-8 Derive equation 6-53 from Figure 6-6.

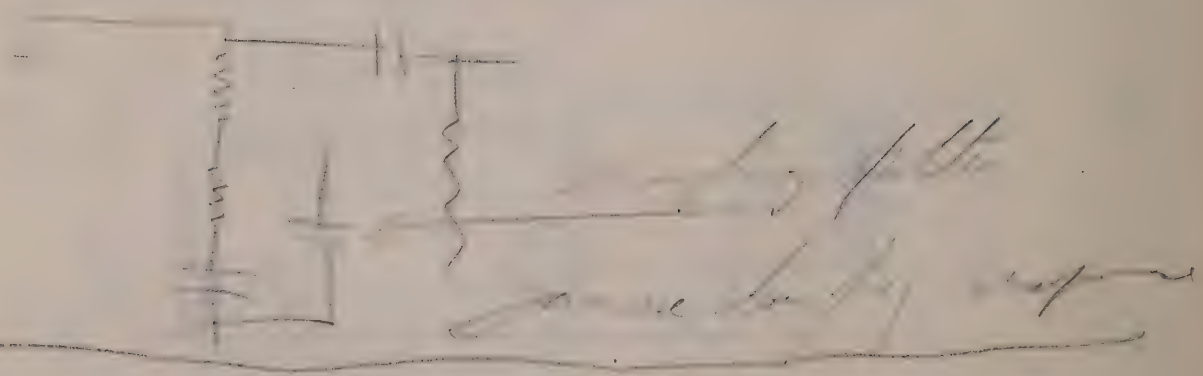
6-9. Compute the ripple fraction for the circuit of Problem 6-7. What is the d-c output voltage?

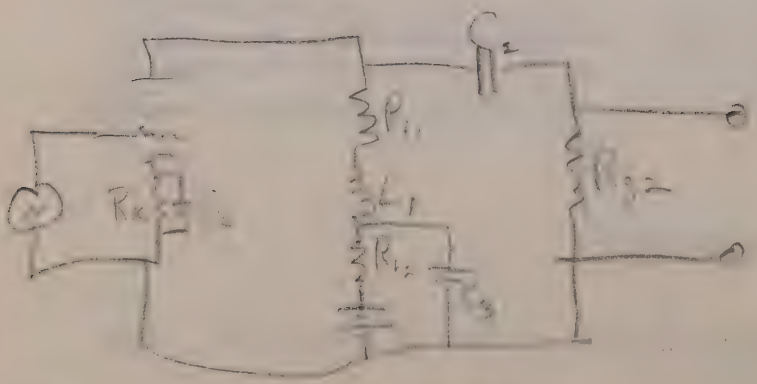
Simplest circuit type shows low freq. response.



Same as high freq response

$$\omega = \frac{1}{R_1 C_1} = \frac{1}{\tau}$$





$$\frac{\omega L_1}{R_1} = .4 \text{ or } \frac{1}{\tau}$$

$$R_1 C_1 = R_2 C_2 = R_3 C_3$$

6-10. A full-wave rectifier circuit uses a Type 83 full-wave mercury-vapor tube and an L-section filter consisting of a 30 henry inductance and a 20-microfarad condenser. The transformer secondary voltage is 500 volts rms, (250 volts to center tap). The load resistance is 500 ohms. Assume that ω = 400 radians per sec., and neglect tube drop.

(a) sketch the rectifier circuit diagram.

(b) sketch the approximate equivalent circuit and write the equations for:
(1) the instantaneous current in the condenser;
(2) the total voltage - d-c and superimposed a-c - across the load.

(c) compute the ripple fraction.

(d) compute the inverse peak voltage across the tube.

6-11. The specifications for a full-wave, single-phase, power supply to operate from 115 volt, 60 cycle mains are as follows:

(a) Output 250 ma. at 1500 volts.

(b) Ripple Fraction not to exceed .001.

The tubes to be used are Type 866-A mercury vapor rectifiers, rated 15 volts arc drop, inverse peak voltage 7500, maximum average current 250 ma.

Specify filter elements and power transformer secondary voltage, assuming choke resistance of 75 ohms and bleeder resistance of 50,000 ohms. Assume that the full load choke inductance may be only one-half the inductance at no-load.

Compute:

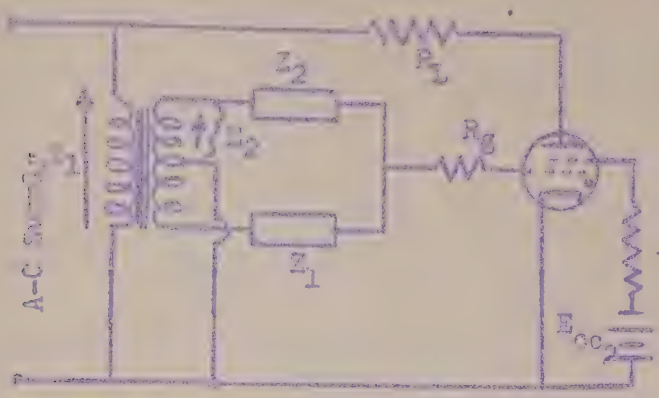
(1) The expected full-load to no-load voltage regulation of the power supply;

(2) the peak inverse voltage on the tube anodes;

(3) the expected percent ripple.

6-12. Derive equations 6-69 and 6-70.

6-13. A thyratron rectifier is connected as shown;



$E_1 = 220$ V., rms

$E_2 = 10$ V., rms

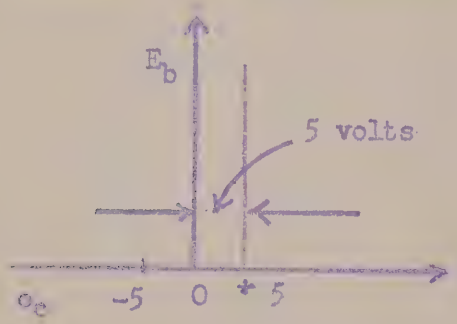
$R_g = 50{,}000$ ohms

$Z_1 = R = 1732$ ohms

$Z_2 = j\omega L = j\,1000$ ohms

$R_L = 50$ ohms

Assume sinusoidal voltages, and that E_2 is in phase with E_1. If the control characteristic of the thyratron is as shown on the accompanying sketch, determine the following:



(a) the ignition angle, α;

(b) the direct load voltage across R_L;

(c) the wave form of the conduction current for one cycle and of the anode-to-grid voltage.

6-14. For the assumed positive sense of voltage rise in Problem 6-13, what should be the values of Z_1 and Z_2 if an R-C phase shift circuit is used and R = 2000 ohms, if it is specified that a 90° phase lag is required for E_2 behind E_1. For this circuit, compute the ignition angle, with the thyratron biased as in 6-13.

E. E. 607 · Assignment Sheet — T. Th. S.

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3	21-31	1-12, 1-13
8	Chapter II, 1-12	2-1, 2-2, 2-5
10	13-24	2-6, 2-7, 2-8
13	Chapter III, 1-14	3-1, 3-3, 3-4
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17	Review	
20	Midterm	
22	31-40	3-7, 3-8
24	40-48	3-9, 3-11
27	48-55	3-12, 3-14
29	55-65	3-15, 3-18
1	Problems	
4	Chapter IV 1-10	4-1, 4-3
6	10-19	4-5, 4-6
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13	Chapter V. 1-11	5-1, 5-3, 5-4
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18	Chapter VI, 1-7	6-1, 6-2
20	7-12	6-3, 6-4
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25	18-24	6-7, 6-8
27	25-30	6-10, 6-12
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1	Review	
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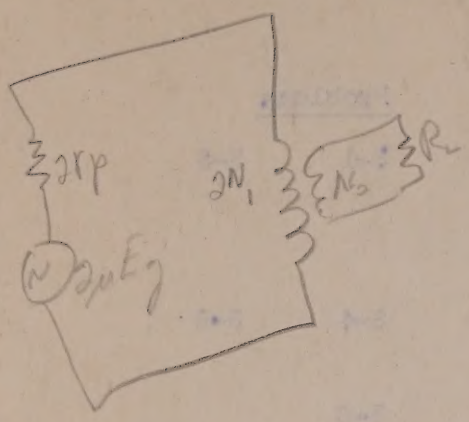
5	Review	

Midterms 300
Final 300
Problems 100
600
Lab 200
800

E. E. 607 Revised Assignment Sheet

(Tue., Th., S.)

Date		Pages		Problems	
April	15	Chapter III -	1-10	3-1	3-3
	17		10-17		
	20	Review			
	22	Midterm			
	24		17-31	3-4	3-5
	27		31-35		
	29		36-40	3-6	
May	1		41-46	3-7	3-8
	4		46-52	3-9	~~3-11~~
	6		52-59	3-12	
	8		59-63	3-14	3-15
	11		63-65	~~3-18~~	
	13	Review			
	15	Midterm			
	18	Chapter IV -	1-6	4-1	4-3
	20		6-12	4-5	
	22		12-18	4-6	
	25	Chapter V -	1-6	5-1	5-3
	27		7-12	5-4	5-5
	29		13-18	5-6	
June	1	Review			
	3	Midterm			
	5	Review			


$2r_p$
$2N_1$
$2N_2$
R_L
$2\mu E_g$